Plasma-like medium: basic definitions and parameters

M.Krasilnikov
"Electrodynamics of plasma and beams" – 2012
L#2

Plasma = gas of charged particles

From Wikipedia: “In physics and chemistry, plasma is a state of matter similar to gas in which a certain portion of the particles is ionized.

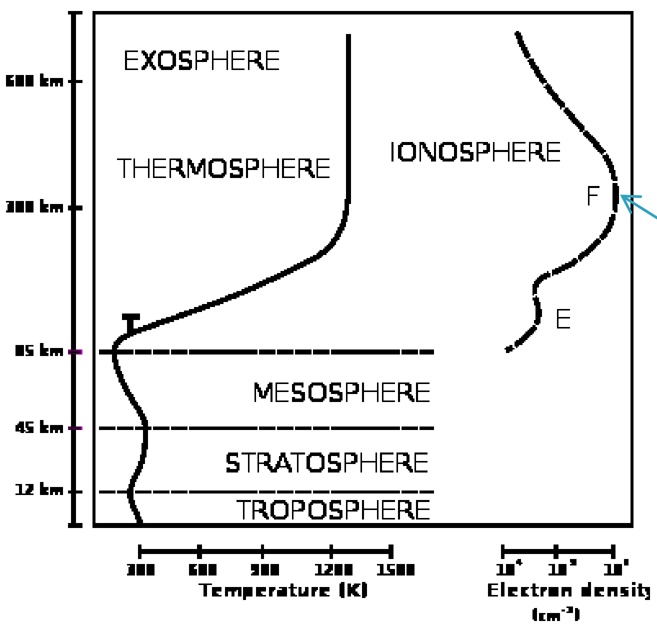
Plasma (preliminary definition?):

- (quasi) neutral → e.g.: electrons + ions, but also + atoms (molecules)
- charged → particle beams

⇒ Fourth state of matter

Irving Langmuir → 1923 electrical discharge in gases

→ paper 1928 the term ‘plasma’ → ‘region containing balanced charges of ions and electrons’.



But even before (1902) Oliver Heaviside proposed the existence of the F-layer of the ionosphere by means of which radio signals are transmitted around the Earth's curvature (confirmed in 1923).

Plasmas:

- ionosphere (e.g. F-layer and other terrestrial)
- space and astrophysical (nuclear fusion, interplanetary and intergalactic medium)
- gas discharge
- particle beams (electron or ion)
- laser produced plasma (LPP)
- solid-state plasma (SSP) (metals and semiconductors)
- …

Plasma parameters - N_α

- plasma=gas → particle density N_α
- charged particles: α – particle sort (type):
  - $\alpha = e$ → electrons
  - $\alpha = i$ → ions (q>0), also holes in a solid-state plasma (SSP)
  - $\alpha = n$ → neutrals (atoms, molecules, or lattice nodes in SSP), N_n=total number of neutrals

Plasma	Typical particle density [m^{-3}]		
	N_e	N_i	N_n
ionosphere (F-layer, h~300km)	10^{12}	$\sim N_e$	$\lesssim 10^{16}$
cosmic (interplanetary)	10^4-10^7	$\sim N_e$	$N_n \ll N_e$
stellar core	10^8-10^{32}	$\sim N_e$	$N_n \ll N_e$
laser produced (LPP)	10^{14}-10^{24}	$\sim N_e{}^*$	
gas discharge	10^{14}-10^{21}	$\sim N_e$	10^{18}-10^{23}
Solid state (SSP) – metals	10^{27}-10^{29}		
Solid state (SSP) – semiconductors	10^{20}-10^{24}		

Plasma parameters - *r*

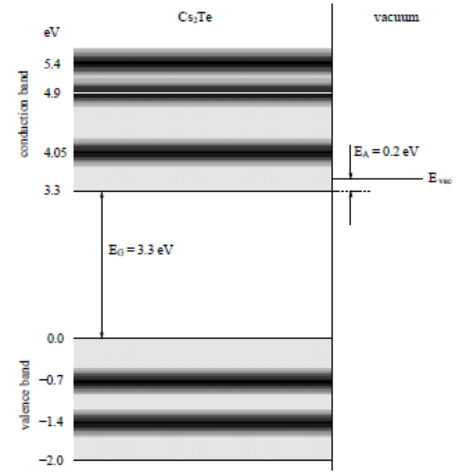
- degree of ionization $\boldsymbol{r} = \frac{\boldsymbol{N_e}}{\boldsymbol{N_n}}$ $\left(\text{sometimes also } \tilde{r} = \frac{N_e}{N_n+N_i} \text{ or } \frac{N_i}{N_n+N_i}\right)$

