Plasma dynamics models

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L#6

Discussed last times…

- Energy of EMF in the medium:

$$\left\langle \frac{dW}{dt} \right\rangle = \frac{i\omega}{4}\varepsilon_0 V \cdot \left(\varepsilon_{ij}{}^*(\omega,\vec{k}) - \varepsilon_{ji}(\omega,\vec{k})\right) \cdot E_i(\omega,\vec{k}) \cdot E_j{}^*(\omega,\vec{k})$$

- Anti-Hermitian part of $\varepsilon_{ij}(\omega,\vec{k})$ → energy dissipation
- Plane wave in the isotropic medium:

$$\left\langle \frac{dW}{dt} \right\rangle = \frac{\omega\varepsilon_0 V}{2} \cdot \left(Im[\varepsilon^l(\omega,k)] \cdot \left|\vec{E}^l\right|^2 + Im[\varepsilon^{tr}(\omega,k)] \cdot \left|\vec{E}^{tr}\right|^2 \right)$$

  - $Im\,\varepsilon^{l,tr}(\omega,k)>0$ → for thermodynamic equilibrium
- Inhomogeneous plasma/wave:
  - nonmonochromatic waves → $\sim\frac{\partial(\omega\varepsilon_{ij})}{\partial\omega}$
  - ponderomotive (Miller) force $\sim\nabla\left|\vec{E}\right|^2$

- Initial and boundary problems

- Kramers-Kronig relations for $\varepsilon_{ij}(\omega,\vec{k})$

Simplest Plasma Models: the Model of Independent Particles

L4 ➔ Single particle approach = "mean" particle model

$$\frac{d\vec{v}_\alpha}{dt} = \frac{e_\alpha}{m_\alpha}\left(\vec{E} + \left[\vec{v}_\alpha \times \vec{B}\right]\right)$$

- isotropic
- ~~w/o collisions~~
- "independent" particles
- no thermal motion
- nonrelativistic case

$$\frac{d\vec{v}_e}{dt} = \frac{e}{m}\left(\vec{E} + \left[\vec{v}_e \times \vec{B}\right]\right) - \nu_{e0}\ \vec{v}_e - \nu_{ei}\left(\vec{v}_e - \vec{v}_i\right)$$

$$\frac{d\vec{v}_i}{dt} = -\frac{e}{M}\left(\vec{E} + \left[\vec{v}_e \times \vec{B}\right]\right) - \nu_{i0}\ \vec{v}_i - \nu_{ie}\cdot\left(\vec{v}_i - \vec{v}_e\right)$$

$\vec{v}_\alpha$ − particle velocity, particle type $\alpha = e, i, 0$

$\nu_{\alpha\beta}$ − effective collision frequency $\alpha\ \beta$ particles

$$m\nu_{ei} = M\nu_{ie}$$

NB: The model is also valid for the solid state plasma

$$\nu_e = \nu_{e0} + \nu_{ei} \sim 1/\tau_e$$

$$\nu_i = \nu_{i0} + \nu_{ie} \sim 1/\tau_{hole}$$

Simplest Plasma Models: the Model of Independent Particles

+Maxwell Equations:

$$\nabla\vec{E} = \frac{1}{\varepsilon_0}\sum_\alpha e_\alpha N_\alpha$$

$$\nabla\vec{B} = 0$$

$$[\nabla\times\vec{E}] = -\frac{\partial\vec{B}}{\partial t}$$

$$[\nabla\times\vec{B}] = \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} + \frac{1}{c^2\varepsilon_0}\sum_\alpha e_\alpha N_\alpha\,\vec{v}_\alpha$$

+Equation of continuity

$$\frac{\partial N_\alpha}{\partial t} + \nabla(N_\alpha\vec{v}_\alpha) = 0$$

main task → to find:

$$j_i(\omega,\vec{k}) = \sigma_{ij}(\omega,\vec{k})E_j(\omega,\vec{k})$$

$$\varepsilon_{ij}(\omega,\vec{k}) = \delta_{ij} + \frac{i}{\varepsilon_0\omega}\sigma_{ij}(\omega,\vec{k})$$

