# Energy of the electromagnetic field in the dispersive medium

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L#5

# Discussed last time…

- Properties of the complex dielectric permittivity
  - Simplest model: $\boldsymbol{\varepsilon^{tr,l}} \Rightarrow \left(\mathbf{1} - \frac{\boldsymbol{\omega_p}^2}{\boldsymbol{\omega}^2}\right)$
- Fields of particles in the dispersive media:
  - Static E-field of a charge q:

$$\Phi(\vec{r}) = \frac{q}{\varepsilon_0 (2\pi)^3} \int d\boldsymbol{k} \, \frac{e^{i\vec{k}(\vec{r}-\vec{r}_0)}}{k_i k_j \varepsilon_{ij}(0,\vec{k})}$$

  - Static B-field of the linear current filament

$$\vec{B}(\omega,\vec{k}) = \frac{i}{\varepsilon_0 c^2} \frac{[\vec{k} \times \vec{j}_0(\vec{k})]}{k^2 - \frac{\omega^2}{c^2} \varepsilon^{tr}(\omega,k)}$$

  - E-field of the fast particle → energy losses

$$E_i(\omega,\vec{k}) = -\frac{i\omega}{\varepsilon_0 k^2} \left\{ \frac{k_i k_j}{\omega^2 \varepsilon^l(\omega,k)} - \frac{k^2 \delta_{ij} - k_i k_j}{k^2 c^2 - \omega^2 \varepsilon^{tr}(\omega,k)} \right\} j_{oj}(\omega,\vec{k})$$

# Energy of the EMF in the medium

$$[\nabla \times \vec{B}] = \frac{1}{c^2\varepsilon_0}\frac{\partial \vec{D}}{\partial t} + \frac{\vec{j}_0}{c^2\varepsilon_0}$$

$$[\nabla \times \vec{E}] = -\frac{\partial \vec{B}}{\partial t}$$

$\ldots \cdot \vec{E})c^2\varepsilon_0$

"-"

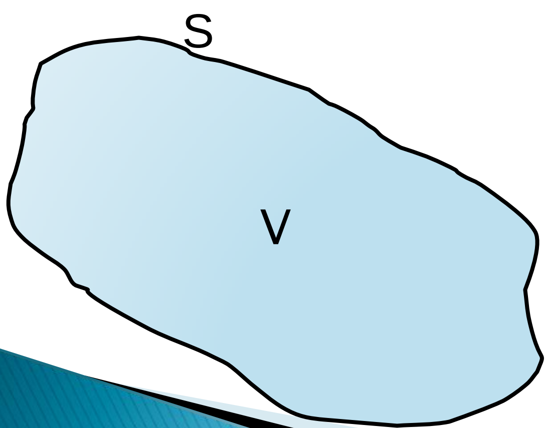
$\ldots \cdot \vec{B})c^2\varepsilon_0$

$$\vec{E}\frac{\partial \vec{D}}{\partial t} + c^2\varepsilon_0\vec{B}\frac{\partial \vec{B}}{\partial t} = c^2\varepsilon_0[\nabla \times \vec{B}]\cdot\vec{E} - c^2\varepsilon_0[\nabla \times \vec{E}]\cdot\vec{B} - \vec{j}_0 \cdot \overrightarrow{E}$$

$$\vec{E}\frac{\partial \vec{D}}{\partial t} + c^2\varepsilon_0\vec{B}\frac{\partial \vec{B}}{\partial t} = -c^2\varepsilon_0\left(\nabla\cdot[\vec{E}\times\vec{B}]\right) - \vec{j}_0\cdot\vec{E}$$

$\int$

$$\int_V d\vec{\boldsymbol{r}}\left(\vec{E}\frac{\partial \vec{D}}{\partial t} + c^2\varepsilon_0\vec{B}\frac{\partial \vec{B}}{\partial t}\right) = -c^2\varepsilon_0\oint_S dS\,[\vec{E}\times\vec{B}] - \int_V d\vec{\boldsymbol{r}}\,(\vec{j}_0\cdot\vec{E})$$