weakly ionized plasma $\boldsymbol{r} \ll \boldsymbol{1}$ [e.g. 10^{-2}.. 10^{-3}] $(\tilde{r} < 1)$

fully ionized plasma $\boldsymbol{r} \to \infty$ $(\tilde{r} = 1)$

Plasma	Typical parameters	
	$\boldsymbol{N_e}$ [m^{-3}]	$\boldsymbol{r}$
ionosphere (F-layer, h~300km)	10^{12}	10^{-4}
cosmic (interplanetary)	10^{4}-10^{7}	$\gg \mathbf{1}$
stellar core	10^{28}-10^{32}	$\gg \mathbf{1}$
laser produced (LPP)	10^{14}-10^{24}	$\sim$
gas discharge	10^{14}-10^{21}	$\sim$

Plasma parameters - e_α and m_α



- Charge
  - electrons $e_e = e = 1.6\times10^{-19}$ C
  - ions $e_i = -Ze$ ($Z \rightarrow$ the multiplicity of ionization)
- Effective mass of carriers
  - electrons
    - "conventional" plasmas: $m_{eff} = m_e = m = 9.11\times10^{-31}$ kg
    - SSP: $m^* = \dfrac{1}{\frac{\partial^2 \mathcal{E}(p)}{\partial p^2}}$, $\mathcal{E}(p)$ – energy of electrons (holes)

      metals: $m_{eff,e} = m^* \approx m_e$

      semiconductors: $m_{eff,e} = m^* \approx (0.1\text{-}0.01)\cdot m_e$
  - Ions
    - "conventional" plasmas: approx. $m_i = M = A\times1.66\times10^{-27}$ kg ($A \rightarrow$ atomic mass)
    - SSP: holes $m_{eff,l} \leq m_e$ (sometimes $> m_e$)

Effective mass (in m_e)			
Group	Material	$m_{eff,e}$	hole m_l
IV	Si (300K)	1.08	0.56
	Ge	0.55	0.37
III–V	GaAs	0.067	0.45
	InSb	0.013	0.6
II–VI	ZnO	0.29	1.21
	ZnSe	0.17	1.44

Plasma parameters - PDF

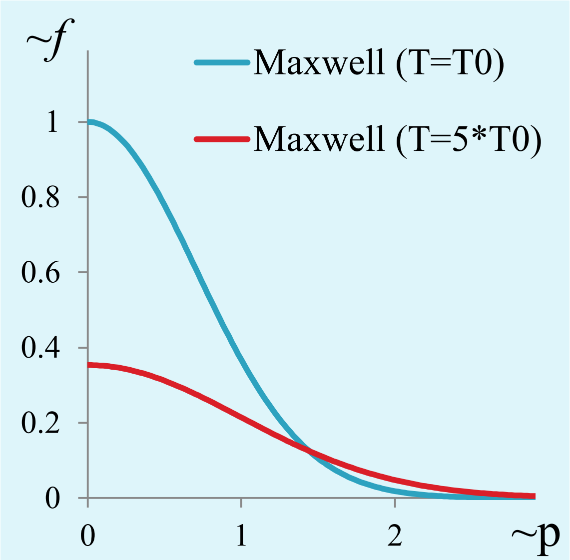
- Particles in plasma in thermal motion → Particle distribution function (PDF)

  e.g. Maxwell $f_{M\alpha} = \frac{N_\alpha}{(2\pi m_\alpha k T_\alpha)^{3/2}} exp\left(-\frac{{p_\alpha}^2}{2 m_\alpha k T_\alpha}\right),$

  Boltzmann constant $k = 1.38 \cdot 10^{-23} J/degK$

  (setting $k = 1$, we measure T_α in eV;

  1eV=11600K= 1.6×10^{-19} J)



- Isothermal ($T_\alpha = T_0$)

  and nonisothermal ($T_e \neq T_i$) plasmas

- Average kinetic energy of the thermal motion:

$$\left\langle \frac{{p_\alpha}^2}{2m_\alpha} \right\rangle = \frac{3}{2} k T_\alpha$$

Plasma parameters - PDF

- From statistical physics:

the Maxwell distribution is valid only for sufficiently high T!

