HEDA2 resolution limitation for longitudinal phase space measurements

- **HEDA2 at PITZ**
- **Matrix formalism for the momentum measurements**
- **Momentum measurement simulations**
- **Resolution of observation screen read-out**
- **Summary**

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DPG Spring meeting
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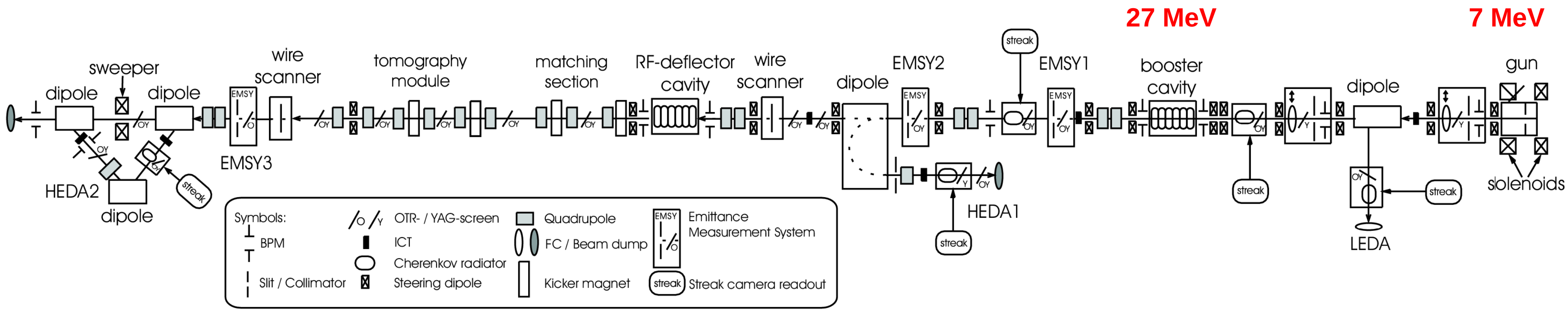
HELMHOLTZ | ASSOCIATION

DESY

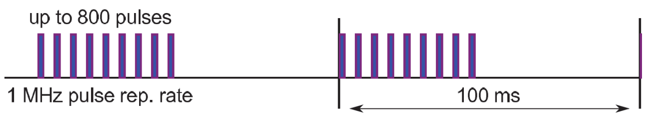
The Photo Injector Test facility, Zeuthen site (PITZ)



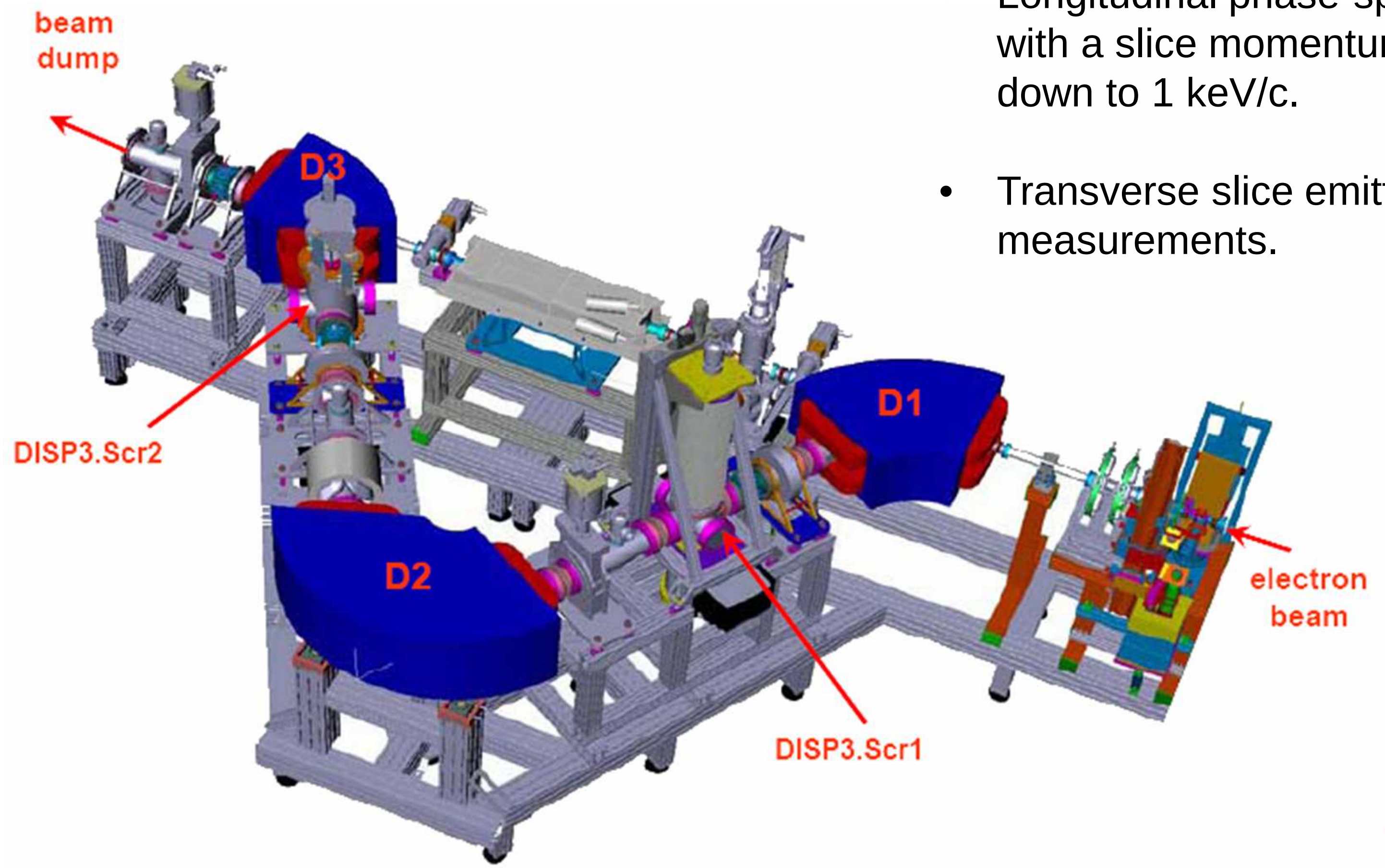
PITZ main parameters:

