“FEL” using pipe with Surface Impedance

pipes with (loss free) surface impedance

comparison with FEL

derivation of FEL gain using wakefield approach (Stupakov)

using pipe with corrugated walls for FEL (Stupakov)

space charge effects (plasma oscillations)

pipes with (loss free) surface impedance

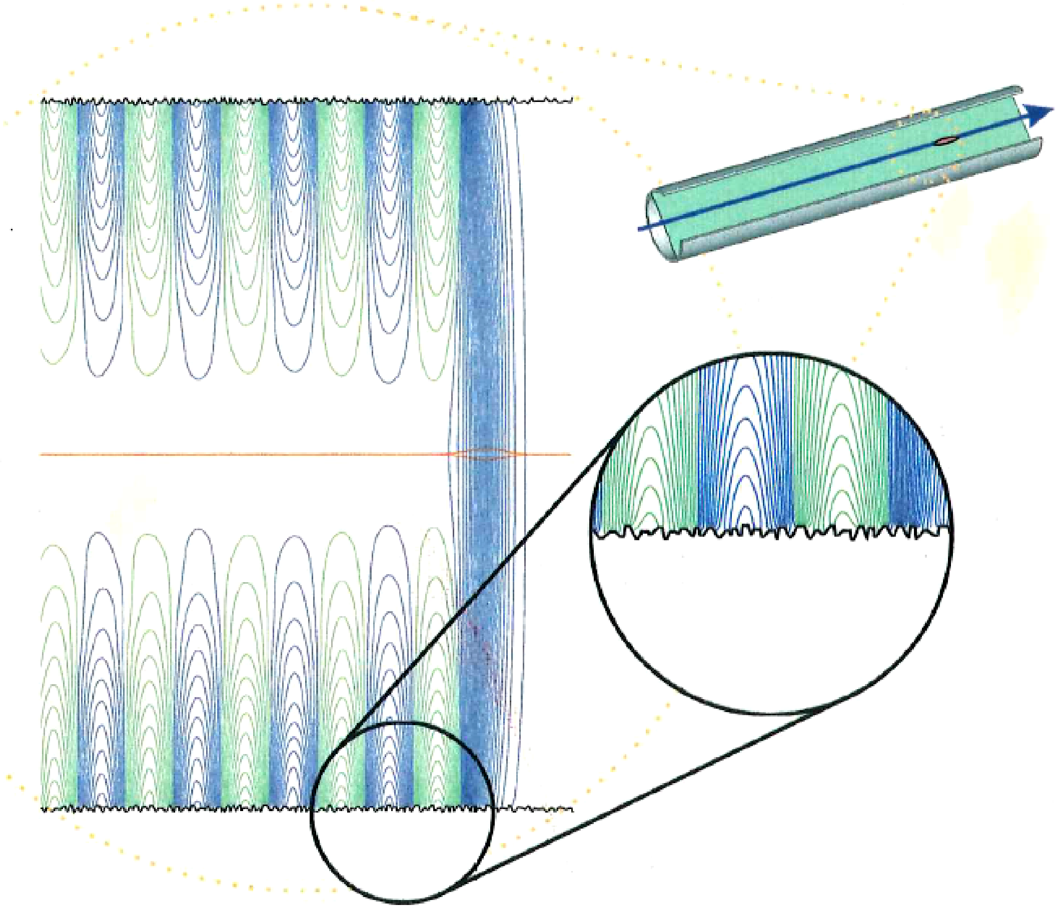
M. Timm, S. Nokhatski and T. Weiland
pipes with surface roughness $\rightarrow$ dielectric layer model
pipes with thin dielectric surface layer

G. Stupakov and K. Bane
pipes with small corrugations

Surface impedance formalism for a metallic beam pipe with small corrugations
PhysRevSTAB 15, 124401 (2012)