- The Fermi degeneracy (following from the Pauli exclusion principle) becomes essential when the Fermi energy $\mathcal{E}_{F\alpha}$ exceeds the thermal one

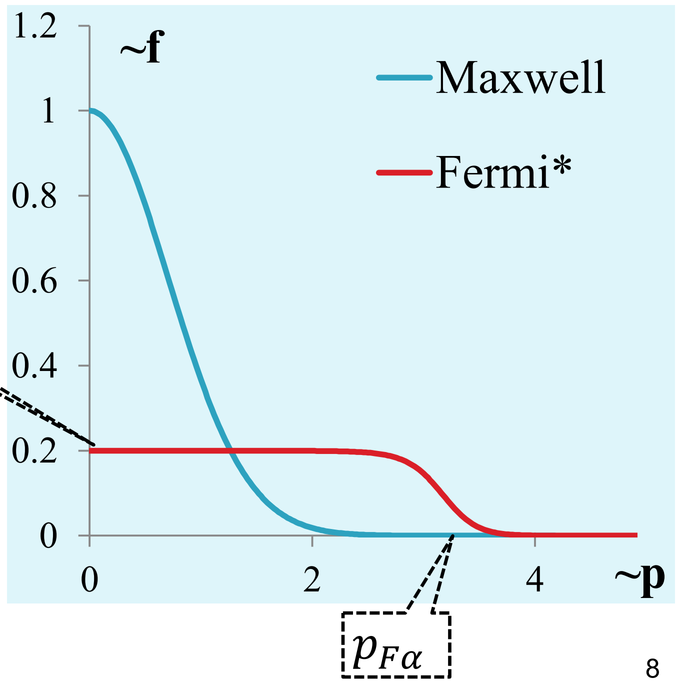
$$\mathcal{E}_{F\alpha} = \frac{{p_{F\alpha}}^2}{2m_\alpha} = \frac{\left(3\pi^2\right)^{2/3}\hbar^2 {N_\alpha}^{2/3}}{2m_\alpha} \gg kT_\alpha$$

Here:
$p_{F\alpha} = (3\pi^2)^{1/3}\hbar {N_\alpha}^{1/3}$
$\hbar = \frac{h}{2\pi} = 1.05 \cdot 10^{-34} J \cdot s$

The PDF of fermions in this case is given by Fermi distribution function:

$$f_{F\alpha} = \frac{\boxed{\frac{2}{(2\pi\hbar)^{3/2}}}}{exp\left(\frac{{p_\alpha}^2/_{2m_\alpha} - \mathcal{E}_{F\alpha}}{kT_\alpha}\right) + 1}$$

$$\left\langle \frac{{p_\alpha}^2}{2m_\alpha} \right\rangle = \frac{3}{5}\mathcal{E}_{F\alpha}$$



Degenerate plasma criterion

- Practical formula: $\mathcal{E}_{F\alpha} = 5 \cdot 10^{-38} \frac{m}{m_\alpha} {N_\alpha}^{2/3}$
- Plasma degenerate if :

$$\mathcal{E}_{F\alpha} = \frac{{p_{F\alpha}}^2}{2m_\alpha} = \frac{(3\pi^2)^{2/3}\hbar^2 {N_\alpha}^{2/3}}{2m_\alpha} \gg kT_\alpha$$

$$\Updownarrow$$

$$N_\alpha \gg \widetilde{N_\alpha} = 5 \cdot 10^{21} \left(\frac{m_\alpha}{m} T_\alpha\right)^{3/2}$$

Plasma	Typical parameters				Remark
	N_e [m^{-3}]	T_e [K]	m_{eff}/m	$\widetilde{N_\alpha}$[m^{-3}]	Degen.?
ionosphere (F-layer, h~300km)	10^{12}	$(3\text{-}5)\cdot10^{3}$	1	$2\cdot10^{27}$	N
stellar core	10^{32}	upto 10^{10}	1	$2\cdot10^{36}$	N
gas discharge	10^{21}	upto 10^{5}	1	$2\cdot10^{29}$	N
Solid state (SSP) – metals	10^{29}	10^{4}	~1	$5\cdot10^{27}$	Y
Solid state (SSP) – semiconductors	10^{20}-10^{24}	10^{2}	0.01	$5\cdot10^{21}$	N/Y

Quasi-neutrality

- Quasi-neutrality
  - → charge neutrality of a plasma overall
  - → while at smaller scales, the positive and negative charges making up the plasma, may give rise to charged regions and electric fields

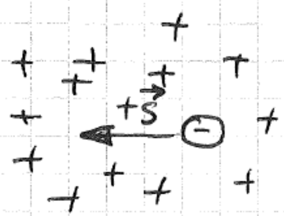
$$\sum_{\alpha} e_{\alpha} N_{\alpha} = 0$$

For single ionized plasma $N_e = N_i$

?? Time and space scale of charge separation??

