Abstract

Self-modulation (SM) of proton beams in plasma has recently gained interest in context with the PWFA experiment proposed by the AWAKE collaboration at CERN. Instrumental for that experiment is the SM of a proton beam to generate bunchlets for resonant wave excitation and efficient acceleration. A fundamental understanding of the underlying physics is vital, and hence an independent experiment has been set up at the beamline of the Photo Injector Test facility at DESY, Zeuthen site (PITZ), to study the SM of electron beams in a plasma.

In this contribution we present simulation results of SM experiments at PITZ using the particle-in-cell code HiPACE. The simulation study is crucial to optimize the beam and plasma parameters for the experiment. Of particular interest is the energy modulation imprinted onto the beam by means of the generated wakefields in the plasma. With the support of simulations, the observation of this information in the experiment can be used to deduce key properties of the accelerating electric fields, such as their magnitude and their phase velocity, both of significant importance for the design of self-modulated plasma-based acceleration experiments.

INTRODUCTION

Plasma wakefield acceleration was proposed as an alternative of conventional acceleration methods due to the large accelerating fields [1]. Accelerating wakefields in excess of 50 GeV/m have been achieved in 85 cm long plasma using a 42 GeV drive electron beam [2]. However, different stages would be required for further acceleration of electrons owing to limited drive beam energy.

Recently, it has been proposed to use short and high energetic proton beams to drive a plasma wave so as to accelerate electron beams to the TeV-energy scale in a single plasma stage [3], rather than in multiple stages, as required with electron drivers. In order to excite large amplitude plasma waves in such a scheme, the driver bunch length ($L_{beam}$) needs to be shorter than the plasma wavelength ($\lambda_p$). However, available proton bunches from the Super Proton Synchrotron at CERN are very long (~12 cm) in comparison with the foreseen plasma wavelength for this experiment ($\lambda_p$ ~1 mm). The AWAKE experiment thus relies on the fact that a long proton bunch ($L_{beam} > \lambda_p$) is self-modulated during its propagation in the plasma, thereby being split into a train of ultra-short sub-bunches with a length on the order of $\lambda_p$, so that the plasma wave is resonantly excited [4]. The self-modulation is a result of transverse two-stream instability, occurring through the coupling of the transverse wakefield with the beam radius evolution. When a long bunch with the beam length $L_{beam} > \lambda_p$ and transverse size $\sigma_r$ enters the plasma, it is radially modulated by the periodic focusing forces, and the beam density modulation ($n_{b0} \propto \sigma_r^2$) provides feedback for the instability to grow. Consequently, this instability self-modulates the long beam into ultra short bunches at the plasma wavelength scale, which resonantly drive the plasma wake. The instability is convective and grows both along the bunch ($\zeta$) and along the plasma ($\zeta$) as illustrated by the number of e-folding growths for a flat-top bunch, [4]:

$$N_{\text{e-folding}} \approx \frac{3\sqrt{2}}{4} \biggl(\frac{n_{b0}}{n_p M_b} \biggr)^{1/3} (k_p \Delta \zeta)^{1/3} (k_p \Delta \zeta)^{2/3}$$

(1)

Where $\omega_p$ is the plasma frequency, $k_p = c/\omega_p$, $\nu \approx 1 - (k_p \sigma_r)^2$, $\sigma_r$ the initial bunch radius, $n_{b0}$ the initial beam density, $M_b$ the bunch particle mass.

The study of the self-modulation of electron beams from the Photo Injector Test Facility at DESY, Zeuthen site (PITZ) offers a unique possibility to demonstrate and optimize the self-modulation instability experimentally and to gain insight into the underlying physics of the involved processes [5]. Self-modulation in PITZ electron beam has been already shown with OSIRIS simulations [6]. The particular interest of this experiment is to see a significant modulation of the beam energy spectrum, which will be resolvable in the experiment. In the following section, results from numerical studies, aiming at the optimization of the PITZ beam and plasma parameters, are presented. Simulations on the self-modulation instability are performed using the particle-in-cell code HiPACE [7]. This code provides the possibility to import beams from the particle tracking code ASTRA [8].

SIMULATION PARAMETERS

A particle tracking code, ASTRA is used to track the beam from the RF photo-electron gun to the entrance of the plasma, 6.2 m downstream [8]. The average energy of the bunch is $E = 22 MeV$ with energy spread of $\Delta E/E = 0.1\%$. The simulation uses a moving window (8.4 mm x 1.7 mm x 1.7 mm) that propagates at the speed of light (c), with resolution of $k_p \Delta z = 0.025$ and $k_p \Delta x = k_p \Delta c = 0.04$. Table 1 shows the beam and plasma parameters for different conditions that are used to study the self-modulations.
The charge of the bunch was tuned from 50\(pC\) to 175\(pC\) and the simulations are performed for both limits in order to explore the growth of the self-modulation instability (SMI). While the low charge may be more appropriate to observe the energy modulation resulting from the occurrence of SMI, the high charge may be more suitable to directly observe the self-modulation instability. As the SMI grows, \(E_z\) saturates and the saturation distances decreases from 70\(mm\) to 40\(mm\) with increasing charge. The simulation result also indicates that within the propagation distance of \(z = 15mm\), \(E_z\) remains almost constant, and this is confirmed by the estimation of \(N_{e\text{-}\text{folding}}\) from Eqn. (1): \(N_{e\text{-}\text{folding}} < 6\). The plasma channel length of PITZ is limited to 10\(cm\) and therefore a significant detectable SMI growth is expected at the plasma exit with higher bunch charges.