Bunch charge	1 pC … 4 nC
Repetition rate	10 Hz
Beam energy after gun	1 … 7 MeV
Beam energy after booster	1 … 27 MeV
Number of bunches	1 … 800
Bunch spacing	1 us
Laser pulse temporal shape	2 ps Gauss … 22 ps flat-top

Laser pulse structure



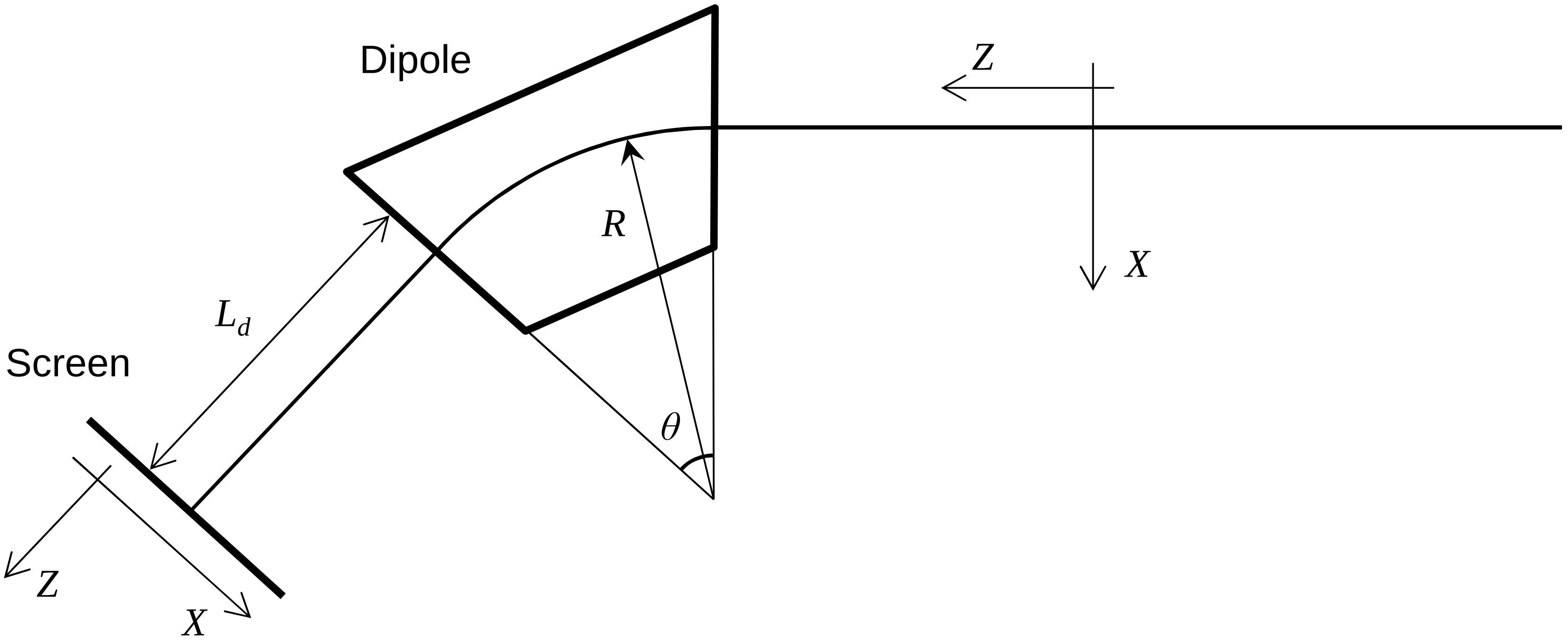
High Energy Dispersive Arm 2 (HEDA2) at PITZ



- High resolution beam momentum measurements up to 40 MeV/c.
- Longitudinal phase-space measurements with a slice momentum spread resolution down to 1 keV/c.
- Transverse slice emittance measurements.

Momentum measurements

For the beam momentum measurements at the first screen after dipole lets define following initial conditions and parameters:



Dipole + drift transport matrix

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ z \\ \delta p \end{pmatrix}_1 = \begin{bmatrix} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & 1 & L_d + R\theta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} x \\ x' \\ y \\ y' \\ z \\ \delta p \end{pmatrix}_0$$

$$x_1 = R_{11}x_0 + R_{12}x'_0 + R_{16}\delta p_0$$

$$R_{11} = cos\theta - sin\theta \frac{L_d}{R}$$

$$R_{12} = L_d \cdot cos\theta + R \cdot sin\theta$$

$$R_{16} = L_d \cdot sin\theta + R \cdot (1 - cos\theta) = D$$

$$z_1 = R_{51}x_0 + R_{52}x'_0 + z_0 + R_{56}\delta p_0$$

$$R_{51} = sin\theta$$

$$R_{52} = R \cdot (1 - cos\theta)$$

$$R_{56} = -R \cdot \theta + R \cdot sin\theta + \frac{R\theta + L_d}{\gamma^2}$$

Dipole limitations

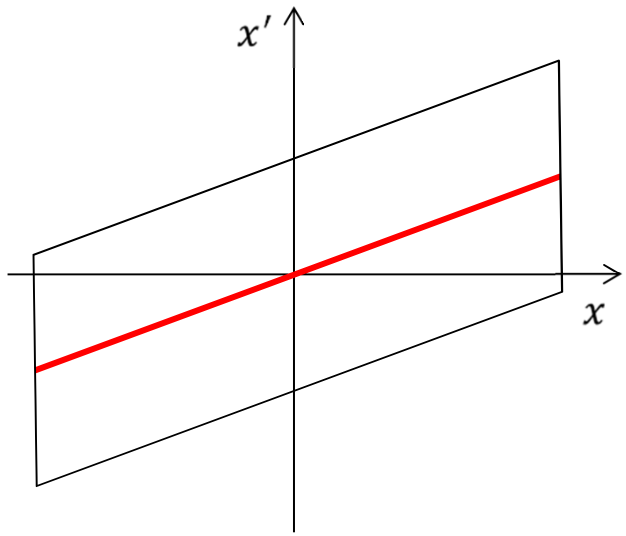
$$x_1 = R_{11}x_0 + R_{12}x'_0 + R_{16}\delta p_0$$