$V \to \infty$

$W$ – energy of EMF

$A$ – work of EMF against the external sources

$$\frac{dW}{dt} = 0 - \frac{dA}{dt}$$

# Energy of the EMF: plane waves

$$\frac{dW}{dt} = \int_V d\vec{\boldsymbol{r}} \left( \vec{E}\frac{\partial \vec{D}}{\partial t} + c^2 \varepsilon_0 \vec{B}\frac{\partial \vec{B}}{\partial t} \right) = -c^2 \varepsilon_0 \oint_S dS \left[ \vec{E} \times \vec{B} \right] - \int_V d\vec{\boldsymbol{r}} \left( \vec{j}_0 \cdot \vec{E} \right)$$

$$\vec{A}(t,\vec{r}) \propto \frac{1}{2}\left[ \vec{A}(\omega,\vec{k}) e^{-i\omega t + i\vec{k}\vec{r}} + \vec{A}^*(\omega,\vec{k}) e^{i\omega t - i\vec{k}\vec{r}} \right]$$

$$\langle \dots \rangle_t$$

Average energy, dissipated in the medium per time unit:

$$\left\langle \frac{dW}{dt} \right\rangle = \frac{i\omega}{4} \int_V d\vec{\boldsymbol{r}} \left( \vec{E}(\omega,\vec{k}) \cdot \vec{D}^*(\omega,\vec{k}) - \vec{E}^*(\omega,\vec{k}) \cdot \vec{D}(\omega,\vec{k}) \right)$$

$$= \frac{i\omega}{4} V \left( \vec{E}(\omega,\vec{k}) \cdot \vec{D}^*(\omega,\vec{k}) - \vec{E}^*(\omega,\vec{k}) \cdot \vec{D}(\omega,\vec{k}) \right) =$$

$$= \frac{i\omega}{4} \varepsilon_0 V \cdot \left( {\varepsilon_{ij}}^*(\omega,\vec{k}) - \varepsilon_{ji}(\omega,\vec{k}) \right) \cdot E_i(\omega,\vec{k}) \cdot {E_j}^*(\omega,\vec{k})$$

**anti-Hermitian part of dielectric permittivity tensor**

# Energy of the EMF: plane waves

Homogeneous isotropic medium: $\varepsilon_{ij}(\omega,\boldsymbol{\vec{k}}) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\varepsilon^{tr}(\omega,k\,) + \frac{k_i k_j}{k^2}\varepsilon^{l}(\omega,k\,)$

$$\left\langle \frac{dW}{dt} \right\rangle = \frac{i\omega}{4}\varepsilon_0 V \cdot \left(\varepsilon_{ij}{}^{*}(\omega,\vec{k}) - \varepsilon_{ji}(\omega,\vec{k})\right) \cdot E_i(\omega,\vec{k}) \cdot E_j{}^{*}(\omega,\vec{k})$$

$$\frac{k_i k_j}{k^2} E_i E_j^{*} \cdot \left(\varepsilon^{l*} - \varepsilon^{l}\right) = \frac{(\vec{k}\vec{E})\cdot(\vec{k}\vec{E}^{*})}{k^2}\cdot\left(\varepsilon^{l*} - \varepsilon^{l}\right)$$

$$\varepsilon^{l*} - \varepsilon^{l} = Re\,\varepsilon^{l} - i\,Im\,\varepsilon^{l} - Re\,\varepsilon^{l} - i\,Im\,\varepsilon^{l} = -2i\,Im\,\varepsilon^{l}$$

$$(\vec{k}\vec{E})\cdot(\vec{k}\vec{E}^{*}) = \left[\vec{k}\cdot\left(Re\,\vec{E} + i\,Im\,\vec{E}\right)\right]\cdot\left[\vec{k}\cdot\left(Re\,\vec{E} - i\,Im\,\vec{E}\right)\right] =$$

