

# AN RF DEFLECTOR FOR THE LONGITUDINAL AND TRANSVERSE BEAM PHASE SPACE ANALYSIS AT PITZ\*

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## Abstract

A detailed characterization of the longitudinal and transverse phase space of the electron beam provided by the Photo Injector Test Facility at DESY in Zeuthen (PITZ) is required to optimize electron beam for Free-Electron Laser (FEL) applications. By means of a RF deflector the transverse slice emittance and the longitudinal phase space can be analysed. The analysis of the prospect diagnostics shows the possibility to achieve a time resolution of about 0.5 ps, and a longitudinal momentum resolution of  $10^{-4}$ . The influence of the deflector on the longitudinal beam phase space is analysed.

## INTRODUCTION

The main research goal of PITZ is the development of electron sources with minimized transverse emittance [1].

At PITZ2 the application of an RF deflector is planned. The deflector position is about 9 m from the gun. The next 3.5 m of space are taken by a tomography module, see Fig.1. At about 15.5 m a spectrometer based on a dipole magnet is positioned. The RF deflector in combination with the tomography module gives possibility to analyse the transverse emittance of longitudinal slices. Using the RF deflector and a dispersive arm the longitudinal beam phase space can be completely reconstructed.

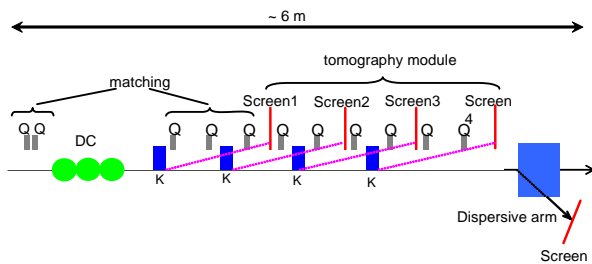


Figure 1: Prospect diagnostic for the beam phase space analysis.

We expect the following beam parameters at PITZ2: bunch length 23 ps, transverse size at the point of the deflecting cavity 3...5 mm (0.7-1.2 mm RMS), slice emittance - about 0.7pi mm mrad, longitudinal momentum 32 MeV/c. The diagnostics components have to provide a time resolution down to 0.5 ps (40 slices), an emittance measurement precision <10% and a precision of the longitudinal momentum spread measurements for 0.5 ps slices of about 1.4 keV/c.

## RF DEFLECTOR

A travelling wave structure was chosen for the RF deflector. It has small field filling time. That permits to analyse single bunch without distortion others. In Fig. 2 you see the general view of the cavity. It has two couplers: input and output, and two pump port on the opposite side for symmetry reasons that reduce the nonuniformity of the field distribution in the coupler cells.

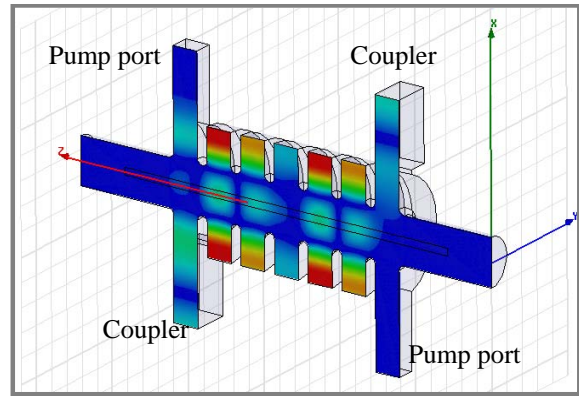


Figure 2: RF deflector based on travelling wave

The RF deflector has to provide a deflecting voltage up to 1.8 MV and should cause minimal distortion of the beam phase space in the measurement directions. In this paper we compare two travelling wave structures operating at 1.3 GHz and 3.0 GHz frequencies. In table 1 you can find the RF power needed for the diagnostics.

	1.3 GHz	3.0 GHz
Longitudinal phase space measurements	1.7 MW (0.8 MV)	0.35 MW (0.35 MV)
Slice emittance measurement, resolution 0.5 ps	9.1 MW (1.8 MV)	1.7 MW (0.8 MV)

Table 1: RF power and corresponding deflecting voltage needed for both kind of diagnostics.

## BEAM LONGITUDINAL MOMENTUM DISTORTION IN THE RF DEFLECTOR

The correct using of the RF deflector requires a negligibly small influence from the RF fields on the measured parameters (transverse slice emittance (X direction), longitudinal phase space (Z direction)). Let us analyse the longitudinal momentum distortion.

