STUDY OF THE DOSE AND BEAM DISTRIBUTION IN WATER DEPENDING ON THE FOCUSING STRENGTH OF THE ELECTRON BEAM

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MOTIVATION

The main motivation:

- Recent publications have shown that it is possible to focus very high-energy electron beams to produce high dose volumes inside a water phantom at depths, and it is possible to change the shape of the dose distribution by changing only the quadrupole configuration[1-3].

What will be the results when electron energy is 22MeV?

- Zakaria has presented the simulation results for 22 MeV electron beam focused in water. In particular, results have shown that in the case of a large RMS beam size, a narrow dose peak can be generated inside the water at depth.

In this case beams with different RMS spot sizes were used. The beams were focused to a point without any quadrupole magnets.

Also in this modeling, water is surrounded only a vacuum, and the exit window is not taken into account.

Therefore, I tried to investigate a more realistic model.

1. Whitmore, L., Mackay, R.I., van Herk, M. *et al.* Focused VHEE (very high energy electron) beams and dose delivery for radiotherapy applications. *Sci Rep* 11, 14013 (2021). https://doi.org/10.1038/s41598-021-93276-8
2. Kokurewicz, K., Brunetti, E., Welsh, G.H. *et al.* Focused very high-energy electron beams as a novel radiotherapy modality for producing high-dose volumetric elements. *Sci Rep* 9, 10837 (2019). https://doi.org/10.1038/s41598-019-46630-w
3. Kokurewicz, K., Brunetti, E., Curcio, A. *et al.* An experimental study of focused very high energy electron beams for radiotherapy. *Commun Phys* 4, 33 (2021). https://doi.org/10.1038/s42005-021-00536-0

GOAL OF THE STUDY

The main purpose of this study was to investigate dose distribution in the water when $22MeV$ electron beam is focused using quadrupole magnets.

A series of Monte Carlo simulations were carried out to figure out the following:

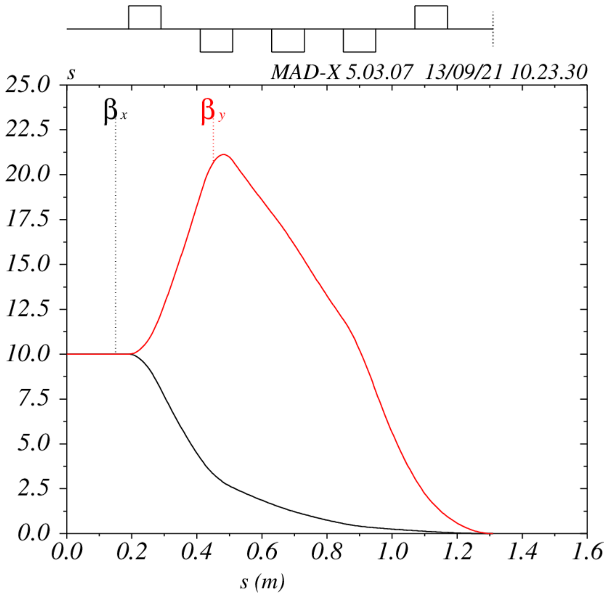
- **Dose distribution for various magnetic strengths.**
- **Dose distribution dependency on the size of the beam at the focal point.**
- **Beam distribution at the entrance to the water surface for various magnetic strengths.**
- **Dose distribution during a focused electron beam with energies from $22MeV$ to $100MeV$.**

Simulation setup

- **Three beam spot size were considered $5mm$, $10mm$ and $20mm$**
- **Water as target with sizes $100 \times 100 \times 60mm^3$, which is located $6cm$ after window**
- **There are five quadrupole magnets in front of the window, and the fifth quadrupole is at a distance of 7 cm from the water.**
- **Three cases were observed**
  - **No exit window and the area around the water is in a vacuum. (case 1)**
  - **Exit window is titanium foil with a thickness of $50\mu m$ (case 2)**
  - **Exit window is two layers material pyrolytic graphite. First layer density is $\rho = 2.0\, g/cm^3$ and thickness is $50\mu m$ and second layer density is $\rho = 1.5\, g/cm^3$ and thickness is $500\mu m$. (case 3)**

DETERMINE MAGNETIC FIELDS

The Mad-X code was used to determine the number, position, and strength of 10 cm long quadrupole magnets to focus a 5 mm, 10 mm, and 20 mm initial RMS beam at a depth of 30 mm in water.