$$= \left(\vec{k}\cdot Re\,\vec{E}\right)^2 + \left(\vec{k}\cdot Im\,\vec{E}\right)^2 = \left(\vec{k}\cdot\vec{E}\right)^2 = \left|\vec{k}\cdot\vec{E}\right|^2$$

$$\left\langle \frac{dW}{dt} \right\rangle = \frac{2\omega\varepsilon_0}{4} V \left\{ Im\,\varepsilon^{l}\cdot\frac{\left|(\vec{k}\vec{E})\right|^2}{k^2} + Im\,\varepsilon^{tr}\cdot\frac{\left|[\vec{k}\times\vec{E}]\right|^2}{k^2} \right\}$$

# Energy of the EMF: plane waves

Homogeneous isotropic medium: $\varepsilon_{ij}(\omega,\vec{\boldsymbol{k}}) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\varepsilon^{tr}(\omega,k) + \frac{k_i k_j}{k^2}\varepsilon^{l}(\omega,k)$

$$\left\langle \frac{dW}{dt} \right\rangle = \frac{i\omega}{4}\varepsilon_0 V \cdot \left(\varepsilon_{ij}{}^{*}(\omega,\vec{k}) - \varepsilon_{ji}(\omega,\vec{k})\right) \cdot E_i(\omega,\vec{k}) \cdot E_j{}^{*}(\omega,\vec{k})$$

$$\left\langle \frac{dW}{dt} \right\rangle = \frac{\omega\varepsilon_0 V}{2} \cdot \left( Im[\varepsilon^{l}(\omega,k)] \cdot \left|\vec{E}^{l}\right|^2 + Im[\varepsilon^{tr}(\omega,k)] \cdot \left|\vec{E}^{tr}\right|^2 \right) = Q$$

➔Heat delivered per unit volume

**absorption of || waves**

**absorption of ⊥ waves**

$$Q > 0 \Leftrightarrow Im\,\varepsilon^{l,tr}(\omega,k) > 0$$

**For medium in thermodynamic equilibrium!**

# Energy of the EMF: nonmonochromatic waves

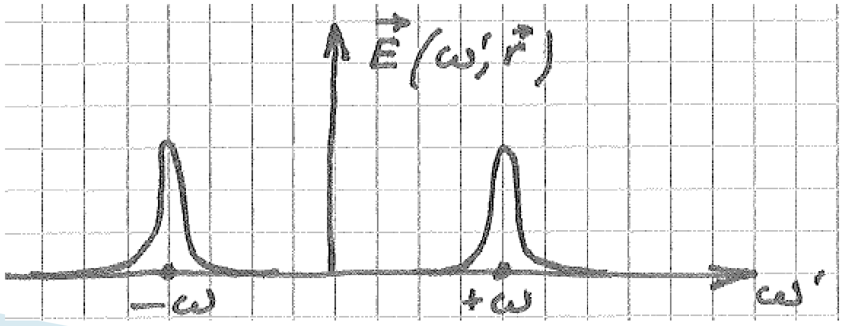
$$\vec{E}(t,\vec{r}) \propto \frac{1}{2}\left[\vec{E}(\omega,\vec{k},t)e^{-i\omega t+i\vec{k}\vec{r}} + \vec{E}^*(\omega,\vec{k},t)e^{i\omega t-i\vec{k}\vec{r}}\right]$$

Slowly varying in time

$$\vec{E}(t,\vec{r}) = \int_{-\infty}^{\infty} d\omega' \, \vec{E}(\omega';\vec{r})\, e^{-i\omega' t}$$

$$\vec{E}(\omega,\vec{k},t) = \int_{0}^{\infty} d\omega' \cdot \vec{E}(\omega';\vec{k}) \cdot e^{i(\omega-\omega')t}$$

$$\vec{E}^*(\omega,\vec{k},t) = \int_{-\infty}^{0} d\omega' \cdot \vec{E}^*(-\omega';\vec{k}) \cdot e^{-i(\omega+\omega')t}$$