Plasma (Langmuir) frequency ω_p

- Time scale of charge separation
- **Problem 1**: Derive a frequency of the homogeneous plasma oscillations due to a small displacement of the electrons w.r.t. ions



unconpensated charge density

$$\rho = \mathrm{div}(eN_e\vec{s}) = eN_e\,\mathrm{div}\,\vec{s}$$

Poisson eq.: $\mathrm{div}\,\vec{E} = \dfrac{\rho}{\varepsilon_0} = \dfrac{eN_e}{\varepsilon_0}\,\mathrm{div}\,\vec{s}$

$$\vec{E}(\vec{s}=0) = 0 \implies \vec{E} = \frac{eN_e}{\varepsilon_0}\vec{s}$$

Eq. of e-motion:

$$m\frac{d^2\vec{s}}{dt^2} = \vec{F} = -e\vec{E} = -\frac{e^2N_e}{\varepsilon_0}\vec{s}$$

$$\frac{d^2\vec{s}}{dt^2} + \underbrace{\frac{e^2N_e}{\varepsilon_0 m}}_{\omega^2}\vec{s} = 0$$

$$\omega_{pe} = \omega_{Le} = \sqrt{\frac{e^2N_e}{\varepsilon_0 m}} \approx \sqrt{3\cdot 10^3 N_e \frac{m}{m_e}}$$

Plasma (Langmuir) frequency ω_p

$$\omega_{pe} = \omega_{Le} = \sqrt{\frac{e^2 N_e}{\varepsilon_0 m}} \approx \sqrt{3 \cdot 10^3 N_e \frac{m}{m_e}}$$

Plasma	Typical parameters		
	N_e[m^{-3}]	$\omega_{pe}[s^{-1}]$	f_{pe}
ionosphere (F-layer, h~300km)	10^{12}	$5.5 \cdot 10^7$	~9 MHz
stellar core (high)	10^{32}	$5.5 \cdot 10^{17}$	90 PHz
laser produced (LPP→PITZ PWA)	10^{21}	$1.7 \cdot 10^{12}$	280 GHz
gas discharge	10^{18}	$5.5 \cdot 10^{10}$	~9 GHz
Solid state (SSP) – metals	10^{27}-10^{29}	$1.7 \cdot (10^{15}$-$10^{16})$	0.3-3 PHz
Solid state (SSP) – semiconductors	10^{20}-10^{24}	$5.5 \cdot (10^{12}$-$10^{14})$	(0.9-90)THz

Debye length r_D - 1

- Space scale of charge separation
- **Problem 2**: Find a potential of a test charged particle q immersed in a spatially homogeneous nonisothermal ($T_e \neq T_i$) plasma.

The charge $q \to \vec{E}$, polarizing the plasma

The Poisson equation for the potential Φ

$$\Delta \Phi = -\frac{\rho(\Phi)}{\varepsilon_0} - \frac{q}{\varepsilon_0}\delta(\vec{r})$$

The density of the induced charge:

$$\rho = \sum_\alpha e_\alpha \tilde{N}_\alpha(\Phi),$$

where $\tilde{N}_\alpha(\Phi)$ is the density of particles α, when the field $\vec{E} = -\vec{\nabla}\Phi$ is applied.

Debye length r_D - 2

The barometric formula:

$$\tilde{N}_\alpha(\Phi) = N_\alpha \cdot \exp\left(-\frac{e_\alpha \Phi}{k T_\alpha}\right) ,$$

N_α - unperturbed particle density.

Assuming $e_\alpha \Phi \ll k T_\alpha$

$$\tilde{N}_\alpha(\Phi) \cong N_\alpha \cdot \left(1 - \frac{e_\alpha \Phi}{k T_\alpha}\right)$$

$$\rho = \sum_\alpha \left(e_\alpha N_\alpha - \frac{e_\alpha^2 N_\alpha}{k T_\alpha} \Phi\right) = -\left(\sum_\alpha \frac{e_\alpha^2 N_\alpha}{k T_\alpha}\right) \Phi$$

where $\sum_\alpha e_\alpha N_\alpha = 0$ used

Debye length r_D - 3

$$\Delta\Phi \simeq \left(\sum_\alpha \frac{e_\alpha^2 N_\alpha}{\varepsilon_0 k T_\alpha}\right)\Phi - \frac{q}{\varepsilon_0}\delta(\vec{r})$$