Beta function in the region of quadrupole magnets.

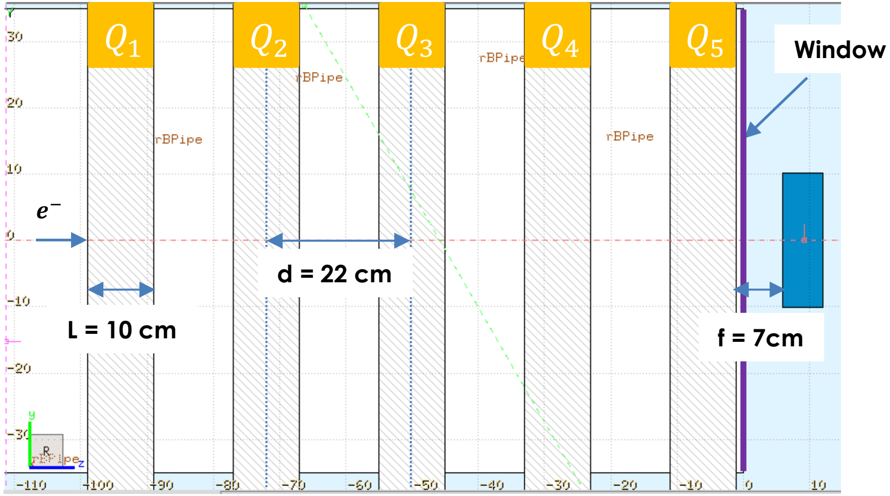
Quadrupole Parameters:

- **Length**: 10 cm
- **Distance from center to center**: 22 cm
- Q_5 **distance from window:** 1cm
- Q_5 **distance from water:** 7cm
- **Focusing Strength k** $[m^{-2}]$: **defined by** Mad-X

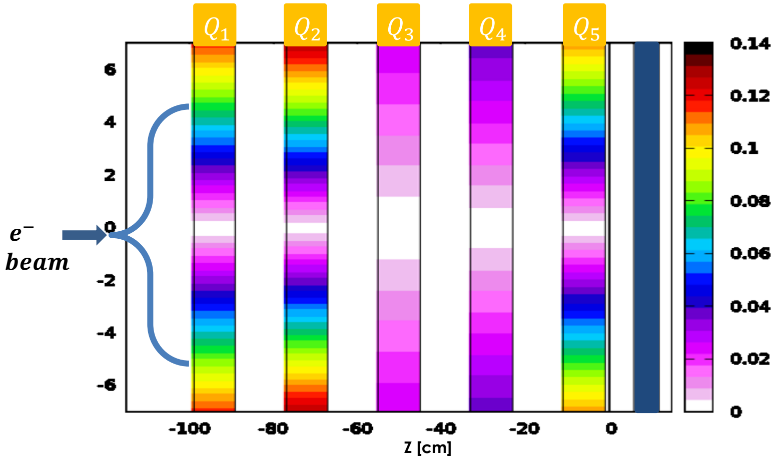
$$k[m^{-2}] = \frac{1}{fL} = \frac{0.299g[T/m]}{\beta E[GeV]}; \quad g = \frac{dB_y}{dx}$$

SIMULATION SETUP

Schematic representation of the geometry and magnetic fields of the simulation model in FLUKA code.



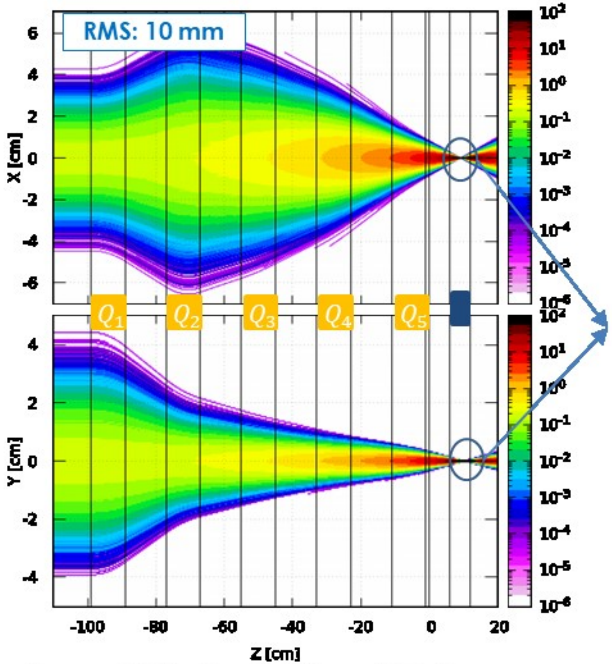
Schematic of the quadrupole set-up used in the symmetric focusing electron beam.