# Energy of the EMF: nonmonochromatic waves

$$\frac{\partial D_i}{\partial t} = -i\omega\varepsilon_0\varepsilon_{ij}(\omega)E_j = \varepsilon_0\frac{\partial}{\partial t}\varepsilon_{ij}(\omega)E_j \rightarrow \varepsilon_0\hat{f}\vec{E}$$

$$\hat{f} = \frac{\partial}{\partial t}\hat{\varepsilon} = \hat{f}(\omega) = \hat{f}(\omega_0 + \alpha)$$

$$\alpha \ll \omega_0$$

$$\hat{f}(\omega) = -i\omega\varepsilon(\omega)$$

$$\alpha = \omega - \omega_0 \ll \omega_0$$

$$\hat{f}(\omega_0 + \alpha) \approx \hat{f}(\omega_0) + \alpha\frac{\partial \hat{f}}{\partial \omega}\bigg|_{\omega = \omega_0}$$

$$-i\alpha = \frac{\partial}{\partial t} \;\Rightarrow\; \alpha = i\frac{\partial}{\partial t}$$

-slowly varying

$$\frac{\partial D_i}{\partial t} \sim -i\omega\varepsilon_0\varepsilon_{ij}(\omega)\cdot E_j(\omega, \vec{k}, t) + \frac{\partial(\omega\varepsilon_{ij}(\omega))}{\partial\omega}\varepsilon_0\frac{\partial E(\omega, \vec{k}, t)}{\partial t}$$

# Energy of the EMF: nonmonochromatic waves

$$\vec{A}(t,\vec{r}) \propto \frac{1}{2}\left[\vec{A}(\omega,\vec{k},t)e^{-i\omega t+i\vec{k}\vec{r}} + \vec{A}^*(\omega,\vec{k},t)e^{i\omega t-i\vec{k}\vec{r}}\right]$$

$$4\left\langle \vec{E}\frac{\partial \vec{D}}{\partial t}\right\rangle = i\omega\varepsilon_0\varepsilon_{ji}^* E_j E_i^* - i\omega\varepsilon_0\varepsilon_{ij} E_j E_i^* + \varepsilon_0 E_j \frac{\partial \omega\varepsilon_{ji}^*}{\partial\omega}\frac{\partial E_i^*}{\partial t} + \varepsilon_0 E_i^* \frac{\partial \omega\varepsilon_{ij}}{\partial\omega}\frac{\partial E_j}{\partial t}$$

For the medium w/o absorption: $\varepsilon_{ji}^* = \varepsilon_{ij}$

$$\frac{1}{V}\left\langle \frac{dW}{dt}\right\rangle = \frac{1}{4}\frac{d}{dt}\left\{\varepsilon_0 \frac{\partial \omega\varepsilon_{ij}}{\partial\omega} E_i^* E_j + c^2\varepsilon_0 B_i^* B_i\right\}$$

$$\frac{U}{V} = \frac{\varepsilon_0}{4}\frac{\partial(\omega\varepsilon_{ij})}{\partial\omega}E_i^*(\omega,\vec{k},t)E_j(\omega,\vec{k},t) + \frac{\varepsilon_0 c^2}{4}B_i^*(\omega,\vec{k},t)B_i(\omega,\vec{k},t)$$

Average energy of the EMF in the medium w/o absorption (Brillouin, 1921)

# Energy of the EMF: nonmonochromatic waves

$$\frac{U}{V} = \frac{\varepsilon_0}{4}\frac{\partial(\omega\varepsilon_{ij})}{\partial\omega}E_i^{\,*}(\omega,\vec{k},t)E_j(\omega,\vec{k},t) + \frac{\varepsilon_0 c^2}{4}B_i^{\,*}(\omega,\vec{k},t)B_i(\omega,\vec{k},t)$$