**INITIAL LONGITUDINAL WAKEFIELDS**

The initial longitudinal wakefield \(E_z\) on the beam axis at \(r = \sigma_r\) can be expressed as [9]

\[
E_z = \frac{\varepsilon_0\hbar\omega}{\varepsilon_0 k_p \cos(k_p\zeta)R(0)} \approx \frac{\varepsilon_0\hbar\omega}{\sqrt{n_p}} R(0) \tag{2}
\]

Where \(R(0)\) is a unitless transverse geometrical factor. \(R(0)\) is an increasing function of \(k_p\sigma_r\), the bunch transverse size \(\sigma_r\) relative to the plasma skin depth \(c/\omega_p\). For the electron bunch with low charge \(Q = 100pC\), Fig. 1 shows the lineout of the periodic \(E_z\) obtained from simulations at 5\(mm\). The amplitude of longitudinal wakefield \(E_z\) decreases from \(\approx 2\) to \(\approx 0.5\) MV/m as the plasma density \(n_p\) is increased, indicating the dominance of the decreasing term \(n_p^{-1}\) over the increasing term \(R(0)\) [9]. When the bunch charge is increased to 175\(pC\), the amplitude of the wakefields also increases \((E_z \propto n_b)\) (not shown here) but the trend is the same as for 100\(pC\) for different plasma densities.

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**THE EVOLUTION OF PEAK ACCELERATING FIELD**

Fig. 2 shows the peak accelerating field \(E_z\) along the propagation distance \(z\) for the bunch charge of 50\(pC\) to 175\(pC\). The initial values \(E_z(z = 0)\) decreases from \(\approx 5\) to \(\approx 1MV/m\) when the bunch charge is reduced from 175\(pC\) to 50\(pC\). Figure 2 shows that the peak accelerating field during the self-modulation occurs earlier for greater bunch charges, indicating a more rapid growth of the self-modulation instability. The SMI grows, \(E_z\) saturates and the saturation distances decreases from 70\(mm\) to 40\(mm\) with increasing charge. The simulation result also indicates that within the propagation distance of \(z = 15mm\), \(E_z\) remains almost constant, and this is confirmed by the estimation of \(N_{e\text{-}\text{folding}}\) from Eqn. (1): \(N_{e\text{-}\text{folding}} < 6\). The plasma channel length of PITZ is limited to 10\(cm\) and therefore a significant detectable SMI growth is expected at the plasma exit with higher bunch charges.

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**DENSITY MODULATION OF THE ELECTRON BUNCH**

Figs. 3, 4, 5 shows the snapshots of the on-axis electron beam density modulation when the beam propagates 15.4\(mm\) in the plasma for different conditions. These snapshots were taken to see the start of the beam modulation for
different conditions with simulation condition 100\(pC\), 26\(\mu m\) and \(L_{beam}/\lambda_p = 6\) taking as reference.

From Fig. 3 as expected, the density modulation is small for lower charge (50\(pC\)). On the contrary, for the high charge case where \(Q = 175\, pC\), the electron bunch starts modulating into small beamlets on the scale of the plasma wavelength, drives the plasma wakefield resonantly. Such radial modulation is the direct evidence of the occurring of the SMI.

Fig. 4 shows the snapshot of the beam density modulation when the beam propagates 15.4\,mm in the plasma for various plasma densities such that \(L_{beam}/\lambda_p = 4.2\) for \(0.5 \times 10^{15}\, cm^{-3}\), 6 for \(1 \times 10^{15}\, cm^{-3}\), 7.3 for \(1.5 \times 10^{15}\, cm^{-3}\) and 8.4 for \(2 \times 10^{15}\, cm^{-3}\).

CONCLUSION

The measurement of the longitudinal phase-space of the originally flat-top PITZ beam after the propagation in plasma is the primary goal for the PITZ experiment, since it reflects the main phenomenological features of the self-modulation. We have shown in initial HiPACE simulations that the long electron bunch available at PITZ drives wakefields with periods dependent on the plasma densities. Simulations indicate that the initial accelerating field amplitudes decrease if the plasma density \(n_p\) is increased. Simulation results suggest that performing the experiment with higher bunch charge, higher plasma density and smaller transverse beam size will lead to an enhancement of the transverse modulation, and therefore to the possibility to experimentally probe and investigate the dynamics of the SMI at the PITZ experiment.

REFERENCES