Quadrupole magnets field distribution in XZ plane (FLUKA)

BEAM DISTRIBUTION

Simulations were carried out for several beam sizes at the focal point.



Beam distribution in **XZ** and **YZ** planes with **RMS 10mm**.

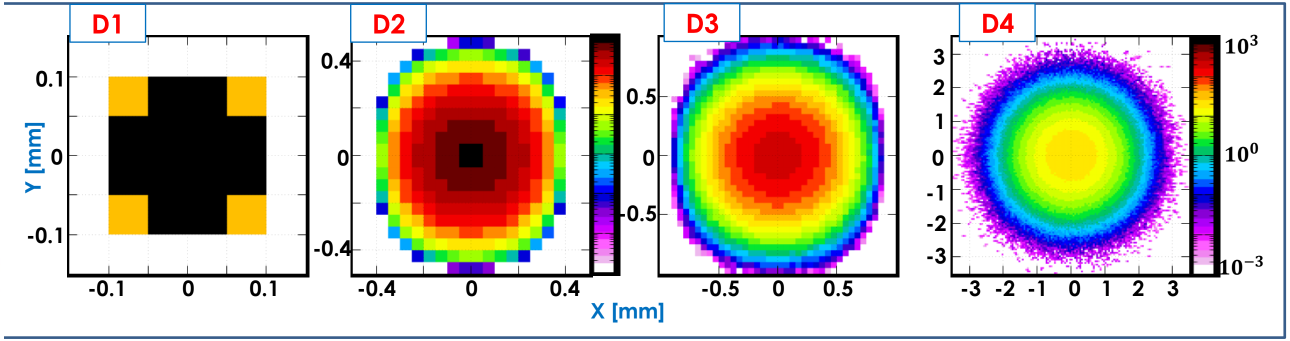
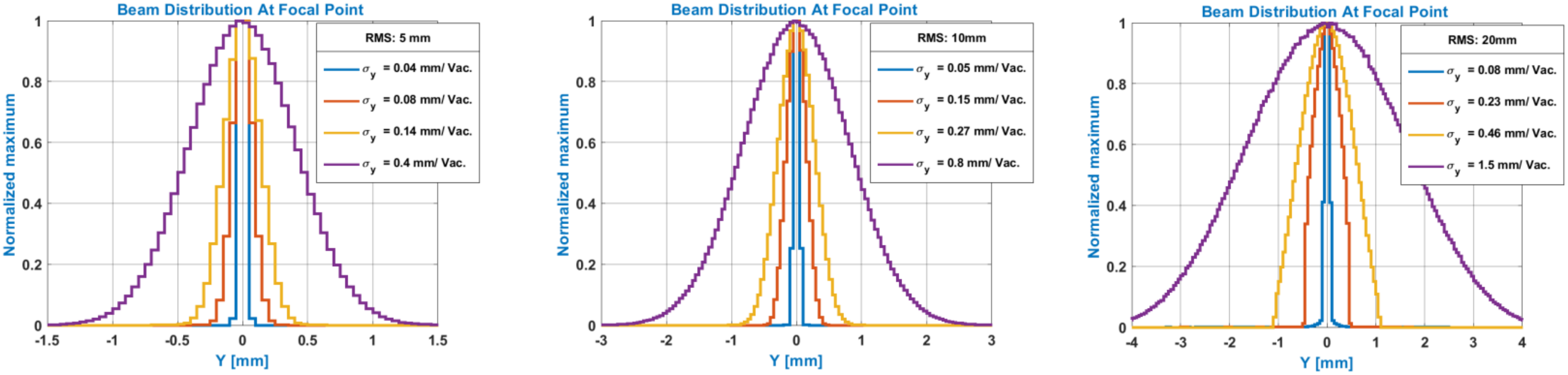


Figure D1 shows the smallest beam profile in the **XY** plane at the focal point that was used in the simulation.
Figure D4 shows the largest beam profile in the **XY** plane.
The **RMS** value of the incident Gaussian parallel beam is **10 mm**, and the energy is **22 MeV**.