Homogeneous isotropic medium: $\varepsilon_{ij}(\omega,\boldsymbol{\vec{k}}) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\varepsilon^{tr}(\omega,k\ ) + \frac{k_i k_j}{k^2}\varepsilon^{l}(\omega,k\ )$

$$\frac{\partial\,\omega\varepsilon_{ij}}{\partial\omega}E_i^{*}E_j = \frac{\partial\,\omega\varepsilon^{tr}}{\partial\omega}\cdot|\vec{E}^{tr}|^2 + \frac{\partial\,\omega\varepsilon^{l}}{\partial\omega}|\vec{E}^{l}|^2$$

$$\vec{B} = \frac{1}{\omega}[\vec{k}\,\vec{E}] \Longrightarrow B_i^{*}B_i = \frac{k^2}{\omega^2}|\vec{E}^{tr}|^2$$

$$\frac{U}{V} = \frac{\varepsilon_0}{4}\left\{|\vec{E}^{l}|^2\frac{\partial\,\omega\varepsilon^{l}}{\partial\omega} + |\vec{E}^{tr}|^2\frac{\partial}{\partial\omega}\left[\omega\left(\varepsilon^{tr} - \frac{k^2c^2}{\omega^2}\right)\right]\right\}$$

# Average force in inhomogeneous plasma

Average force affecting the plasma in the inhomogeneous high-frequency field - ?

$\varepsilon_{ij}(\omega, \vec{\boldsymbol{k}}) = \varepsilon_{ij}(\omega, \vec{\boldsymbol{k}}, n_{\boldsymbol{\alpha}})$ – function of the plasma density $n_\alpha$

Density variation → variation in $\varepsilon_{ij}(\omega, \vec{\boldsymbol{k}})$ → EMF energy variation

$$\delta\varepsilon_{ij} = \frac{\partial \varepsilon_{ij}}{\partial n_\alpha}\delta n_\alpha$$

$$\frac{\overline{\delta W}}{V} = \frac{1}{2}\overline{\vec{E}\cdot\delta\vec{D}} = \frac{\varepsilon_0}{8}\left\{E_i E_j{}^* \delta\varepsilon_{ij}{}^* + E_j E_i{}^* \delta\varepsilon_{ij}\right\}$$

If $\varepsilon_{ij}(\omega, \vec{\boldsymbol{k}})$ →Hermitian ➔ $\delta\varepsilon_{ij}{}^* = \delta\varepsilon_{ji}$

$$\frac{\overline{\delta W}}{V} = \frac{\varepsilon_0}{4}\delta\varepsilon_{ij}E_i{}^* E_j = \frac{\varepsilon_0}{4}\frac{\partial \varepsilon_{ij}}{\partial n_\alpha}\delta n_\alpha E_i{}^* E_j$$

# Average force in inhomogeneous plasma

$$\frac{\overline{\delta W}}{V} = \frac{\varepsilon_0}{4}\delta\varepsilon_{ij}E_i{}^*E_j = \frac{\varepsilon_0}{4}\frac{\partial\varepsilon_{ij}}{\partial n_\alpha}\delta n_\alpha E_i{}^*E_j$$

if $\delta n_\alpha = 1$ → $U_{pot} = -\frac{\overline{\delta W}}{V}$ - potential energy of a particle in the EM wave $\vec{E}$

Average force acting on a particle → $\vec{F}_{av} = \nabla U_{pot} = \frac{\varepsilon_0}{4}\frac{\partial\varepsilon_{ij}}{\partial n}\nabla\left(E_i{}^*E_j\right)$

For isotropic plasma → $\vec{F}_{av} = \frac{\varepsilon_0}{4}\left\{\frac{\partial\varepsilon^l}{\partial n}\nabla\left|\vec{E}^l\right|^2 + \frac{\partial\varepsilon^{tr}}{\partial n}\nabla\left|\vec{E}^{tr}\right|^2\right\}$