PhD Thesis (2000) **Martin Brüne Timm**

Wake Fields of Short Ultra-Relativistic Electron Bunches



wave

$$H_\varphi = -\frac{\alpha}{\mu_0} I_0'(\alpha r)\exp\left(ik_p\left(z - v_p t\right)\right)$$

$$E_z = -i\frac{c^2\alpha^2}{k_p v_p} I_0(\alpha r)\exp\left(ik_p\left(z - v_p t\right)\right)$$

$$E_r = -\frac{\alpha c^2}{v_p} I_0'(\alpha r)\exp\left(ik_p\left(z - v_p t\right)\right)$$

with

$$v = v_p = \beta_p c$$

$$k_p = \frac{\omega}{v_p}$$

$$\alpha = k_p / \gamma$$

asymptotic behaviour for $r \le r_b$

$$\alpha r_b << 1$$

using

$$I_0(x) \approx 1 + \left(\frac{x}{2}\right)^2$$

$$I_0'(x) \approx \frac{x}{2} + \frac{x^3}{16}$$

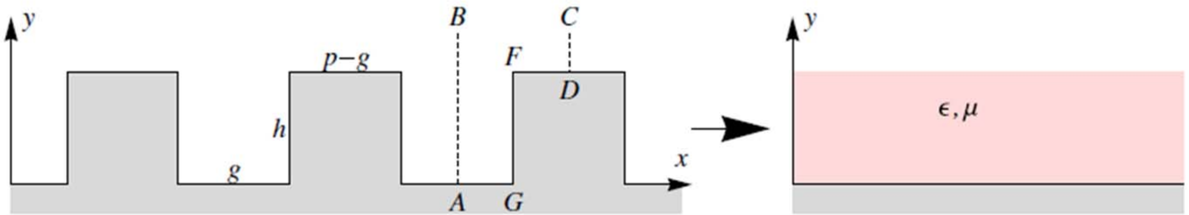
$$H_\varphi \approx -\frac{\alpha^2 r}{2\mu_0}\left(1 + \frac{(\alpha r)^2}{8}\right)\exp\left(ik_p\left(z - v_p t\right)\right)$$

$$E_z \approx -i\frac{c^2\alpha^2}{\omega}\left(1 + \frac{(\alpha r)^2}{4}\right)\exp\left(ik_p\left(z - v_p t\right)\right)$$

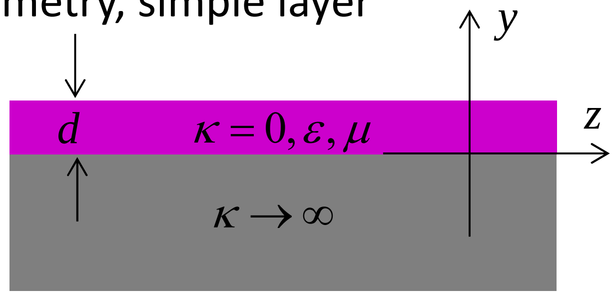
$$E_r \approx -\frac{\alpha^2 c^2 r}{2 v_p}\left(1 + \frac{(\alpha r)^2}{8}\right)\exp\left(ik_p\left(z - v_p t\right)\right)$$

boundary condition $Z_b(\omega) = -\left.\frac{E_z}{H_\varphi}\right|_{r=r_b}$ radius of pipe r_b

G. Stupakov and K. Bane



planar geometry, simple layer



$$Z_s = -\frac{E_z}{H_x} \rightarrow i\omega d\left(\frac{k_p^2}{\omega^2\varepsilon} - \mu\right)$$

notation $f(t) = \mathrm{Re}\{\tilde{f}\exp(-i\omega t)\}$

→ phase velocity

$$\beta_p = \frac{1}{\sqrt{2\left(\frac{2c}{r_b\omega}\right)^2\left(\frac{i\omega r_b Z_b(\omega)}{2\mu_0 c^2} - 1\right) + 1}}$$