BEAM DISTRIBUTION



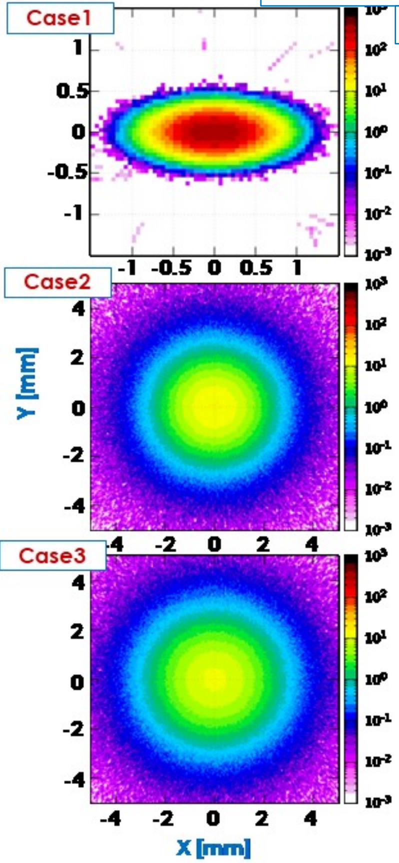
The graphs shown the transverse spatial distributions of the electron beam at the focal point. The incident beam for **RMS 5mm, 10mm** and **20mm**.