$\frac{\partial\varepsilon^{l,tr}}{\partial n}$

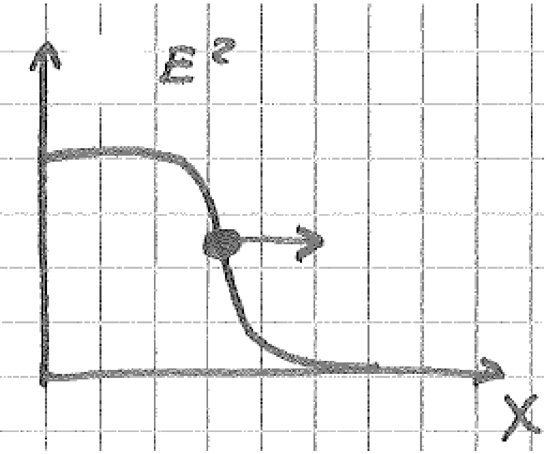
- >0 → attraction to the high field region
- <0 → repulsion from the high field region

# Miller force

$$\vec{F}_{av} = \frac{\varepsilon_0}{4}\left\{\frac{\partial \varepsilon^l}{\partial n}\nabla\left|\vec{E}^l\right|^2 + \frac{\partial \varepsilon^{tr}}{\partial n}\nabla\left|\vec{E}^{tr}\right|^2\right\}$$

$$\varepsilon^{tr,l} \Rightarrow \left(1 - \frac{{\omega_p}^2}{\omega^2}\right)$$

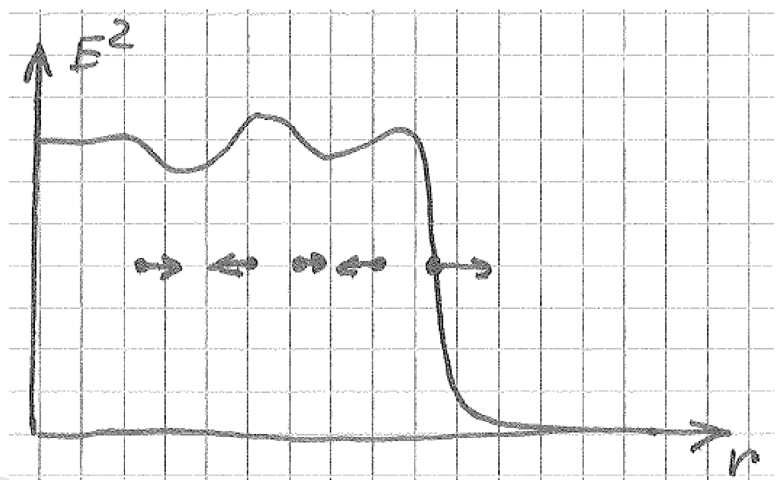
$${\omega_p}^2 = \frac{e^2 n}{\varepsilon_0 m}$$




$$\frac{\partial \varepsilon^{l,tr}}{\partial n} = -\frac{e^2}{\varepsilon_0 m \omega^2}$$

**Ponderomotive** (Miller) force:

$$\vec{F}_M = -\frac{e^2}{4m\omega^2}\nabla|\vec{E}|^2$$



# Initial and boundary value problems

Electromagnetic waves in medium → dispersion equation (eigenvalue problem):

$$\Lambda(\omega,\vec{k}) = \left| k^2\delta_{ij} - k_i k_j - \frac{\omega^2}{c^2}\varepsilon_{ij}(\omega,\vec{k}) \right| = 0$$

Initial Value Problem **(IVP)**:

$\omega_n(\vec{k})$ - eigen frequencies

➔at $t = 0$ perturbation by external source, then $(t > 0)$ it is switched off ➔ medium response?