Applying the Fourier transformation

$$\Phi(\vec{r}) = \int \Phi(\vec{k}) \cdot e^{i\vec{k}\vec{r}} d\vec{k}$$

$$\delta(\vec{r}) = \frac{1}{(2\pi)^3}\int e^{i\vec{k}\vec{r}} d\vec{k}$$

Denoting $\dfrac{1}{r_D^2} = \sum_\alpha \dfrac{e_\alpha^2 N_\alpha}{\varepsilon_0 k T_\alpha} = \dfrac{1}{r_{De}^2} + \dfrac{1}{r_{Di}^2}$

Debye length r_D - 4

$$\Delta\Phi = \left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}\right) \int \Phi(\vec{k})\, e^{ik_x x + ik_y y + ik_z z}\, dk_x dk_y dk_z =$$

$$= \int \left(-k_x^2 - k_y^2 - k_z^2\right) \Phi(\vec{k})\, e^{i\vec{k}\vec{r}}\, d\vec{k} =$$

$$= -\int k^2\, \Phi(\vec{k})\, e^{i\vec{k}\vec{r}}\, d\vec{k}$$

The Poisson equation takes a form:

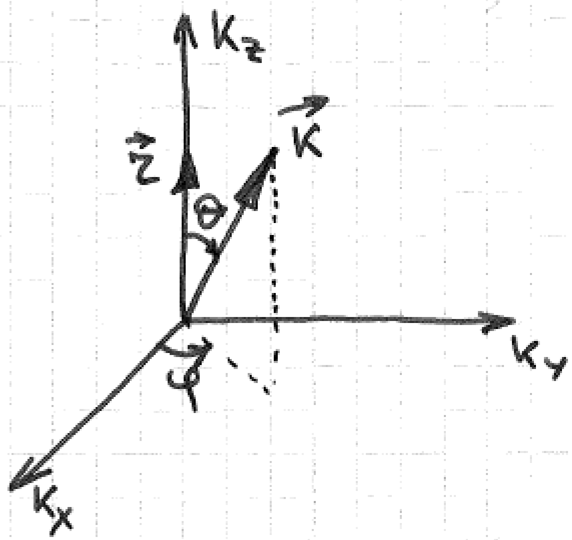
$$-k^2 \Phi(\vec{k}) = \frac{1}{r_D^2} \Phi(\vec{k}) - \frac{q}{\varepsilon_0} \frac{1}{(2\pi)^3}$$

$$\Phi(\vec{k}) = \frac{q}{\varepsilon_0 (2\pi)^3} \frac{1}{k^2 + r_D^{-2}}$$

Debye length r_D - 5

$$\Phi(\vec{r}) = \frac{q}{\varepsilon_0 \cdot (2\pi)^3} \int \frac{e^{i\vec{k}\vec{r}}}{k^2 + \frac{1}{r_D^2}} d\vec{k}$$

In spherical $\vec{k}$-coordinates



$$\vec{k} \cdot \vec{r} = k \cdot r \cdot \cos\Theta$$

$$d\vec{k} = k^2 \sin\Theta \, dk \, d\Theta \, d\varphi$$

$$0 \leq k < \infty$$
$$0 \leq \varphi \leq 2\pi$$
$$0 \leq \Theta \leq \pi$$

$$\int d\vec{k} \Rightarrow \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\Theta \, d\Theta \int_0^{\infty} dk \cdot \ldots$$

Debye length r_D - 6

$$\Phi(\vec{r}) = \frac{q}{\varepsilon_0 \cdot (2\pi)^3} \int \frac{e^{i\vec{k}\vec{r}}}{k^2 + \frac{1}{r_D^2}} \, d\vec{k}$$

$$\Phi(\vec{r}) = \frac{q}{\varepsilon_0 \cdot (2\pi)^3} \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta d\theta \int_0^{\infty} dk \, \frac{e^{ikr\cos\theta} \cdot k^2}{k^2 + \frac{1}{r_D^2}} =$$

$$= \frac{q}{\varepsilon_0 \cdot (2\pi)^2} \int_0^{\infty} \frac{k^2 dk}{k^2 + \frac{1}{r_D^2}} \int_0^{\pi} e^{ikr\cos\theta} \cdot \sin\theta \cdot d\theta$$

$$\int_0^{\pi} e^{ikr\cdot\cos\theta} \cdot \sin\theta \cdot d\theta = -\int_1^{-1} e^{ikr\cdot\cos\theta} \, d(\cos\theta) =$$

$$= -\frac{1}{ikr} e^{ikr\cdot\cos\theta} \Big|_{\theta=0}^{\theta=\pi} = -\frac{1}{ikr}\left(e^{-ikr} - e^{ikr}\right)$$