$R_{11} = -0.516$

$R_{12} = 0.867$ [m]

$R_{16} = 0.905$ [m]

$$|R_{11}x_0 + R_{12}x'_0| < R_{16}\delta p_0$$



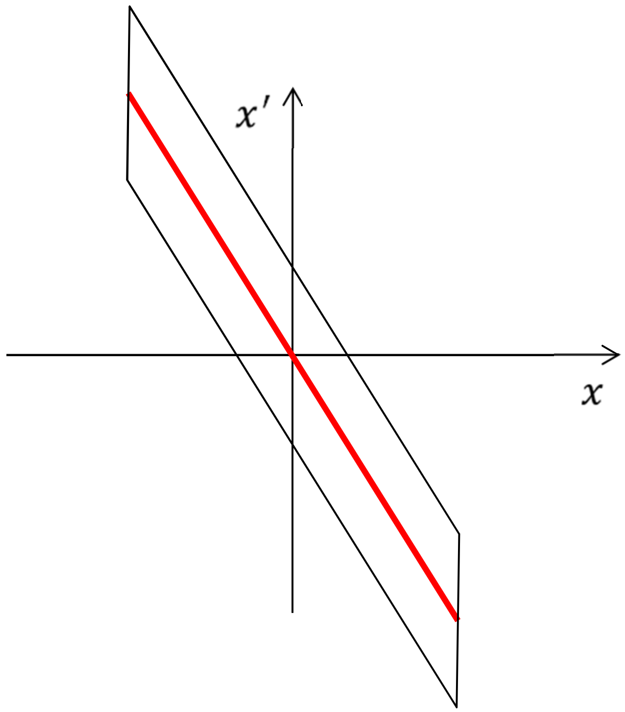
$$z_1 = R_{51}x_0 + R_{52}x'_0 + z_0 + R_{56}\delta p_0$$

$R_{51} = 0.866$

$R_{52} = 0.298$ [m]

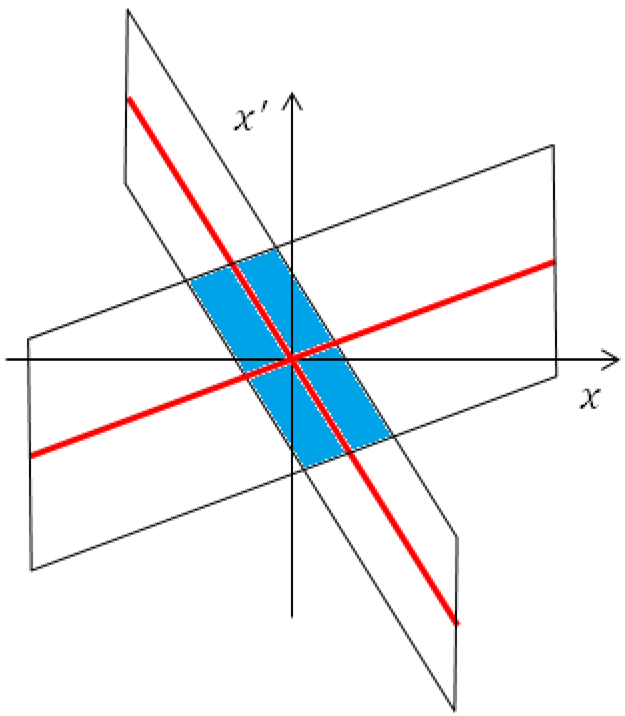
$R_{56} = -0.107$ [m]

$$|R_{51}x_0 + R_{52}x'_0 + R_{56}\delta p_0| < z_0$$

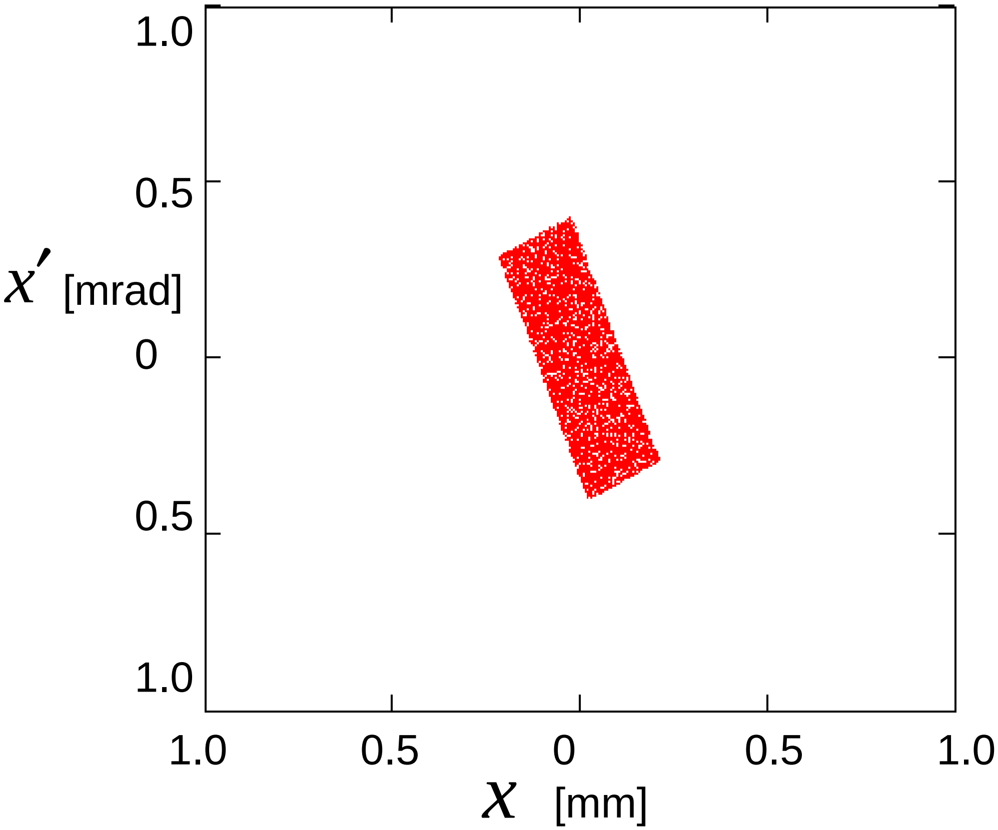


Solution for system of inequations

$$\begin{cases} |R_{11}x_0 + R_{12}x'_0| < R_{16}\delta p_0 \\ |R_{51}x_0 + R_{52}x'_0 + R_{56}\delta p_0| < z_0 \end{cases}$$