Simplest Plasma Models: the Model of Independent Particles

Linear approximation:

1. Fields and currents are small enough
2. No external sources
3. Weakly ionized (rear) non-relativistic plasma

$$\nu_e \approx \nu_{e0} \gg \nu_{ei}$$
$$\nu_i \approx \nu_{i0} \gg \nu_{ie}$$

$$-i\omega\vec{v}_e = \frac{e}{m}\left(\vec{E} + [\vec{v}_e \times \vec{B}]\right) - \nu_{e0}\,\vec{v}_e - \nu_{ei}\,(\vec{v}_e - \vec{v}_i)$$
$$-i\omega\vec{v}_i = -\frac{e}{M}\left(\vec{E} + [\vec{v}_e \times \vec{B}]\right) - \nu_{i0}\,\vec{v}_i - \nu_{ie}\cdot(\vec{v}_i - \vec{v}_e)$$

$$\vec{v}_e = \frac{ie}{m}\frac{\vec{E}}{\omega + i\nu_e}$$
$$\vec{v}_i = -\frac{ie}{M}\frac{\vec{E}}{\omega + i\nu_i}$$

Simplest Plasma Models: the Model of Independent Particles

$$\vec{v}_e = \frac{ie}{m}\frac{\vec{E}}{\omega + i\nu_e}$$

$$\vec{v}_i = -\frac{ie}{M}\frac{\vec{E}}{\omega + i\nu_i}$$

$$\vec{j} = \sum_\alpha e_\alpha N_\alpha \vec{v}_\alpha = \sum_\alpha e_\alpha N_\alpha \frac{ie_\alpha}{m_\alpha}\frac{\vec{E}}{\omega + i\nu_\alpha} = \sum_\alpha \frac{i{e_\alpha}^2 N_\alpha}{m_\alpha(\omega + i\nu_\alpha)}\vec{E} = \sigma\vec{E}$$

$$\sigma_{ij}(\omega, \vec{k}) = \delta_{ij}\sigma$$

$$\sigma = \sum_\alpha \frac{i{e_\alpha}^2 N_\alpha}{m_\alpha(\omega + i\nu_\alpha)}$$

Simplest Plasma Models: the Model of Independent Particles

$$\sigma_{ij}(\omega,\vec{k}) = \delta_{ij}\sigma \qquad \sigma = \sum_{\alpha}\frac{i{e_\alpha}^2 N_\alpha}{m_\alpha(\omega + i\nu_\alpha)}$$

$$\varepsilon_{ij}(\omega,\vec{k}) = \delta_{ij} + \frac{i}{\varepsilon_0\omega}\sigma_{ij}(\omega,\vec{k})$$

$$\varepsilon_{ij}(\omega,\vec{k}) = \delta_{ij}\varepsilon \qquad \varepsilon = 1 - \sum_{\alpha}\frac{{\omega_{p\alpha}}^2}{\omega(\omega + i\nu_\alpha)} = \varepsilon^{l,tr}$$

$$\omega_{p\alpha} = \sqrt{\frac{{e_\alpha}^2 N_\alpha}{\varepsilon_0 m_\alpha}}$$ – plasma (Langmuir) frequency of particle of type α