Debye length r_D - 7

$$\Phi(\vec{r}) = \frac{q}{\varepsilon_0 \cdot (2\pi)^2} \int_0^\infty \frac{k^2 dk}{k^2 + \frac{1}{r_D^2}} \cdot \left(\frac{+1}{ikr}\right)\left(e^{ikr} - e^{-ikr}\right) =$$

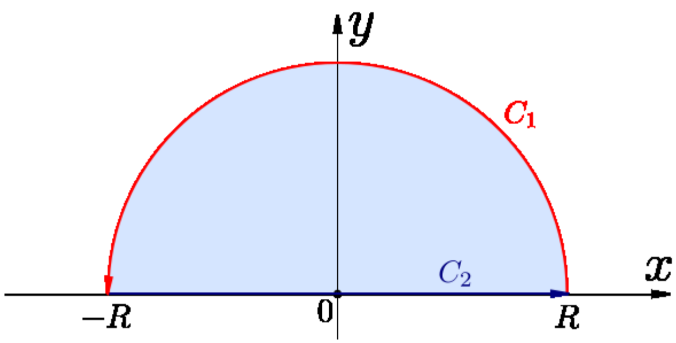
$$= \frac{q}{4\pi\varepsilon_0 r} \cdot \frac{1}{i\pi} \int_0^\infty \frac{k\,dk}{k^2 + \frac{1}{r_D^2}} \left(e^{ikr} - e^{-ikr}\right)$$

$$\int_0^\infty \frac{k\,dk}{k^2 + \frac{1}{r_D^2}} e^{-ikr} = \begin{bmatrix} k_1 = -k \\ k\,dk = k_1 dk_1 \end{bmatrix} = \int_0^{-\infty} \frac{k_1 dk_1}{k_1^2 + \frac{1}{r_D^2}} e^{ik_1 r} =$$

$$= -\int_{-\infty}^{0} \frac{k\,dk}{k^2 + \frac{1}{r_D^2}} e^{ikr}$$

$$\boxed{\Phi(\vec{r}) = \frac{q}{4\pi\varepsilon_0 r} \cdot \frac{1}{i\pi} \int_{-\infty}^{\infty} \frac{k\,dk}{k^2 + \frac{1}{r_D^2}} e^{ikr}}$$

M: Jordan's lemma



Consider a complex-valued, continuous function *f*, defined on a semicircular contour

$$C_R = \{z : z = Re^{i\theta}, \theta \in [0, \pi]\}$$

of radius *R* > 0 lying in the upper half-plane, centred at the origin. If the function *f* is of the form

$$f(z) = e^{iaz} g(z)\,, \quad z \in C_R,$$

with a parameter *a* > 0, then Jordan's lemma states the following upper bound for the contour integral:

$$\left| \int_{C_R} f(z)\, dz \right| \le \frac{\pi}{a} \max_{\theta \in [0,\pi]} \left| g\left(Re^{i\theta}\right) \right| .$$

Application of Jordan's lemma

Jordan's lemma yields a simple way to calculate the integral along the real axis of functions $f(z) = e^{iaz} g(z)$ holomorphic on the upper half-plane and continuous on the closed upper half-plane, except possibly at a finite number of non-real points z_1, z_2, ..., z_n. Consider the closed contour *C*, which is the concatenation of the paths C_1 and C_2 shown in the picture. By definition,

$$\oint_C f(z)\, dz = \int_{C_1} f(z)\, dz + \int_{C_2} f(z)\, dz\,.$$

Since on C_2 the variable *z* is real, the second integral is real:

$$\int_{C_2} f(z)\, dz = \int_{-R}^{R} f(x)\, dx\,.$$

The left-hand side may be computed using the residue theorem to get, for all *R* larger than the maximum of $|z_1|$, $|z_2|$, ..., $|z_n|$,

$$\oint_C f(z)\, dz = 2\pi i \sum_{k=1}^{n} \operatorname{Res}(f, z_k)\,,$$

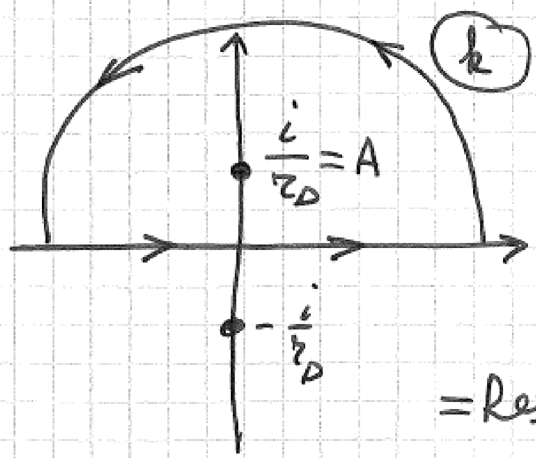
$$\boxed{\int_{-\infty}^{\infty} e^{iax} g(x)\, dx = 2\pi i \sum_{k=1}^{n} Res(e^{iaz} g(z), z_k)}$$