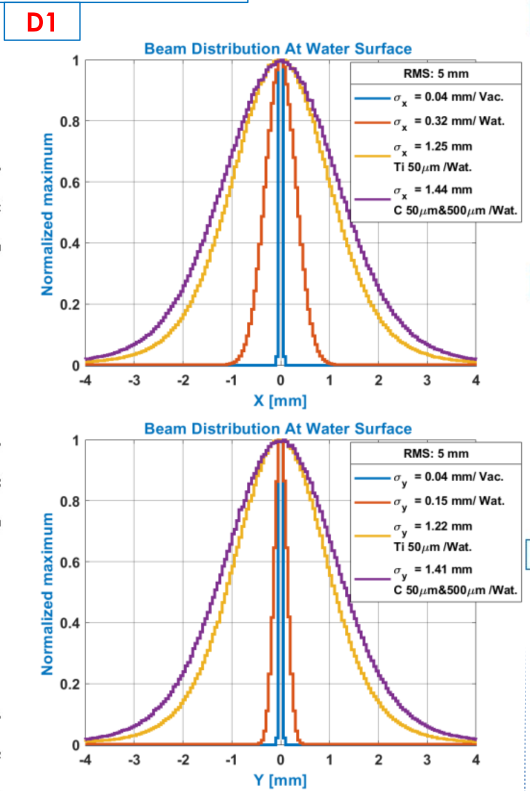
RMS: 5 mm

BEAM PROFILE ON WATER SURFACE

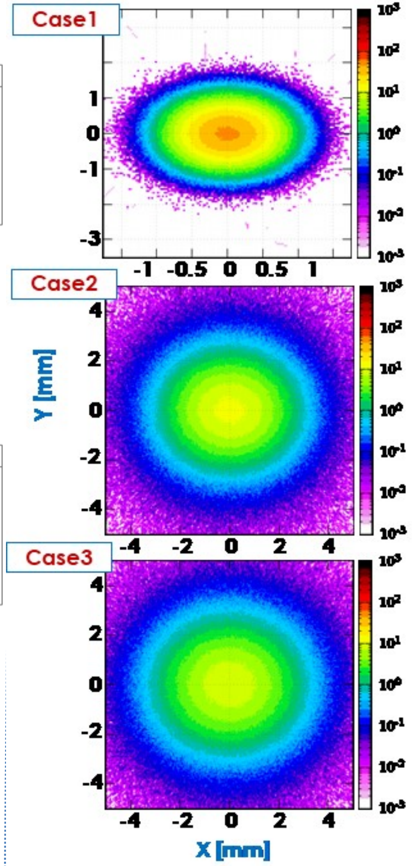
Focused center of water

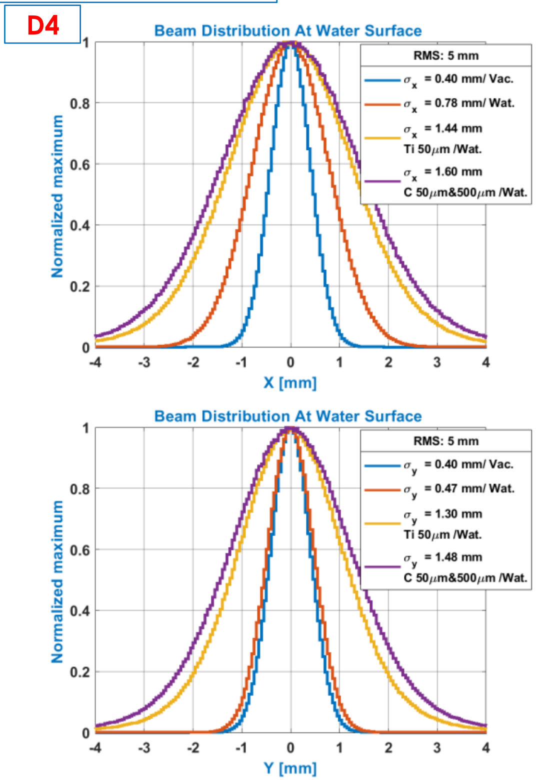
Beam Profile on Water Surface





Beam Profile on Water Surface





Beam profiles before entering to the water phantom area in **XY** plane for **RMS 5mm**. The focusing point is **RMS** $\sigma_x = \sigma_y = 0.04mm$ [D1].

Beam profiles before entering to the water phantom area in **XY RMS 5mm**. The focusing point **RMS** $\sigma_x = \sigma_y = 0.4mm$ [F4].

BEAM PROFILE ON WATER SURFACE

Incident Beam RMS 5mm / Focused Center of Water			
Vacuum (mm)	Water(mm)	Ti $50\mu m$ & Water (mm)	C $50/500\mu m$& Water (mm)
[D1] $\sigma_x = \sigma_y = 0.04$	$\sigma_x = 0.32$ $\sigma_y = 0.15$	$\sigma_x = 1.25$ $\sigma_y = 1.22$	$\sigma_x = 1.44$ $\sigma_y = 1.41$
[D2] $\sigma_x = \sigma_y = 0.08$	$\sigma_x = 0.36$ $\sigma_y = 0.19$	$\sigma_x = 1.26$ $\sigma_y = 1.22$	$\sigma_x = 1.45$ $\sigma_y = 1.42$
[D3] $\sigma_x = \sigma_y = 0.14$	$\sigma_x = 0.42$ $\sigma_y = 0.25$	$\sigma_x = 1.28$ $\sigma_y = 1.24$	$\sigma_x = 1.47$ $\sigma_y = 1.43$
[D4] $\sigma_x = \sigma_y = 0.40$	$\sigma_x = 0.78$ $\sigma_y = 0.47$	$\sigma_x = 1.44$ $\sigma_y = 1.30$	$\sigma_x = 1.60$ $\sigma_y = 1.48$

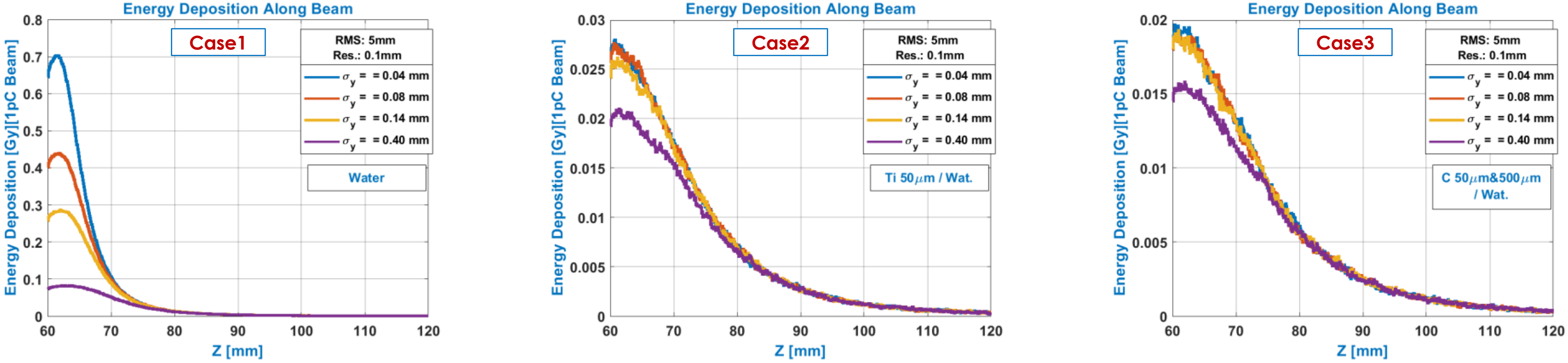
The table shows that by increasing the size of the beam at the focal point from **D1** to **D4** (by means of changing the magnetic fields), the electron beam size RMS in the horizontal and vertical directions **at the entrance to the water surface** increases by **2.4** and **3.1** times, respectively.