Numerical example



$$\Delta\delta p_0 = \frac{10\ keV}{25\ MeV} = 4 \cdot 10^{-4}$$

$$\Delta z = 0.1\ mm\ (0.33\ ps)$$

$$\varepsilon = \sqrt{\langle x^2 \rangle \cdot \langle x'^2 \rangle - \langle x \cdot x' \rangle^2}$$

$$\varepsilon_n = \gamma \cdot \beta \cdot \varepsilon = 0.6 mm \cdot mrad$$

Dipole resolution limitation by beam pipe aperture

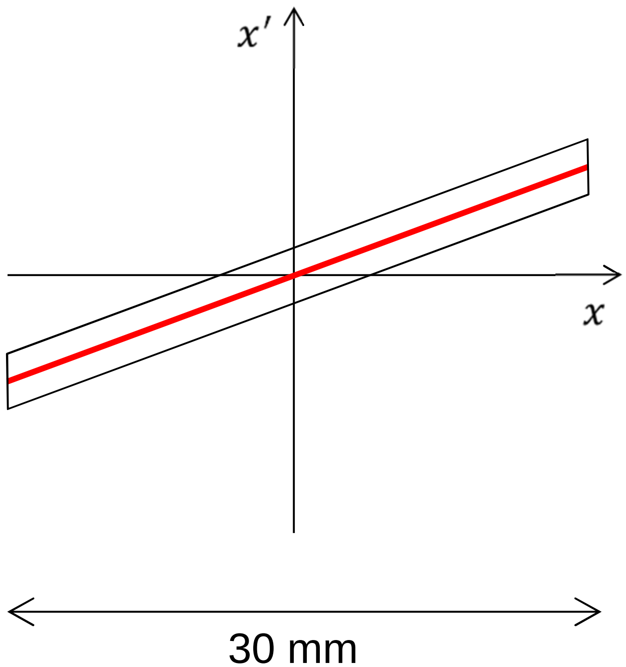
$$x_1 = R_{11}x_0 + R_{12}x'_0 + R_{16}\delta p_0$$

$$R_{11} = -0.516$$

$$R_{12} = 0.867$$

$$R_{16} = 0.905$$

$$|R_{11}x_0 + R_{12}x'_0| < R_{16}\delta p_0$$



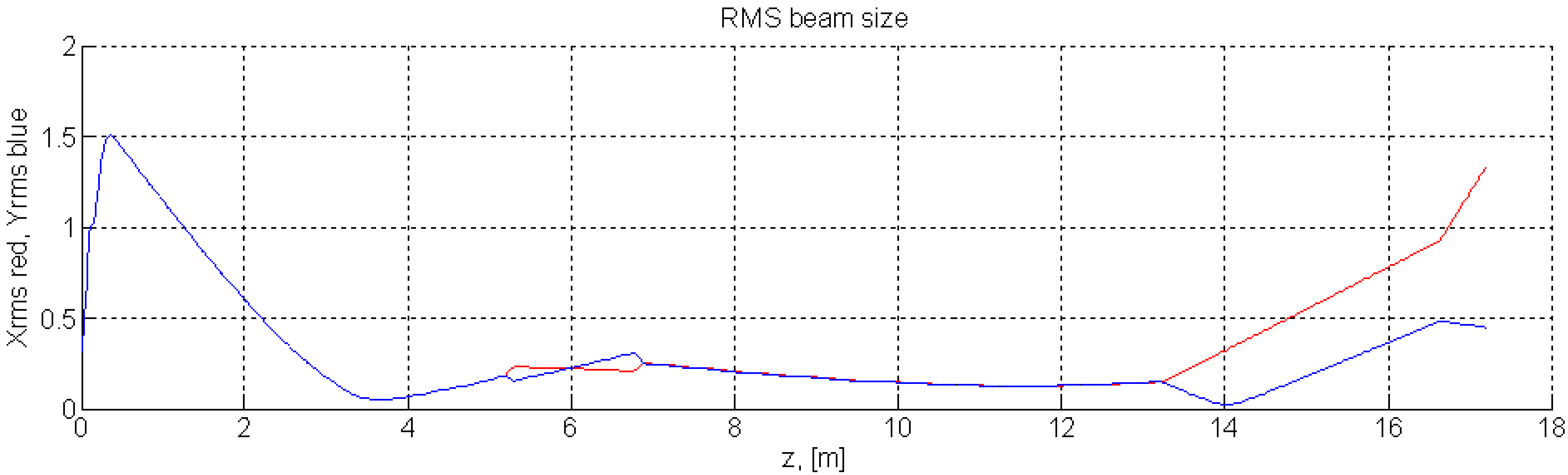
If we are not interesting in the particle longitudinal coordinate (within the bunch) after the dipole

$$x'_{\max} = \frac{\pi \cdot \varepsilon_n / \gamma \cdot \beta}{2 \cdot 2 \cdot x_{\max}}$$

$$x'_{max} \approx \frac{\pi \cdot 0.6 \cdot \frac{10^{-6}}{45}}{2 \cdot 3 \cdot 10^{-2}} = 0.7 \cdot 10^{-6}$$