Debye length r_D - 8

$$\Phi(\vec{r}) = \frac{q}{4\pi\varepsilon_0 r}\cdot\frac{1}{i\pi}\int_{-\infty}^{\infty}\frac{k\,dk}{k^2+\frac{1}{r_D^2}}\,e^{ikr}$$

$$\int_{-\infty}^{\infty} e^{iax}g(x)\,dx = 2\pi i\sum_{k=1}^{n} Res\left(e^{iaz}g(z), z_k\right)$$

$$\int_{-\infty}^{\infty}\frac{k e^{ikr}}{k^2+\frac{1}{r_D^2}}\,dk = \left[\text{Jordan's lemma}\right] = 2\pi i\, Res\left\{\frac{k e^{ikr}}{k^2+\frac{1}{r_D^2}}\right\}$$



$r > 0 \rightarrow$ upper contour

$(\mathrm{Im}\,k > 0)$

$$Res\left\{\frac{z e^{izr}}{z^2+A^2}\right\} = \left[\begin{matrix} k \rightarrow z \\ \frac{1}{r_D} \rightarrow A \end{matrix}\right] =$$

$$= Res\left\{\frac{\frac{z e^{izr}}{z+iA}}{z-iA}\right\} = \left.\frac{z e^{izr}}{z+iA}\right|_{z=iA} =$$

$$= \frac{iA\cdot e^{-rA}}{iA+iA} = \frac{1}{2}e^{-rA} \rightarrow \frac{1}{2}e^{-\frac{r}{r_D}}$$

Debye length r_D - 9

$$\boxed{\Phi(\vec{r}) = \frac{q}{4\pi\varepsilon_0 r} \cdot \frac{1}{i\pi} \int_{-\infty}^{\infty} \frac{k\,dk}{k^2 + \frac{1}{r_D^2}} e^{ikr}}$$

$$\frac{1}{2} e^{-\frac{r}{r_D}}$$

$$\int_{-\infty}^{\infty} \frac{k e^{ikr}}{k^2 + \frac{1}{r_D^2}} dk = 2\pi i \,\mathrm{Res}\left\{ \frac{k e^{ikr}}{k^2 + \frac{1}{r_D^2}} \right\}$$

$$\Phi(r) = \frac{q}{4\pi\varepsilon_0 r} \cdot \frac{1}{i\pi} \cdot 2\pi i \cdot \frac{1}{2} e^{-\frac{r}{r_D}}$$

$$\boxed{\Phi(r) = \frac{q}{4\pi\varepsilon_0 r} e^{-\frac{r}{r_D}}}$$

Debye length r_D

$$\Phi(r) = \frac{q}{4\pi\varepsilon_0 r} e^{-\frac{r}{r_D}}$$

$$\frac{1}{r_D{}^2} = \sum_\alpha \frac{e_\alpha{}^2 N_\alpha}{\varepsilon_0 k T_\alpha} = \frac{1}{r_{De}{}^2} + \frac{1}{r_{Di}{}^2}$$

$$r_{De} = \sqrt{\frac{\varepsilon_0 k T_e}{e^2 N_e}} \approx 69 \sqrt{\frac{T_e[K]}{N_e[m^{-3}]}}$$

Plasma	Typical parameters		
	$\boldsymbol{N_e}$ [m^{-3}]	$\boldsymbol{T_e}$ $[\boldsymbol{K}]$	r_{De}[m]
ionosphere (F-layer, h~300km)	10^{12}	$(3\text{-}5)\cdot 10^3$	$4\text{-}5\cdot 10^{-3}$
gas discharge	10^{18}-10^{21}	upto 10^5	$2\cdot 10^{-5}$ $7\cdot 10^{-7}$
LPP plasma	10^{21}	$2\cdot 10^4$ (~2eV)	~$3\cdot 10^{-7}$
Solid state (SSP) – metals	10^{28} -10^{29}	$\mathcal{E}_{Fe} = (1-5)eV$	10^{-9} -10^{-8}