Simplest Plasma Models: the Model of Independent Particles

High frequency transverse EM waves in plasma: $\omega \gg \nu_e$

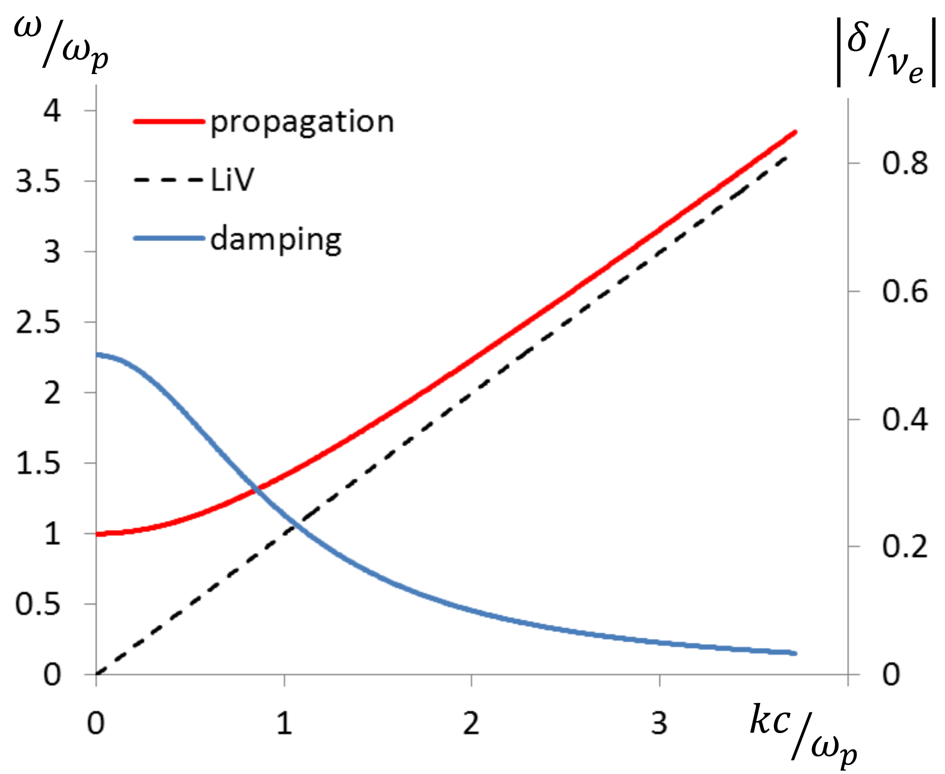
$$\varepsilon^{tr} = 1 - \frac{{\omega_{pe}}^2}{\omega(\omega + i\nu_e)} - \frac{{\omega_{pi}}^2}{\omega(\omega + i\nu_i)} \approx 1 - \frac{{\omega_{pe}}^2}{\omega^2}\left(1 - i\frac{\nu_e}{\omega}\right)$$

$$k^2 - \frac{\omega^2}{c^2}\varepsilon^{tr}(\omega, k) = 0$$

$$\omega \to \omega + i\delta$$

$$\omega^2 = {\omega_{pe}}^2 + k^2c^2$$

$$\delta = -\frac{\nu_e}{2}\frac{{\omega_{pe}}^2}{{\omega_{pe}}^2 + k^2c^2}$$



Simplest Plasma Models: the Model of Independent Particles

Low frequency case $\omega \ll \nu_e$

$$\varepsilon^{tr} = 1 - \frac{{\omega_{pe}}^2}{\omega(\omega + i\nu_e)} - \frac{{\omega_{pi}}^2}{\omega(\omega + i\nu_i)} \approx 1 + i\frac{{\omega_{pe}}^2}{\nu_e \omega}$$

$$\varepsilon_{ij}(\omega, \vec{k}) = \delta_{ij} + \frac{i}{\varepsilon_0 \omega}\sigma_{ij}(\omega, \vec{k})$$

$$\sigma_{ij}(\omega, \vec{k}) = \delta_{ij}\sigma \qquad \varepsilon_{ij}(\omega, \vec{k}) = \delta_{ij}\varepsilon$$

$$\Rightarrow \quad \varepsilon = 1 + \frac{i}{\varepsilon_0 \omega}\sigma$$

$$\sigma = \frac{\varepsilon_0 {\omega_{pe}}^2}{\nu_e} = \frac{e^2 N_e}{m\nu_e} = \text{const}(\omega)$$

$$k^2 = \frac{\omega^2}{c^2}\varepsilon^{tr}(\omega, k) = \frac{\omega^2}{c^2} + i\frac{\omega\sigma}{\varepsilon_0 c^2} \approx i\frac{\omega\sigma}{\varepsilon_0 c^2} \quad \Rightarrow \quad k = \sqrt{i\frac{\omega\sigma}{\varepsilon_0 c^2}} = \frac{1+i}{\sqrt{2}}\sqrt{\frac{\omega\sigma}{\varepsilon_0 c^2}}$$

$$\lambda_{sk} = \frac{1}{Im\ k} = c\sqrt{\frac{2\varepsilon_0}{\omega\sigma}} \sim \frac{1}{\sqrt{\omega}}$$

$\Rightarrow$ Normal skin-effect!