In the case of **exit windows**, the beam size at the entrance water surface is practically the same for different beam sizes at the focal point.

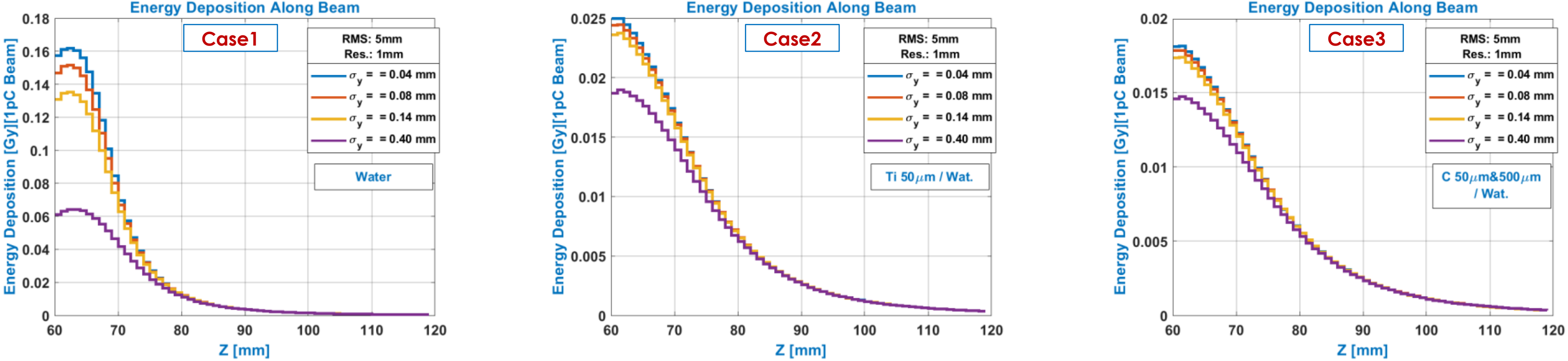
RMS: 5 mm

DOSE DISTRIBUTIONS

Focused center of water



Energy deposition along the beam path where two transverse dimensions are integrated for central $\mathbf{0.1 \times 0.1mm^2}$ space.



Energy deposition along the beam path where two transverse dimensions are integrated for central $\mathbf{1 \times 1mm^2}$ space. The incident beam **RMS is 5mm.**

Line colors correspond for different **beam sizes at the focal point [D1,D2,D3 and D4].**