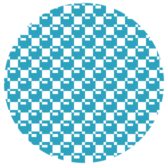
$$r_{De} = \sqrt{\frac{6\varepsilon_0 \mathcal{E}_{Fe}}{e^2 N_e}}$$

Gas approximation, plasma parameter

- Gas approximation: the average potential energy of particles is smaller than their average kinetic energy → weak interaction of particles

- For the Coulomb interaction: $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\langle r\rangle} \sim \frac{e^2 N^{1/3}}{4\pi\varepsilon_0} \ll kT$

- **Plasma parameter**: $\eta = \frac{e^2 N^{1/3}}{4\pi\varepsilon_0 kT} \sim \left(\frac{\langle r\rangle}{r_{De}}\right)^2 \ll 1$



- Plasma parameter, if degenerate: $\eta = \frac{e^2 N^{1/3}}{4\pi\varepsilon_0 \mathcal{E}_{F\alpha}} \sim \left(\frac{\hbar\omega_{pe}}{\mathcal{E}_{Fe}}\right)^2 \sim \left(\frac{\langle r\rangle}{r_{De}}\right)^2 \ll 1$

Plasma	Typical parameters		
	N_e [m^{-3}]	T_e $[K]$	η
ionosphere (F-layer, h~300km)	10^{12}	$3\cdot10^{3}$	$6\cdot10^{-5}$
gas discharge	10^{21}	upto 10^{5}	$2\cdot10^{-3}$
Solid state (SSP) – metals	10^{28} -10^{29}	$\mathcal{E}_{Fe} = (1-5)eV$	2-1

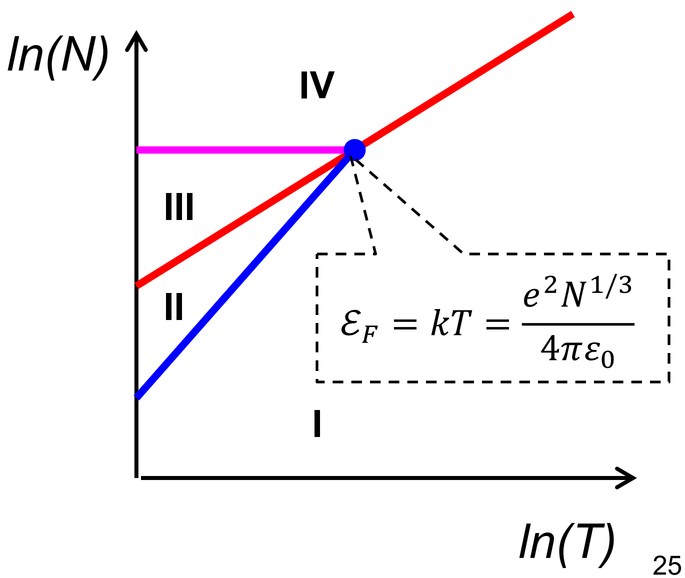
Plasma classification

Problem 3: Plot N(T) logarithmic diagram for degenerate/nondegenerate gas/fluid plasma

- The degeneracy condition $\mathcal{E}_F = \frac{(3\pi^2)^{2/3}\hbar^2 N^{2/3}}{2m} = kT$ → line 1
- Gas approximation for the nondegenerate plasma $\eta = \frac{e^2 N^{1/3}}{4\pi\varepsilon_0 kT} < 1$ → line 2
- Gas approximation for the degenerate plasma

$$\eta = \frac{e^2 N^{1/3}}{4\pi\varepsilon_0 \mathcal{E}_F} = \frac{2m\, e^2}{4\pi\varepsilon_0 (3\pi^2)^{2/3}\hbar^2 N^{1/3}} < 1$$ → line 3

- Regions:
  - I → nondegenerate gas
  - II → nondegenerate (classical) fluid
  - III → degenerate (quantum) fluid
  - IV → degenerate (quantum) gas



Summary

- Plasma:
  - gas of ionized particles
  - $N_{e,i,n}, r, m_{eff}$
  - degenerate/ nondegenerate
  - quasi-neutrality
  - plasma frequency
  - Debye length: Debye screening of a test particle
  - Gas approximation, plasma parameter

- Next: Tensor of complex dielectric permittivity
  - for isotropic medium

$$\varepsilon_{ij}(\omega,\vec{\boldsymbol{k}}) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\varepsilon^{tr}(\omega,k\ ) + \frac{k_i k_j}{k^2}\varepsilon^{l}(\omega,k\ )$$