Simplest Plasma Models: the Model of Independent Particles

High frequency longitudinal waves in plasma: $\omega \gg \nu_e$

$$\varepsilon^l = 1 - \frac{{\omega_{pe}}^2}{\omega(\omega + i\nu_e)} - \frac{{\omega_{pi}}^2}{\omega(\omega + i\nu_i)} \approx 1 - \frac{{\omega_{pe}}^2}{\omega^2}\left(1 - i\frac{\nu_e}{\omega}\right)$$

$$\varepsilon^l(\omega, k) = 0$$

$$\omega \to \omega + i\delta$$

$$\omega^2 = {\omega_{pe}}^2$$
$$\delta = -\frac{\nu_e}{2}$$

Model of Independent Particles

"++":

- simple and straightforward
- several phenomena could be described (Langmuir || waves, transverse EM waves, including damping)

"—":

- static limit for || plasma oscillations ($\omega \to 0 \Rightarrow \varepsilon \to \infty;\ \Phi \to 0?$)
- Non-dissipative skin-effect $\sim c/\omega_p$) + anomalous skin-effect (($\lambda_{sk} \sim \omega^{-1/3}$))
- Can not explain low frequency longitudinal waves with linear dispersion (experimentally observed)

Kinetic equation with self-consistent field

Kinetic approach to plasma

- particle distribution function for **N** particles:

$$f_N(t, \overrightarrow{r_1}, \overrightarrow{r_2}, \dots, \overrightarrow{r_N}, \overrightarrow{p_1}, \overrightarrow{p_2}, \dots, \overrightarrow{p_N})$$

- for noninteracting particles (**collisionless** plasma)

$$f_N(t, \overrightarrow{r_1}, \overrightarrow{r_2}, \dots, \overrightarrow{r_N}, \overrightarrow{p_1}, \overrightarrow{p_2}, \dots, \overrightarrow{p_N}) = \prod_{i=1}^{N} f(t, \overrightarrow{r_i}, \overrightarrow{p_i})$$

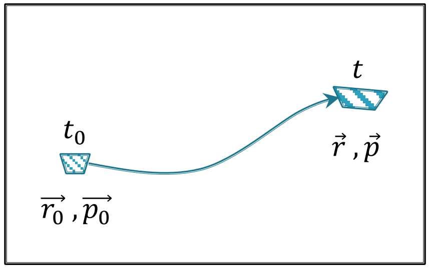
- **probability** that the particle is within the volume $d\vec{r}\, d\vec{p}$ around the point $\vec{r}\,, \vec{p}$ of the phase space at the *t* time moment:

$$f(t, \vec{r}, \vec{p})\, d\vec{r}\, d\vec{p}$$

- **normalization** → NoP:

$$\int f(t, \vec{r}, \vec{p})\, d\vec{r}\, d\vec{p} = N$$

Kinetic equation for $f(t, \overrightarrow{r_i}, \overrightarrow{p_i})$



$$\frac{d\vec{r}}{dt} = \vec{v}\,, \quad \frac{d\vec{p}}{dt} = \vec{F} = e\left(\vec{E} + [\vec{v} \times \vec{B}]\right)$$

$$\vec{r}(t_0) = \overrightarrow{r_0}\,, \qquad \vec{p}(t_0) = \overrightarrow{p_0}$$

$$d\overrightarrow{r_0} \cdot d\overrightarrow{p_0} \to d\vec{r} \cdot d\vec{p}, \quad f(t_0, \overrightarrow{r_0}, \overrightarrow{p_0}) \to f(t, \vec{r}(t), \vec{p}(t))$$