$\delta p_0 \approx 0.7 \cdot 10^{-6} \rightarrow$ **16 eV**

ASTRA Simulation: beam transport, RMS beam size

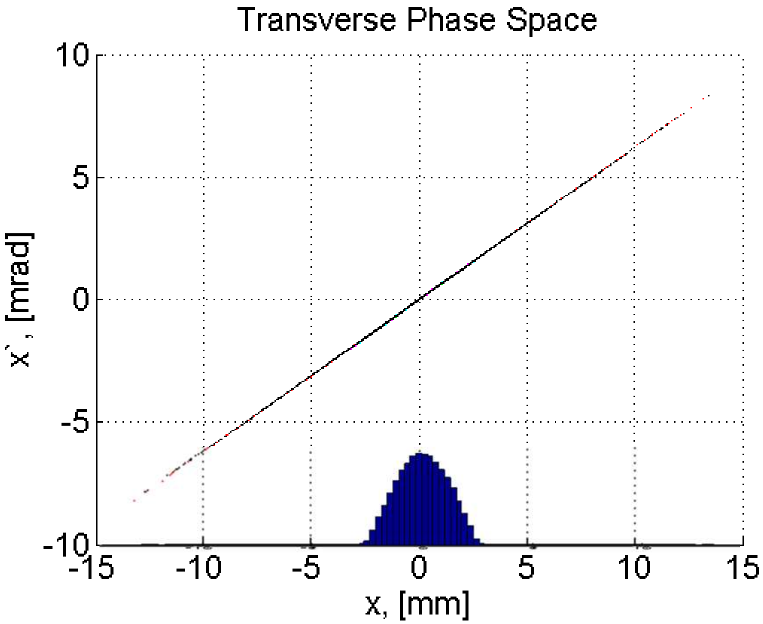


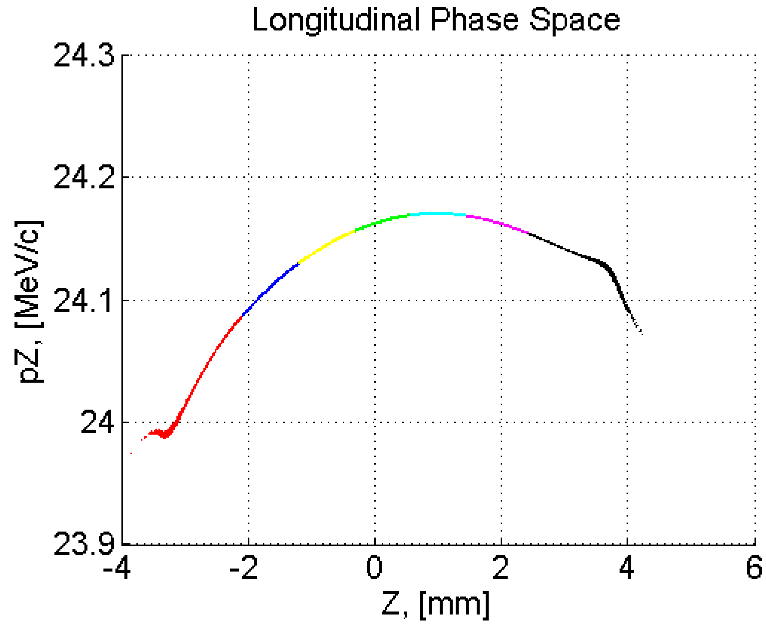
Gun&solenoid Booster Quads Quads Dipole

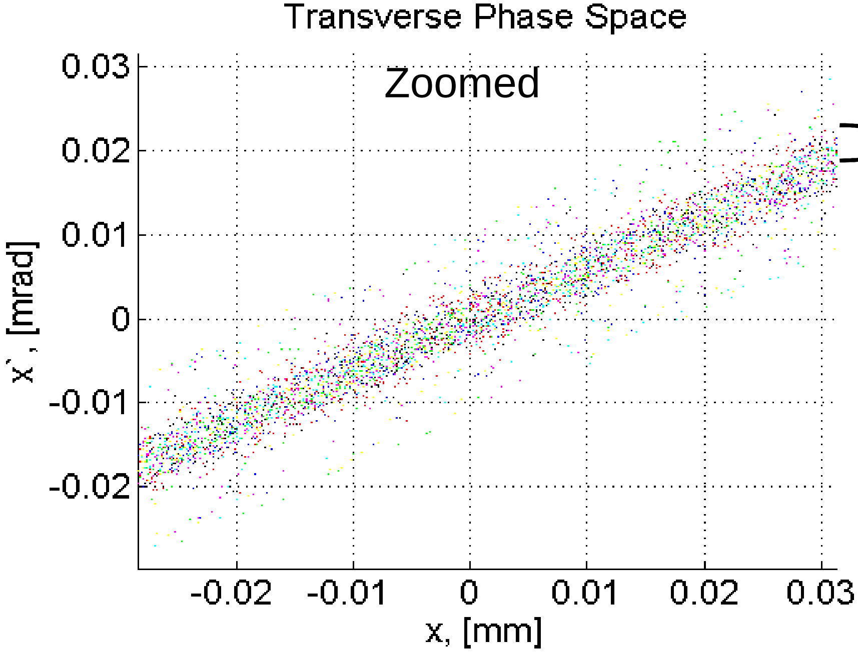
ASTRA simulation parameters:

- 100 pC bunch charge,
- Gun solenoid optimized to have minimum emittance after the accelerating structure,
- Quadrupoles at the end of beamline were optimized to get highest momentum resolution.