$$Z_b(\omega) \approx -i\omega L$$

$$L = L(g, h, p) \approx \mu_0 \frac{gh}{p}$$

phase- and groop velocity

$$w = \frac{\omega r_b}{2c} \qquad \text{normalized frequency}$$

$$x = \frac{2L}{r_b \mu_0} \qquad \text{normalized surface parameter}$$

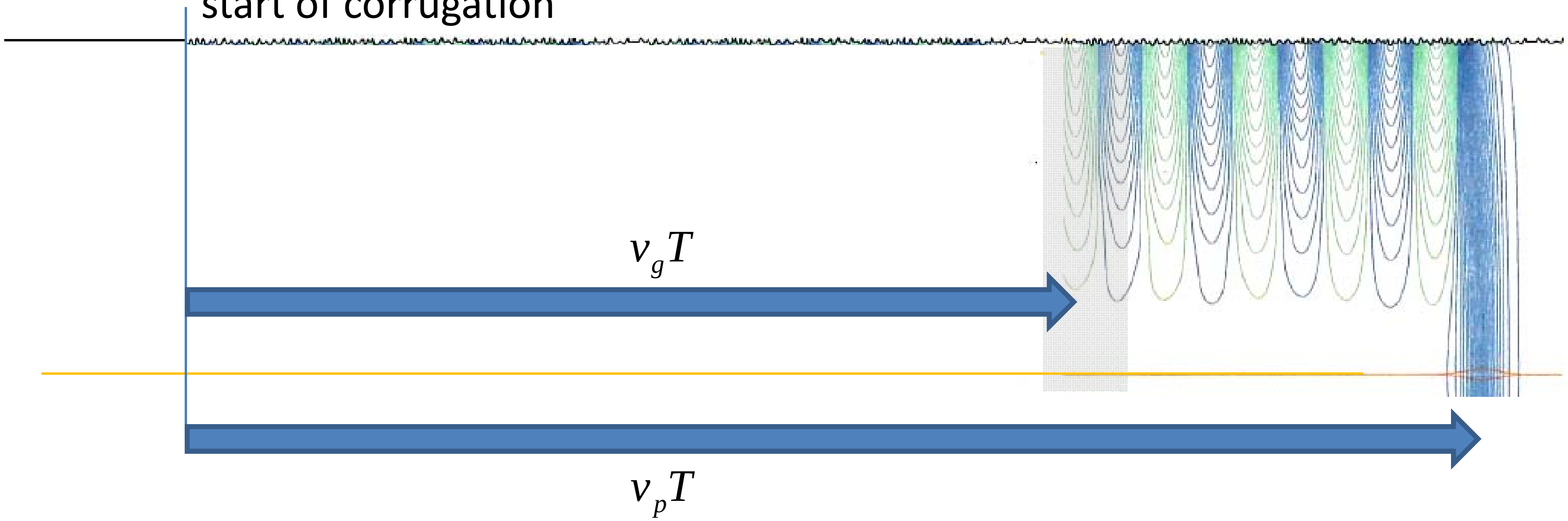
$$\beta_p(w, x) = \frac{1}{\sqrt{1 + 2\left(x - \frac{1}{w^2}\right)}}$$

$$\beta_g = \beta_p \left(1 - \frac{\omega}{\beta_p} \frac{\partial \beta_p}{\partial \omega}\right)^{-1} = \frac{1}{(1+2x)\beta_p}$$

required surface parameter (resonance condition)

$$x(\beta_p, w) = \frac{1}{w^2} + \frac{1}{2}\left[\frac{1}{\beta_p^2} - 1\right] = \frac{1}{w^2} + \frac{1/2}{\gamma^2 - 1}$$

simplified wake



$$W(Z,z)=\begin{cases} 2\kappa\cos(k_0\beta_p z) & (\beta_g/\beta_p-1)Z<z<0 \\ \kappa & z=0 \\ 0 & \text{otherwise} \end{cases}$$

κ loss-parameter (energy loss per length)

$$\kappa=\frac{E_{z0}^2}{4W'\left(1-\beta_g/\beta_p\right)}$$

Z (upper case) = beamline coordinate

z (lower case) = bunch coordinate

comparison with FEL

wave behind bunch

pipe with surface impedance

$$W(Z,z)=\begin{cases}2\kappa\cos(k_0\beta_p z) & (\beta_g/\beta_p-1)Z<z<0\\ \kappa & z=0\\ 0 & \text{otherwise}\end{cases}$$

$$\frac{\partial z}{\partial Z}=\frac{1}{\gamma^2}\left(\frac{\Delta\gamma}{\gamma}\right)$$ longitudinal dispersion

wave before bunch

undulator

$$W(Z,z)=\begin{cases}2\kappa\cos(k_{ph}\overline{\beta}_z z) & 0<z<(\overline{\beta}_z^{-1}-1)Z\\ \kappa & z=0\\ 0 & \text{otherwise}\end{cases}$$

$$\frac{\partial z}{\partial Z}=\frac{2k_u}{k_{ph}}\left(\frac{\Delta\gamma}{\gamma}\right)$$ or $$\frac{\partial\psi}{\partial Z}=2k_u\left(\frac{\Delta\gamma}{\gamma}\right)$$

derivation of FEL gain using wakefield approach

Derivation of FEL gain using wakefield approach
G. Stupakov and S. Krinsky, PAC 2003