- Assuming the invariance of the particle number (no ionization, no recombination, no collisions) a full particle number in the phase space volume is constant:

$$f(t, \vec{r}(t), \vec{p}(t))\, d\vec{r}\, d\vec{p} = f(t_0, \overrightarrow{r_0}, \overrightarrow{p_0})\, d\overrightarrow{r_0}\, d\overrightarrow{p_0} = const$$

- According to the Liouville's theorem the phase space volume is preserved:

$$d\overrightarrow{r_0} \cdot d\overrightarrow{p_0} \;= 1 \cdot\; d\vec{r} \cdot d\vec{p}$$

➔the particle distribution function along the phase trajectory is constant:

$$f(t, \vec{r}(t), \vec{p}(t)) = const$$

Kinetic equation for $f(t, \overrightarrow{r_i}, \overrightarrow{p_i})$

$$\frac{df(t,\vec{r}(t),\vec{p}(t))}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \vec{r}}\frac{d\vec{r}(t)}{dt} + \frac{\partial f}{\partial \vec{p}}\frac{d\vec{p}(t)}{dt} = 0$$

- Combining with (1.5)

$$\frac{d\vec{r}}{dt} = \vec{v}\,, \quad \frac{d\vec{p}}{dt} = \vec{F} = e\left(\vec{E} + \left[\vec{v}\times\vec{B}\right]\right)$$

- **Vlasov equation:**

$$\frac{\partial f}{\partial t} + \vec{v}\frac{\partial f}{\partial \vec{r}} + e\left(\vec{E} + \left[\vec{v}\times\vec{B}\right]\right)\frac{\partial f}{\partial \vec{p}} = 0$$

- +Maxwell equations for the electromagnetic fields:

$$\nabla\vec{E} = \frac{\rho}{\varepsilon_0}, \quad \nabla\vec{B} = 0, \quad \left[\nabla\times\vec{E}\right] = -\frac{\partial\vec{B}}{\partial t}\,, \quad \left[\nabla\times\vec{B}\right] = \mu_0\vec{j} + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}\,,$$

  - charge and current densities are calculated as

$$\rho(t,\vec{r}) = e\int f(t,\vec{r},\vec{p})d\vec{p}\,, \qquad \vec{j} = e\int \vec{v}f(t,\vec{r},\vec{p})d\vec{p}$$

  - particle density $n(t,\vec{r})$:

$$\int f(t,\vec{r},\vec{p})d\vec{p} = n(t,\vec{r}),$$
$$\int n(t,\vec{r})d\vec{r} = N$$

NB: only variables $t,\vec{r},\vec{p}$ in (1.12) are independent $\vec{v} = c\frac{\vec{p}}{\sqrt{m^2c^2+p^2}}$

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = 299792458\frac{m}{s}$$
$$\varepsilon_0 = 8.854\cdot 10^{-12}\frac{A\,s}{V\,m}$$
$$\mu_0 = 4\pi\cdot 10^{-7}\frac{V\,s}{A\,m}$$

Dielectric permittivity from the kinetic model

- Collisionless homogeneous isotropic plasma

- Particles in plasma in thermal motion → Particle distribution function (PDF)

→Maxwell $f_{0\alpha} = f_{M\alpha} = \frac{N_\alpha}{(2\pi m_\alpha k T_\alpha)^{3/2}} exp\left(-\frac{{p_\alpha}^2}{2m_\alpha k T_\alpha}\right)$ for sufficiently high T

→Fermi distribution function $f_{0\alpha} = f_{F\alpha} = \frac{\frac{2}{(2\pi\hbar)^{3/2}}}{exp\left(\frac{{p_\alpha}^2/_{2m_\alpha} - \varepsilon_{F\alpha}}{kT_\alpha}\right)+1}$, if $\varepsilon_{F\alpha} = \frac{{p_{F\alpha}}^2}{2m_\alpha} = \frac{(3\pi^2)^{2/3}\hbar^2 {N_\alpha}^{2/3}}{2m_\alpha} \gg kT_\alpha$

- Perturbed PDF:

$$f_\alpha(\vec{p},\vec{r},t) = f_{0\alpha}(p) + \delta f_\alpha(\vec{p},\vec{r},t)$$