Simulation: phase spaces before dipole (17.0 m)



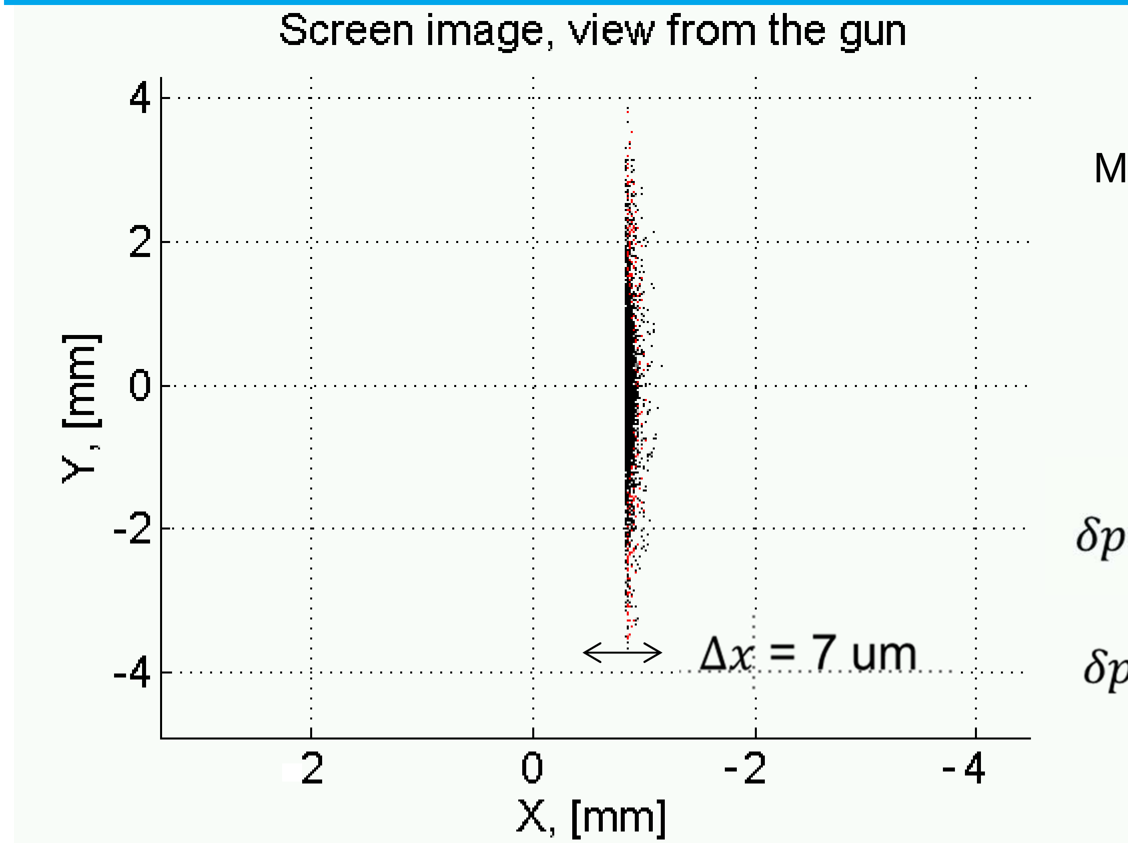




$\Delta x' \approx 5 \cdot 10^{-6} \approx \delta p \rightarrow$ **0.12 keV**

DESY

Simulation: screen image after dipole (18.5m)

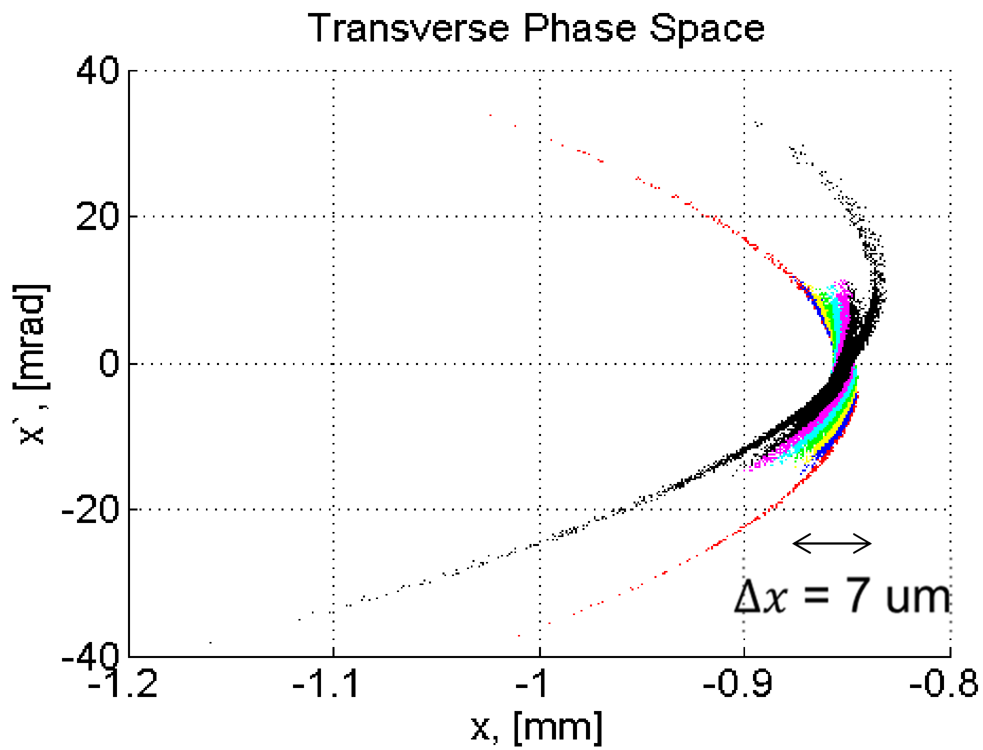


Monoenergetic beam:

$$\delta p = \frac{\Delta x}{D} = \frac{7 \cdot 10^{-6}}{0.9} \approx 8 \cdot 10^{-6}$$