### Linear longitudinal momentum distortion.

The transport matrix of an ideal RF deflecting cavity looks like:

$$\begin{pmatrix} y \\ y' \\ z \\ dp_z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & a & 0 \\ 0 & 0 & 1 & 0 \\ a & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_0 \\ y'_0 \\ z_0 \\ dp_{z,0} \end{pmatrix} \quad (1)$$

$dp_z$  – energy spread in relative units.

From Panofsky-Wenzel theorem we can find the relation between the transverse deflecting voltage and the longitudinal electrical field gradient in the RF deflector:

$$V_y = i \frac{c}{\omega} \int \nabla_y E_z e^{i \frac{\omega}{c} z} dz \quad (2)$$

where  $\omega$  is the RF frequency,  $c$  - the velocity of light,  $y$  - the deflecting direction,  $z$  - the longitudinal position in the deflector,  $E_z$  - the longitudinal electric field,  $V_y$  - the integrated transverse deflecting voltage and the integration is performed over the deflector length.

Using the Panofsky-Wenzel theorem we can find the change of the energy for a single particle after passing the deflector:

$$dp_z = \frac{dE}{E} = \frac{\omega}{c} \cdot \frac{V_{y \max}}{E/e} \cdot y_0 \quad (3)$$

where  $V_{y \max}$  is amplitude value of the deflecting voltage,  $E$  – particle energy.

Let's look at the couple of deflector and a screen, Fig.3.

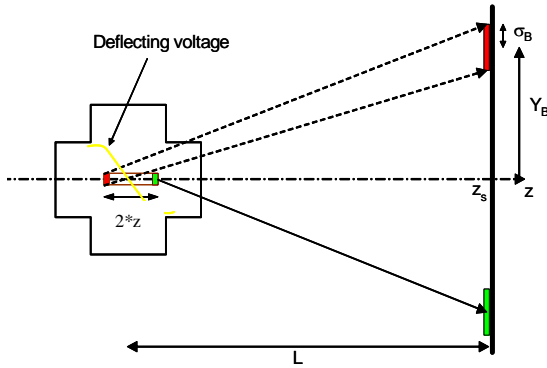


Figure 3: The principal of the RF deflector work.

The vertical beam size is:

$$y_B = \frac{2 \cdot \pi \cdot f \cdot L \cdot z_0 \cdot V_{y \max}}{c \cdot E / e} \quad (5)$$

The number of slices which we can resolve is described by the formula:

$$N_{slices} = \frac{y_B}{y_0} = \frac{2 \cdot \pi \cdot f \cdot L \cdot z_0 \cdot V_{y \max}}{y_0 \cdot c \cdot E / e} \quad (6)$$

From the last expression we can derive  $V_{y \max}$  and put it in formula (3). Finally we have following expression for energy spread:

$$dp_z = \frac{dE}{E} = \frac{N_{slices} \cdot y_0^2}{L \cdot z_0} \quad (7)$$

So we have shown that the energy spread doesn't depend on the RF frequency and beam energy. Let us continue the linear analysis for a 1.3 GHz deflector.

For PITZ two cases of RF cavity operation are presented, see Table 1. They require to different distances  $L$  from the

deflector to a screen. For longitudinal phase space analysis in the dispersive section  $L \sim 6m$ . The second case is transverse slice emittance measurements in the tomography module and  $L_{\min} \sim 2.5m$ . At PITZ we want to use the maximum possible screen size in vertical direction – about 30 mm. That means that the beam size at the output of the deflector ( $L_{\text{deflector}} \sim 1m$ ) is already different from the entrance. For the case of longitudinal phase space diagnostic the maximum vertical beam size at the end of the deflector is about 1.2 mm, for the case of transverse slice emittance measurements – up to 7.5 mm. For the outermost slice we have a maximum momentum distortion according to Fig. 4.

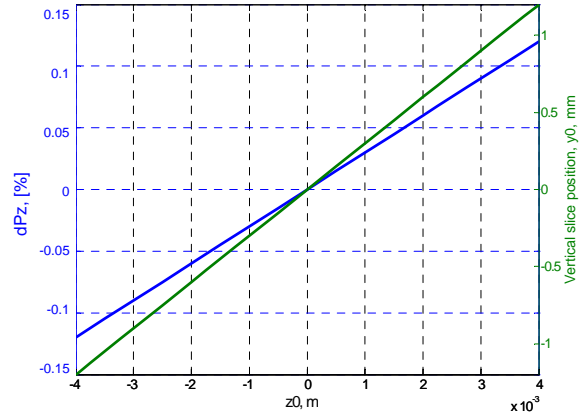


Figure 4: Momentum distortion for different slice positions along the bunch (blue line), Vertical slice position (green line). Longitudinal phase space analysis diagnostic.