- Assumptions → perturbations (including induced fields are small)

Dielectric permittivity from the kinetic model

- Vlasov equation $$\frac{\partial f_\alpha}{\partial t} + \vec{v}\frac{\partial f_\alpha}{\partial \vec{r}} + e_\alpha\left(\vec{E} + \left[\vec{v}\times\vec{B}\right]\right)\frac{\partial f_\alpha}{\partial \vec{p}} = 0$$

- For the PDF perturbation (using $\left[\vec{v}\times\vec{B}\right]\cdot\frac{\partial f_\alpha}{\partial \vec{p}}$=0):

$$\frac{\partial \delta f_\alpha}{\partial t} + \vec{v}\frac{\partial \delta f_\alpha}{\partial \vec{r}} + e_\alpha\vec{E}\frac{\partial f_{0\alpha}}{\partial \vec{p}} = 0$$

- Linearity → $\delta f_\alpha(t,\vec{r}) \propto \delta f_\alpha(\omega,\vec{k})e^{-i\omega t+i\vec{k}\vec{r}}$ →

$$\boldsymbol{\delta f_\alpha(\omega,\vec{k}) = -i\frac{e_\alpha\vec{E}\cdot\frac{\partial f_{0\alpha}}{\partial \vec{p}}}{\omega-\vec{k}\vec{v}}}$$

Dielectric permittivity from the kinetic model

- PDF perturbation: $\boldsymbol{\delta f_{\alpha}(\omega,\vec{k}) = -i\frac{e_\alpha \vec{E}\cdot\frac{\partial f_{0\alpha}}{\partial \vec{p}}}{\omega-\vec{k}\vec{v}}}$

- Induced current density:

$$\vec{j} = \sum_\alpha e_\alpha \int \vec{v} f_\alpha(t,\vec{r},\vec{p})d\vec{p} = \sum_\alpha e_\alpha \int \vec{v}\,\delta f_\alpha(t,\vec{r},\vec{p})d\vec{p}$$

- For Fourier components:

$$j_i(\omega,\vec{k}) = -i\sum_\alpha {e_\alpha}^2 \int v_i \frac{\left(E_j(\omega,\vec{k})\cdot\frac{\partial f_{0\alpha}}{\partial p_j}\right)}{\omega-\vec{k}\vec{v}}d\vec{p} = \underbrace{\left\{-i\sum_\alpha {e_\alpha}^2 \int \frac{v_i\left(\frac{\partial f_{0\alpha}}{\partial p_j}\right)}{\omega-\vec{k}\vec{v}}d\vec{p}\right\}}_{\boldsymbol{\sigma_{ij}(\omega,\vec{k})}} E_j(\omega,\vec{k})$$

$$\boldsymbol{\sigma_{ij}(\omega,\vec{k}) = -i\sum_\alpha {e_\alpha}^2 \int \frac{v_i\left(\frac{\partial f_{0\alpha}}{\partial p_j}\right)}{\omega-\vec{k}\vec{v}}d\vec{p}}$$

Dielectric permittivity from the kinetic model

- Conductivity tensor: $$\sigma_{ij}(\omega,\vec{k}) = -i\sum_\alpha e_\alpha{}^2 \int \frac{v_i\left(\frac{\partial f_{0\alpha}}{\partial p_j}\right)}{\omega-\vec{k}\vec{v}}\,d\vec{p}$$

- Dielectric permittivity tensor:

$$\varepsilon_{ij}(\omega,\vec{k}) = \delta_{ij} + \frac{i}{\varepsilon_0\omega}\sigma_{ij}(\omega,\vec{k})$$