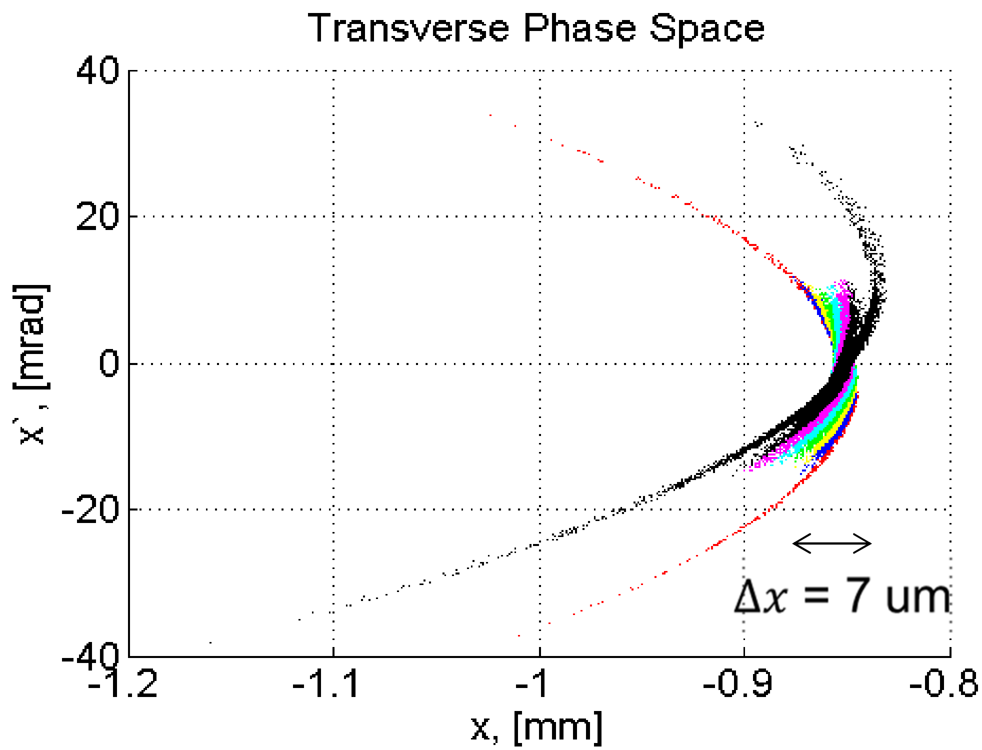
$\delta p \rightarrow$ **0.2 keV**

Simulation: phase space after dipole (18.5m)



Monoenergetic beam:

Simulation: phase space after dipole (18.5m)

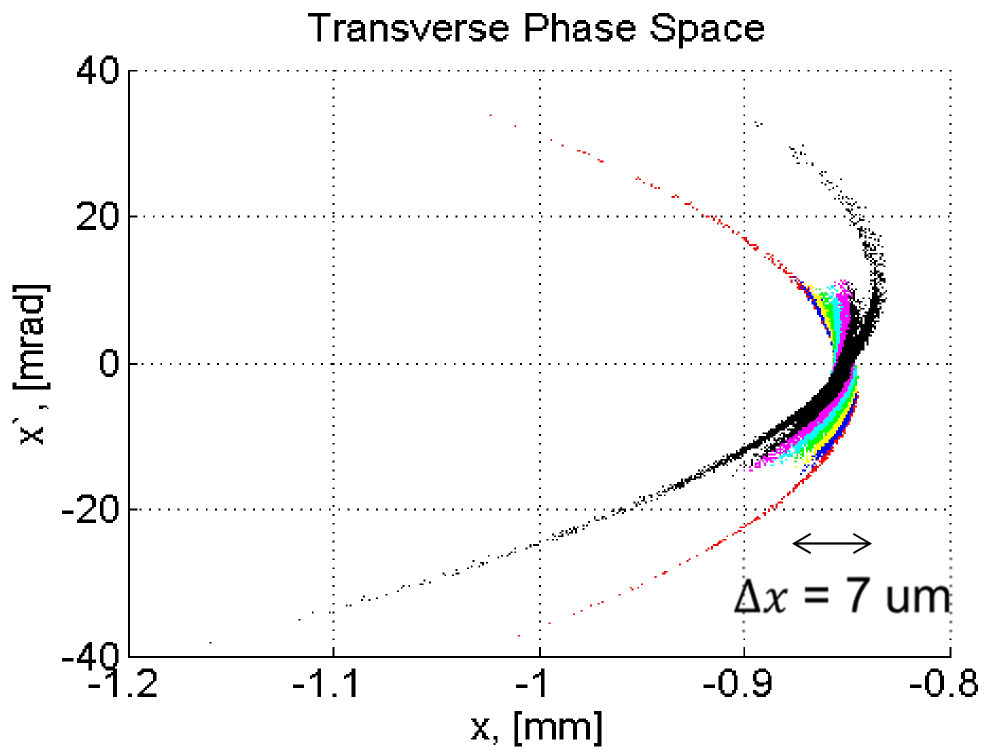


Monoenergetic beam:

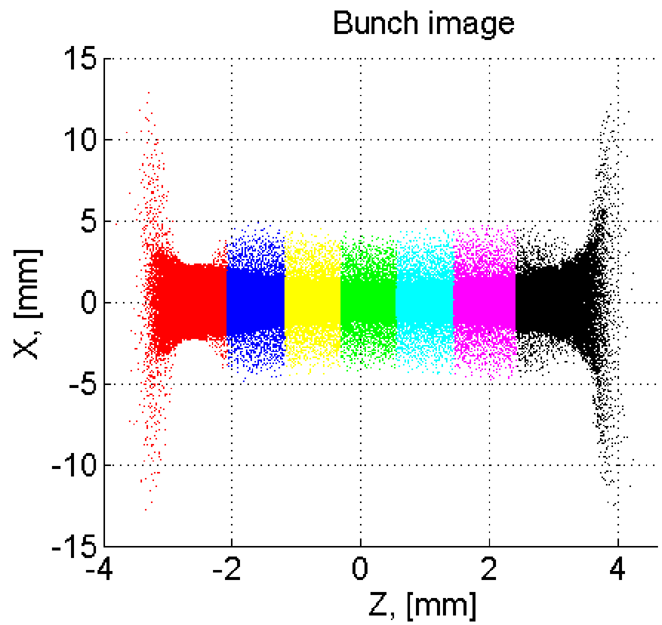
$$R_{11} = cos\theta \quad \longrightarrow \quad R_{11} = cos\theta + \frac{x_0}{2R} sin^2\alpha + \cdots$$

$$R_{12} = R \cdot sin\theta \quad \longrightarrow \quad R_{12} = R \cdot sin\theta + \cdots$$