$\vec{k} \rightarrow \omega_n(\vec{k})$ → $v_{ph}, v_g$

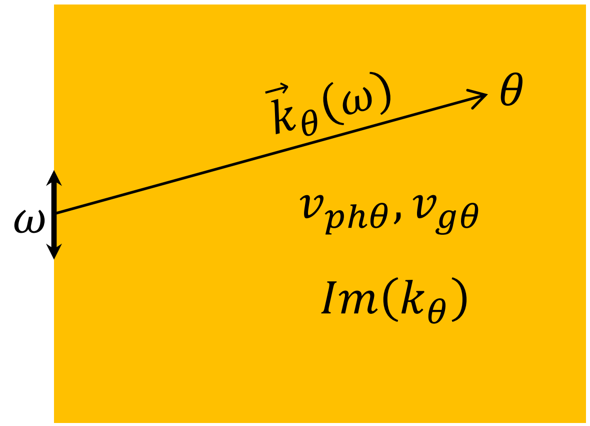
$\vec{k} \rightarrow \omega_n(\vec{k})$ → $\delta_n = Im(\omega_n)$

$\vec{k} \rightarrow \omega_n(\vec{k})$ → $\vec{E}(\vec{k})$ → initial spatial distribution

Boundary Value Problem **(BVP)**:

$\vec{k}_n(\omega)$ - eigen wave vectors

➔local perturbation (at $\vec{r} = 0$) - by external source ➔ propagation of the signal in the medium?



# Initial value problem

$$\Lambda(\omega,\vec{k}) = \left|k^2\delta_{ij} - k_i k_j - \frac{\omega^2}{c^2}\varepsilon_{ij}(\omega,\vec{k})\right| = 0$$

Assumptions:

- $Re\Lambda(\omega,\vec{k}) \gg Im\Lambda(\omega,\vec{k})$
- $\omega \rightarrow \omega + i\delta$

$$Re\Lambda(\omega,\vec{k}) + i\delta\frac{\partial Re\Lambda(\omega,\vec{k})}{\partial\omega} + i\,Im\Lambda(\omega,\vec{k}) - \delta\frac{\partial Im\Lambda(\omega,\vec{k})}{\partial\omega} = 0$$

=0

$$\boldsymbol{\delta = -\frac{Im\Lambda(\omega,\vec{k})}{\frac{\partial Re\Lambda(\omega,\vec{k})}{\partial\omega}}}$$

≤0 → damping decrement

>0 → instability increment (non-equilibrium medium)

Homogeneous isotropic medium: $\varepsilon_{ij}(\omega,\vec{\boldsymbol{k}}) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\varepsilon^{tr}(\omega,k) + \frac{k_i k_j}{k^2}\varepsilon^{l}(\omega,k)$

Longitudinal waves ⇒ $Re\,\varepsilon^l(\omega,k) = 0;\quad \delta^l(k) = -\frac{Im\,\varepsilon^l(\omega,k)}{\frac{\partial Re\,\varepsilon^l(\omega,k)}{\partial\omega}}$

Transverse waves ⇒ $k^2 = \frac{\omega^2}{c^2}Re\,\varepsilon^{tr}(\omega,k);\quad \delta^{tr}(k) = -\frac{Im\,\omega^2\varepsilon^{tr}(\omega,k)}{\frac{\partial}{\partial\omega}\omega^2 Re\,\varepsilon^l(\omega,k)}$

| |
|---|
| $\frac{\partial Re\,\varepsilon^l(\omega,k)}{\partial\omega} > 0$ |
| $\frac{\partial}{\partial\omega}\omega^2 Re\,\varepsilon^l(\omega,k) > 0$ |
| For equil. medium |

# Initial and boundary value problems

Initial Value Problem **(IVP)**:
$\omega_n(\vec{k})$ - eigen frequencies

Boundary Value Problem **(BVP)**:
$\vec{k}_n(\omega)$ - eigen wave vectors

$$\delta_n = Im(\omega_n) \quad \Longleftrightarrow ? \quad Im(k_\theta)$$

$$\delta = -\frac{Im\Lambda(\omega,\vec{k})}{\frac{\partial Re\Lambda(\omega,\vec{k})}{\partial\omega}}$$

$$Im(k_\theta) = -\frac{Im\Lambda(\omega,\vec{k})}{\frac{\partial Re\Lambda(\omega,\vec{k})}{\partial k_\theta}}$$