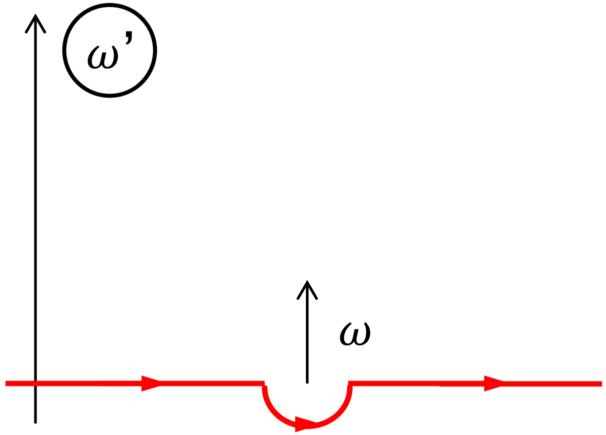
$$\varepsilon_{ij}(\omega,\vec{k}) = \delta_{ij} + \sum_\alpha \frac{e_\alpha{}^2}{\varepsilon_0\omega}\int \frac{v_i\left(\frac{\partial f_{0\alpha}}{\partial p_j}\right)}{\omega-\vec{k}\vec{v}}\,d\vec{p}$$

Dielectric permittivity from the kinetic model

- Dielectric permittivity tensor:

$$\varepsilon_{ij}(\omega,\vec{k}) = \delta_{ij} + \sum_{\alpha} \frac{{e_\alpha}^2}{\varepsilon_0 \omega} \int \frac{v_i \left(\frac{\partial f_{0\alpha}}{\partial p_j}\right)}{\omega - \vec{k}\vec{v}} d\vec{p}$$

- ? Singularity at $\omega = \vec{k}\vec{v}$ → Landau prescription



- **M:**

$$\lim_{\nu \to 0} \frac{1}{x + i\nu} = \frac{\mathcal{V}.\mathcal{P}.}{x} - i\pi\delta(x)$$

$$\varepsilon_{ij}(\omega,\vec{k}) = \delta_{ij} + \sum_{\alpha} \frac{{e_\alpha}^2}{\varepsilon_0 \omega} \int d\vec{p}\; v_i \left(\frac{\partial f_{0\alpha}}{\partial p_j}\right) \left[\frac{\mathcal{V}.\mathcal{P}.}{\omega - \vec{k}\vec{v}} - i\pi\delta(\omega - \vec{k}\vec{v})\right]$$

Dielectric permittivity from the kinetic model

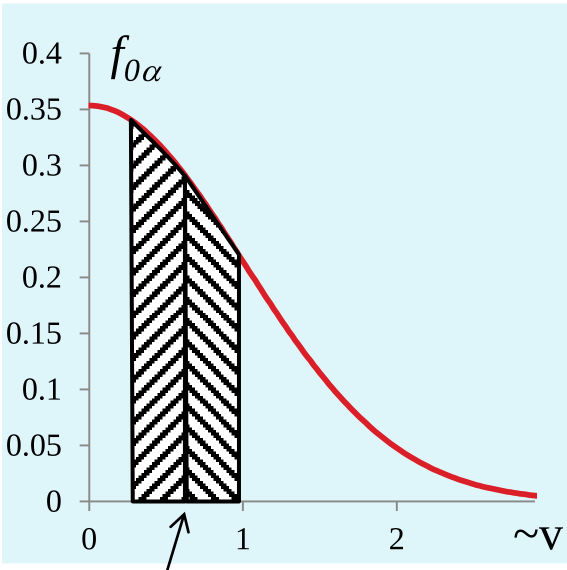
$$\varepsilon_{ij}(\omega,\vec{k}) = \delta_{ij} + \sum_{\alpha} \frac{{e_\alpha}^2}{\varepsilon_0 \omega} \int d\vec{p}\; v_i \left(\frac{\partial f_{0\alpha}}{\partial p_j}\right) \left[\frac{\mathcal{V}.\mathcal{P}.}{\omega - \vec{k}\vec{v}} - i\pi\delta(\omega - \vec{k}\vec{v})\right]$$

Hermitian

Anti-Hermitian

$\omega = \vec{k}\vec{v} \rightarrow$
Cherenkov wave absorption by plasma particles

Even w/o collisions!