Simulation: phase space after dipole (18.5m)

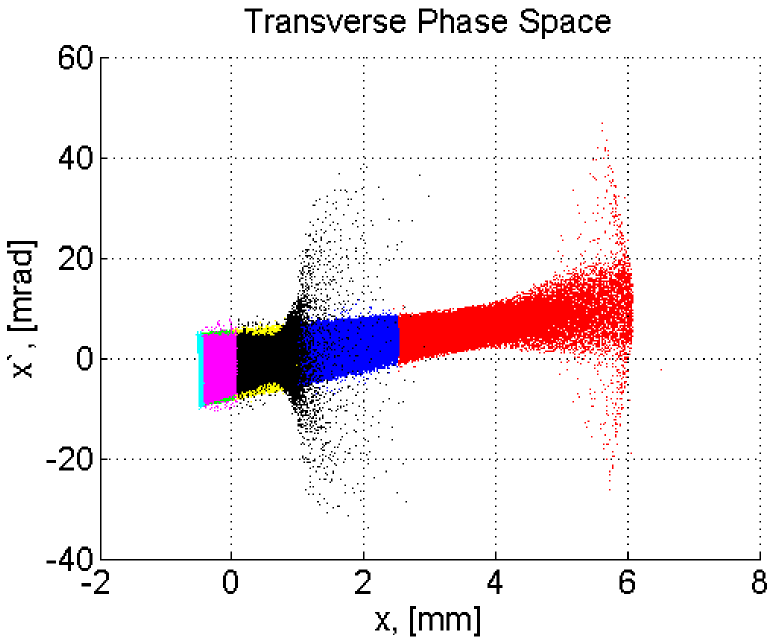


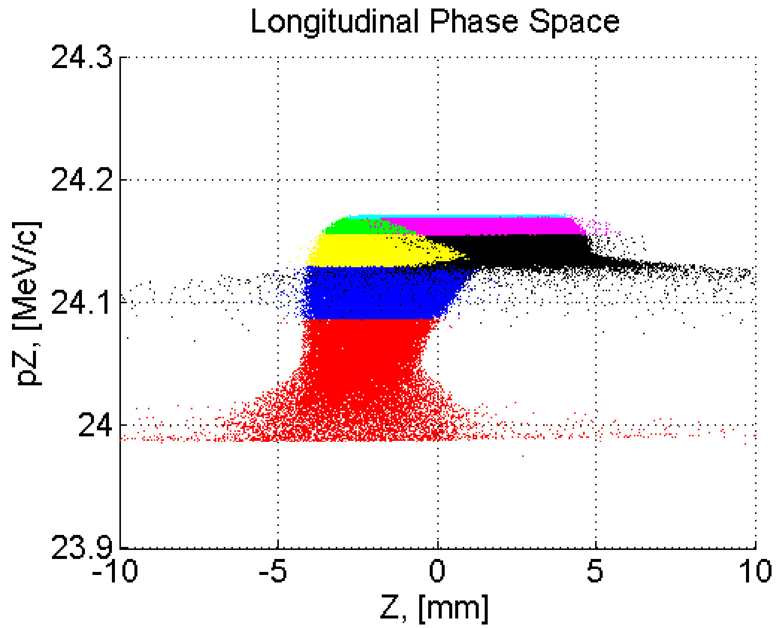
Monoenergetic beam:

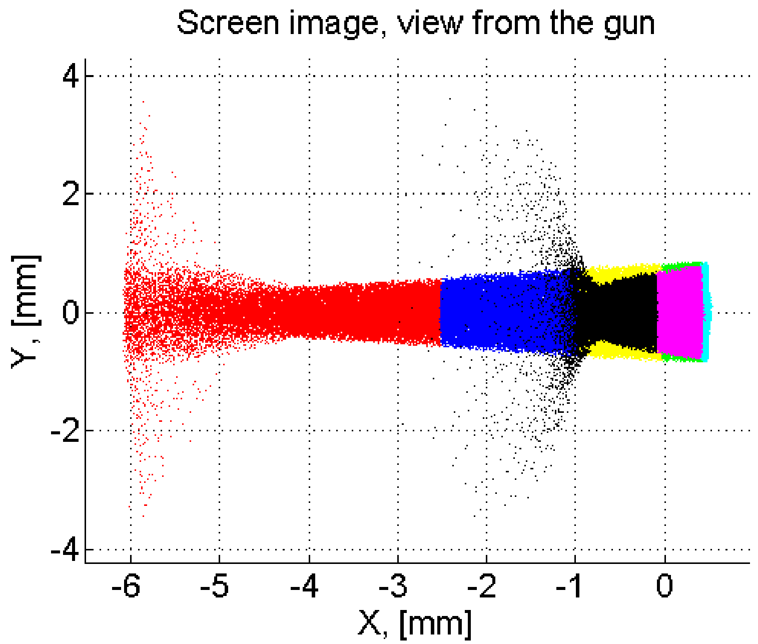


Color map corresponds to 7 longitudinal slices with equal charge

Simulation: phase spaces after dipole (18.5m)







Screen read out resolution

YAG screen calibration (2x2 camera bin mode):

- lens f80 -> 126 um/pixel
- lens f200 -> 37 um/pixel

1 pixel -> 37 um -> $\frac{\Delta p}{p} = 4 \cdot 10^{-5}$ -> **1 keV**

1 keV per pixel for **2x2 bin mode** or
0.5 keV per pixel for **full frame mode**.

Possible errors in optics alignment:

50 um resolution (15* um if you are lucky enough)

1.3 keV (**0.4* keV**)

Summary

For the HEDA2 momentum measurements linear model gives resolution of **16 eV**, for **0.6 mm*mrad** emittance.

ASTRA simulation (particles tracking) gives for such measurements **0.2 keV** momentum resolution.

Screen read-out limiting this resolution by the optics alignment from **0.4* keV** to **1.3 keV**, and by the video camera pixel size: **0.5 keV** (18 um/pixel).