For the case of longitudinal phase space diagnostic the distortion of the longitudinal energy is maximum for the first and the last slices,  $dP_z(\text{linear}) \sim 1.2 \cdot 10^{-3}$  (for Prf=1.7MW). For the transverse slice emittance measurements we can show that the momentum distortion in relative units  $dP_z(\text{linear}) \sim 27 \cdot 10^{-3}$  (for Prf=9.1MW) for the first and the last slices.

For measurements of the longitudinal phase space using the RF deflector and the dipole magnet we can unfold the original momentum distribution from the one introduced by the RF deflector. The energy spread of the single bunch can be described by:

$$\sigma_E'^2 = \sigma_E^2 + \left[ \frac{\omega}{c} V_{y \max} \cos(\omega t) \right]^2 \sigma_y^2 = \sigma_E^2 + \Delta \sigma_{defl}^2 \quad (8)$$

where  $\sigma_E$  is the slice energy spread before the deflector and  $\sigma_y$  is the average rms transverse slice dimension in the deflector. The last equation is strictly valid if the  $y$  and  $E$  plane of each slice are uncorrelated. The slice energy spread then can be “reconstructed” knowing the deflecting voltage, the frequency of operation and the average transverse beam dimension ( $\sigma_y$ ) in the deflector:

$$\sigma_E = \sqrt{\sigma_E'^2 - \left[ \frac{\omega}{c} V_{y \max} \cos(\omega t) \right]^2 \sigma_y^2} \quad (9)$$

## Nonlinearity of the electro-magnetic fields in the RF deflector.

According the linear theory ([2]) the  $E_z$  component of the electrical field along the axis X for fixed Y and Z coordinates is constant. We have made numerical calculations of the real electro-magnetic field distribution in the RF deflector for the two different frequencies 1.3 GHz and 3.0 GHz starting with the cavities geometry and using the CST MicroWave Studio software. We were comparing the  $E_z$  distributions along the axis X at fixed Y position. In the Fig.5 the relative values  $E_z(x,y=1mm,z)/E_z(x=0mm,y=1mm,z)$  for two different TW structures are shown. The black lines show the region of the bunch.

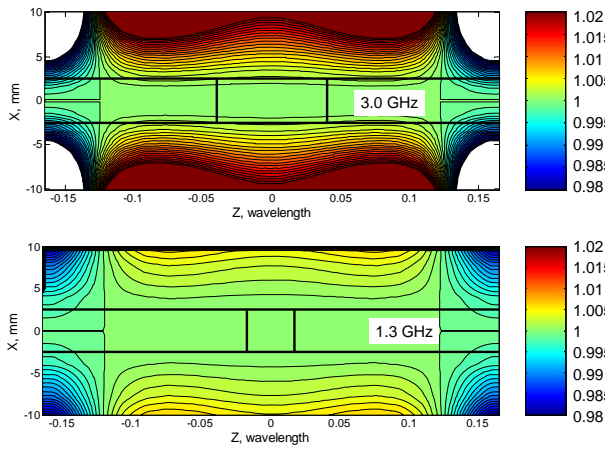


Figure 5: Relative values of  $E_z$ .

One can see that in the case of the 3.0 GHz TW structure the deviation of  $E_z$  from constant is up to several percents. For the 1.3 GHz structure this deviation is smaller – smaller then one percent.

This effect of not constant  $E_z$  dependence vs X can not be taken into account during the longitudinal phase space measurements (see previous paragraph). The final result of the momentum distortion due to this nonlinear effect we can estimate:

$$dE/E_{final} = dE/E + dE/E \cdot \delta \quad (10)$$

where  $\frac{dE}{E} = \frac{N_{slices} \cdot y_0^2}{L \cdot z_0}$  - linear momentum distortion for

the off axis particles (in Y direction),  $\delta$  - coefficient corresponding to the nonuniformity of  $U_z$  in X direction,  $U_z = \int E_z(x,y) dl$  – integral along the slice trajectory in the cavity.