$v = \omega/k$

Dielectric permittivity from the kinetic model

$$\varepsilon_{ij}(\omega,\vec{k}) = \delta_{ij} + \sum_{\alpha} \frac{{e_{\alpha}}^{2}}{\varepsilon_0 \omega} \int d\vec{p}\; v_i \left(\frac{\partial f_{0\alpha}}{\partial p_j}\right) \left[\frac{\mathcal{V}.\mathcal{P}.}{\omega - \vec{k}\vec{v}} - i\pi\delta(\omega - \vec{k}\vec{v})\right]$$

Homogeneous isotropic (nonrelativistic) plasma:

$$\varepsilon_{ij}(\omega,\boldsymbol{\vec{k}}) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\varepsilon^{tr}(\omega,k\ ) + \frac{k_i k_j}{k^2}\varepsilon^{l}(\omega,k\ )$$

$$\varepsilon^{l}(\omega,k\ ) = 1 + \sum_{\alpha} \frac{{e_{\alpha}}^{2}}{\varepsilon_0 \omega k^2} \int d\vec{p}\; \frac{\left(\vec{k}\vec{v}\right)^2}{\omega - \vec{k}\vec{v}} \left(\frac{\partial f_{0\alpha}}{\partial \mathcal{E}_{\alpha}}\right)$$

$$\varepsilon^{tr}(\omega,k\ ) = 1 + \sum_{\alpha} \frac{{e_{\alpha}}^{2}}{2\varepsilon_0 \omega k^2} \int d\vec{p}\; \frac{\left[\vec{k} \times \vec{v}\right]^2}{\omega - \vec{k}\vec{v}} \left(\frac{\partial f_{0\alpha}}{\partial \mathcal{E}_{\alpha}}\right)$$

$$\mathcal{E}_{\alpha} = \frac{{p_{\alpha}}^{2}}{2m_{\alpha}}$$

Dielectric permittivity from the kinetic model

Homogeneous isotropic plasma:

$$\varepsilon^{l}(\omega,k) = 1 + \sum_{\alpha} \frac{e_{\alpha}{}^{2}}{\varepsilon_0 \omega k^2} \int d\vec{p}\, \frac{\left(\vec{k}\vec{v}\right)^2}{\omega - \vec{k}\vec{v}} \left(\frac{\partial f_{0\alpha}}{\partial \mathcal{E}_{\alpha}}\right)$$

$$\varepsilon^{tr}(\omega,k) = 1 + \sum_{\alpha} \frac{e_{\alpha}{}^{2}}{2\varepsilon_0 \omega k^2} \int d\vec{p}\, \frac{\left[\vec{k}\times\vec{v}\right]^2}{\omega - \vec{k}\vec{v}} \left(\frac{\partial f_{0\alpha}}{\partial \mathcal{E}_{\alpha}}\right)$$

- Maxwell $f_{0\alpha}(\mathcal{E}_{\alpha}) = f_{M\alpha} = \frac{N_{\alpha}}{(2\pi m_{\alpha} k T_{\alpha})^{3/2}} exp\left(-\frac{\mathcal{E}_{\alpha}}{kT_{\alpha}}\right)$
- Fermi distribution function $f_{0\alpha}(\mathcal{E}_{\alpha}) = f_{F\alpha} = \frac{\frac{2}{(2\pi\hbar)^{3/2}}}{exp\left(\frac{\mathcal{E}_{\alpha} - \mathcal{E}_{F\alpha}}{kT_{\alpha}}\right)+1}$

L7

$$\mathcal{E}_{\alpha} = \frac{p_{\alpha}{}^{2}}{2m_{\alpha}}$$

Summary

- Simplest Plasma Models: the Model of Independent Particles:
  - Including collisions
  - Spectra for waves in plasma + damping constants for high and low frequency cases (normal skin-effect)
  - Difficulties of the model

- Kinetic approach to plasma:
  - Vlasov equation
  - Dielectric permittivity from the kinetic model
  - Landau prescription
  - Cherenkov wave absorption
  - Longitudinal and transverse dielectric permittivity for the homogeneous isotropic plasma

- Next: Dielectric permittivity:
  - Classical (Maxwell) and degenerate (Fermi) plasma
  - Spectra of plasma wave derived from kinetic model
  - Particle collisions