group velocity: $v_{g\theta} = \frac{\partial\omega}{\partial k_\theta} = -\frac{\frac{\partial Re\Lambda(\omega,\vec{k})}{\partial k_\theta}}{\frac{\partial Re\Lambda(\omega,\vec{k})}{\partial\omega}} = -\frac{\delta}{Im(k_\theta)}$

$$Im(k_\theta) = -\frac{\delta}{v_{g\theta}}$$

phase velocity: $\vec{v}_{ph} = \frac{\omega\vec{k}}{k^2}$

$(\vec{v}_{ph}\cdot\vec{v}_g)$
- >0 → forward wave with positive dispersion
- <0 → backward wave with negative dispersion

# Kramers-Kronig relations for $\varepsilon$

$$E_i(\omega,\vec{k}) = -\frac{i\omega}{\varepsilon_0 k^2}\left\{\frac{k_i k_j}{\omega^2\varepsilon^l(\omega,k)} - \frac{k^2\delta_{ij} - k_i k_j}{k^2c^2 - \omega^2\varepsilon^{tr}(\omega,k)}\right\} j_{oj}(\omega,\vec{k})$$

Electric field in the isotropic medium from external source (problem 4.3)

Medium response functions

$\vec{j}_0$ → casualty principle → $\vec{E}$

$\frac{1}{\varepsilon^l(\omega,k)}$ and $\frac{1}{k^2c^2-\omega^2\varepsilon^{tr}(\omega,k)}$ → analytical functions in the upper half of the complex $\omega$-plane

They should satisfy the Cauchy Theorem for an analytical function $\chi(\omega)$

$$0 = \oint_\Gamma \frac{\chi(\omega')d\omega'}{\omega'-\omega} = \mathcal{V}.\mathcal{P}.\int_{-\infty}^{\infty}\frac{\chi(\omega')d\omega'}{\omega'-\omega} - i\pi\,\chi(\omega)$$

$\chi(\omega) = \frac{1}{\varepsilon^l(\omega,k)} - 1$ ➔

$$\frac{1}{\varepsilon^l(\omega,k)} - 1 = \frac{1}{i\pi}\mathcal{V}.\mathcal{P}.\int_{-\infty}^{\infty}\frac{\frac{1}{\varepsilon^l(\omega',k)} - 1}{\omega'-\omega}d\omega'$$

$$\boldsymbol{Re\frac{1}{\varepsilon^l(\omega,k)} - 1 = \frac{1}{\pi}\mathcal{V}.\mathcal{P}.\int_{-\infty}^{\infty}\frac{Im\frac{1}{\varepsilon^l(\omega',k)}}{\omega'-\omega}d\omega'}$$

$$\boldsymbol{Im\frac{1}{\varepsilon^l(\omega,k)} - 1 = \frac{1}{\pi}\mathcal{V}.\mathcal{P}.\int_{-\infty}^{\infty}\frac{Re\frac{1}{\varepsilon^l(\omega',k)} - 1}{\omega'-\omega}d\omega'}$$

# Summary

- Energy of EMF in the medium
- Anti-Hermitian part of $\varepsilon_{ij}(\omega, \vec{k})$ → energy dissipation
- Plane wave in the isotropic medium:
  - $Im\ \varepsilon^{l,tr}(\omega, k)>0$ → for thermodynamic equilibrium
- Inhomogeneous plasma/wave:
  - nonmonochromatic waves → $\sim\frac{\partial(\omega\varepsilon_{ij})}{\partial\omega}$
  - ponderomotive (Miller) force $\sim\nabla\left|\vec{E}\right|^2$
- Initial and boundary problems
- Kramers-Kronig relations for $\varepsilon_{ij}(\omega, \vec{k})$

- Next: Plasma dynamics
  - Simplest model → single particle approximation
  - The hydrodynamic model
  - Kinetic equations with self-consistent field (Boltzmann-Landau, Fokker-Plank, Lenard-Balescu, BGK)