To estimate the value of  $\delta$  we can use the distributions of the  $dE_z$  (look at the Fig. 5). The beam size in X direction is about 3-5 mm (full size). For the 3.0GHz cavity nonuniformity of  $E_z$  is about 1%, so we can estimate  $\delta \sim 0.02$ . For the 1.3 GHz cavity  $\delta$  is less than 0.005.

Let us make the estimations of the momentum distortion for 1.3 GHz and 3.0 GHz structures for the longitudinal phase space diagnostic (large distance deflector-screen  $\sim 6m$ , vertical beam size on the exit of the cavity  $y_0=1.2$  mm, longitudinal half bunch length  $z_0 = 4$  mm):

$$dp_z(nonlinear\_x) / 1.3GHzz / = \frac{N_{slices} \cdot y_0^2}{L \cdot z_0} \cdot \delta_{1.3GHz} = \frac{20 \cdot (1.2 \cdot 10^{-3})^2}{6 \cdot 4 \cdot 10^{-3}} \cdot 0.005 = 0.6 \cdot 10^{-5}$$

$$dp_z(nonlinear\_x) / 3.0GHzz / = \frac{N_{slices} \cdot y_0^2}{L \cdot z_0} \cdot \delta_{3.0GHz} = \frac{20 \cdot (1.2 \cdot 10^{-3})^2}{6 \cdot 4 \cdot 10^{-3}} \cdot 0.01 = 1.2 \cdot 10^{-5}$$

Another nonlinear effect comes from the finite length of the bunch. In Fig.6 the accelerating voltage along the bunch is shown in relative units.

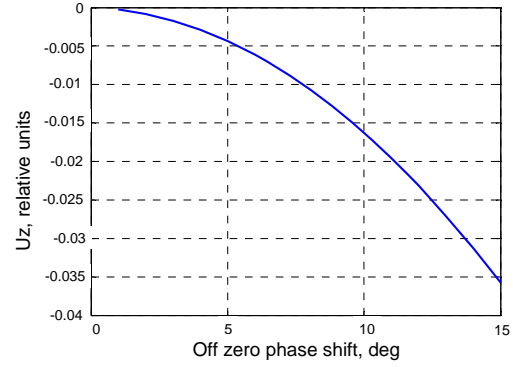


Figure 6:  $U_z$  vs off phase.

For different slices along the bunch we have different RF phases. The difference of  $U_z$  between the center and the edge of the bunch is about 3.5 % for 3GHz structure and  $\sim 0.5$  % for 1.3 GHz cavity. That gives the maximum momentum distortion for the first and last slices:

$$dp_z(longitudinal) / 3.0GHzz / = dp_z(linear) \cdot 0.035 = 3.6 \cdot 10^{-5}$$

$$dp_z(longitudinal) / 1.3GHzz / = dp_z(linear) \cdot 0.005 = 0.6 \cdot 10^{-5}$$

The sum of both processes (transverse nonlinear  $E_z$  distribution and longitudinal nonlinear  $E_z$  distribution) is:

$$dp_z(sum) / 3.0GHzz / = \sqrt{dp_z(nonlinear\_x)^2 + dp_z(longitudinal)^2} = 3.8 \cdot 10^{-5}$$

$$dp_z(sum) / 1.3GHzz / = \sqrt{dp_z(nonlinear\_x)^2 + dp_z(longitudinal)^2} = 0.85 \cdot 10^{-5}$$

## DISCUSSION

The study of 1.3 GHz and 3.0 GHz cavities for phase space diagnostics at PITZ shows that longitudinal distortions in the RF structures are not expected from linear theory. But nonlinear  $E_z$  distribution in the cavity due to the real geometry of the TW structures will cause longitudinal distortions.

The analysis shows that a distortion of a single slice in a 1.3GHz structure sums up to  $dpz \sim 0.85 \cdot 10^{-5}$  and for a 3.0GHz structure yields  $dpz \sim 3.8 \cdot 10^{-5}$ .  $4 \cdot 10^{-5}$  corresponds to the value of the original momentum spread in a slice (0.5ps) and the minimum non-correlated momentum spread is  $\sim 2 \cdot 10^{-5}$  in relative units. The distortion from the 3 GHz cavity would be of the same order resulting in worse resolution during longitudinal phase space measurements.

## REFERENCES

- [1] F.Stephan, D.Kraemer, I.Will and A.Novokhatski, Proc. FEL2k, Durham, USA, August 2000
- [2] O.H. Altermueller et. al. "Investigation of traveling-wave separators for the Stanford Two-Mile Linear Accelerator" SLAC-PUB-17, 1963