longitudinal charge density $\lambda(Z,z)=\int f(Z,z,\gamma)d\gamma$

instantaneous longitudinal wake $E_{\parallel}(Z,z)=\int W(Z,u)\lambda(Z,z-u)du$

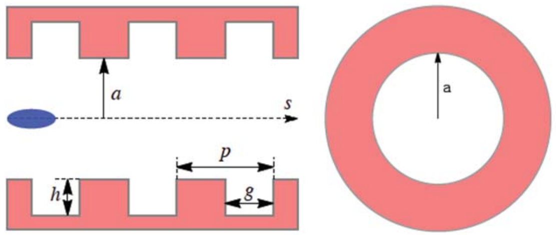
trick: use wake with retarded source distribution

$$E_{\parallel}(Z,z)=\int W(Z,u)\lambda\left(Z-\frac{\bar{v}_z u}{c-\bar{v}_z},z-u\right)du$$

$\rightarrow$ power gain length, etc

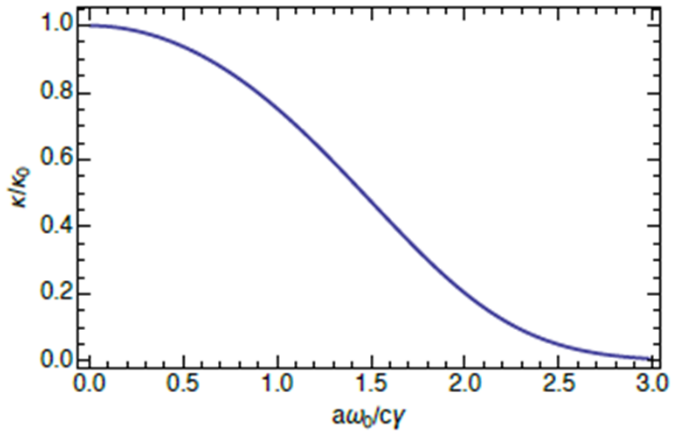
using pipe with corrugated walls for FEL

Using pipe with corrugated walls for a sub-terahertz FEL
G. Stupakov, SLAC-PUB-16171, December 2014



power gain length (cold beam, on resonance)

$$L_g = \frac{\gamma}{\sqrt{3}} \sqrt[3]{\frac{I_A}{I k_u \kappa}} \quad \text{with} \quad k_u = \frac{2\pi}{\lambda_u} \qquad k_u = \frac{2\pi}{\lambda_u} = \frac{\omega}{v_p}\left(1 - \frac{v_g}{v_p}\right)$$



Using pipe with corrugated walls for a sub-terahertz FEL
G. Stupakov, SLAC-PUB-16171, December 2014

IV. NUMERICAL EXAMPLE

TABLE I. Corrugation and beam parameters

Pipe radius, mm	2
Depth h, μm	50
Period p, μm	40
Gap g, μm ?	10
Bunch charge, nC	1
Energy, MeV	5
Bunch length, ps	10

$I = 100\ \mathrm{A}$

$f = 0.34\ \mathrm{THz}$

$1 - \beta_p = 0.0052$

$1 - \beta_g = 0.053$

gain length $L_g \approx 7\ \mathrm{cm}$

saturation power $P_{sat} \approx 6.7\ \mathrm{MW}$

slip condition $\frac{\lambda_u}{v_p}\left|v_p - v_g\right| = \lambda_p \rightarrow \quad \lambda_p \approx 0.88\ \mathrm{mm}$

$\lambda_u \approx 1.8\ \mathrm{cm}$

note: $\frac{\partial \psi}{\partial Z} = k_p \frac{\partial z}{\partial Z} \neq 2k_u\left(\frac{\Delta\gamma}{\gamma}\right)$

argument of modified Bessel function $\alpha r_b = \frac{k_p r_b}{\gamma} = 2\pi \frac{r_b}{\gamma\lambda_p} \approx 1.46$

space charge effects (plasma oscillations)

period length of plasma oscillation

$$S_p = \sqrt{\frac{I_A}{I}\frac{Z_0}{|Z'|}\frac{\lambda\gamma^3}{2}}$$

with $Z'(\omega) = \dfrac{-iZ_0}{2\pi\sigma_r\gamma\beta} F\left(\dfrac{\omega}{\omega_r}\right)$

$$F(\Omega) = \frac{\Omega}{2}\int_{\Omega^2}^{\infty}\frac{\exp\left(\Omega^2 - u\right)}{u}du$$

$$\omega_r = \gamma\beta c/\sigma_r \qquad \lambda = \frac{2\pi c}{\omega}$$

$$I_A \approx 17\,\mathrm{kA}$$

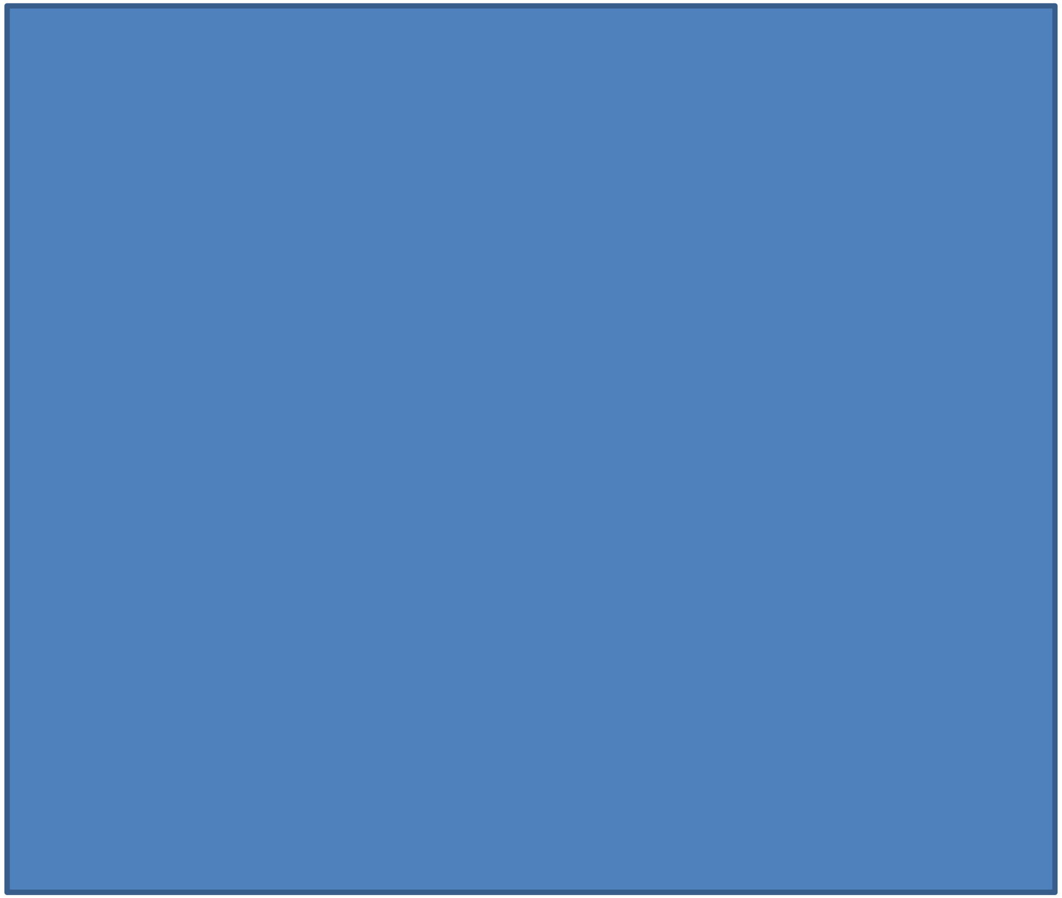
with parameters of example and $\sigma_r \approx 0.44\,\mathrm{mm}$

(for $\langle\beta_{\mathrm{Twiss}}\rangle \approx 2\,\mathrm{m}$ and $\gamma\varepsilon \approx 1\,\mu\mathrm{m}$)

$$\rightarrow S_p \approx 2.6\,\mathrm{m}$$

saturation length $L_{\mathrm{sat}}/L_g \approx 10\cdots 20$

but $0.25\,S_p/L_g \approx 10$



accurate

$$\frac{\alpha r_b I_0(\alpha r_b)}{I_0'(\alpha r_b)} = i\omega\varepsilon_0 r_b Z(\omega) = \omega^2 \varepsilon_0 r_b L \rightarrow \alpha(\omega)$$

$$\alpha = \frac{\omega}{c}\sqrt{\frac{1}{\beta_p^2 - 1}} \rightarrow \beta_p(\omega)$$

$$\beta_g = \beta_p \left(1 - \frac{\omega}{\beta_p}\frac{\partial \beta_p}{\partial \omega}\right)^{-1}$$

$$L = d\left(\mu - \frac{1}{\varepsilon c \beta_p^2}\right)$$