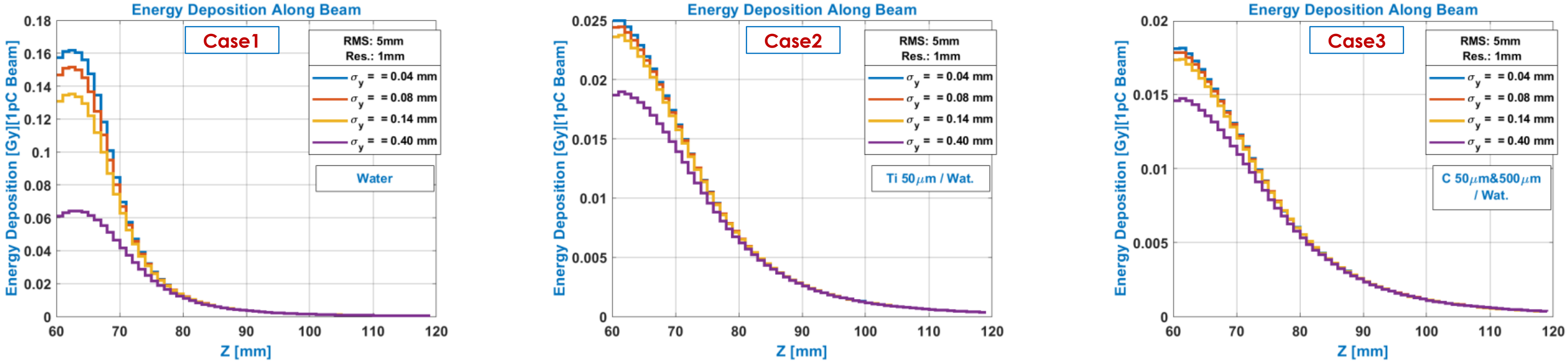
RMS: 5 mm

DOSE DISTRIBUTIONS

Focused center of water

Focused initial beam RMS 5 mm:

- **In case on exit window FLUKA Monte Carlo simulations demonstrate that**
  - ➢ **the energy deposition along the path of the beam peaks at a depth of 3 mm in water.**
  - ➢ **The energy deposition, depending on the size of the beam at the focal point, leads to a change in the peak value, but the position remains almost unchanged.**
- **In case exit window (on both)**
  - ➢ **the energy deposition along the path of the beam no have peaks.**
  - ➢ **the energy deposition behaves like the case parallel beam.**



Energy deposition along the beam path where two transverse dimensions are integrated for central $\mathbf{1 \times 1mm^2}$ space. The incident beam **RMS is 5mm.**
Line colors correspond for different **beam sizes at the focal point [D1,D2,D3 and D4].**

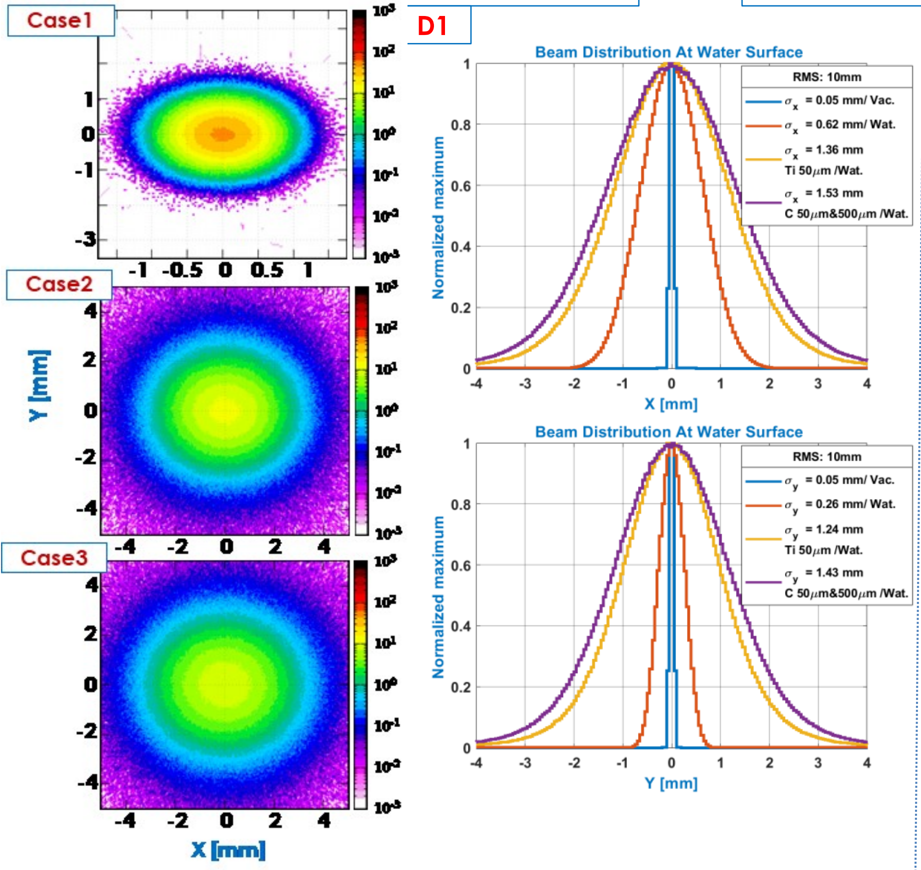
RMS: 10 mm

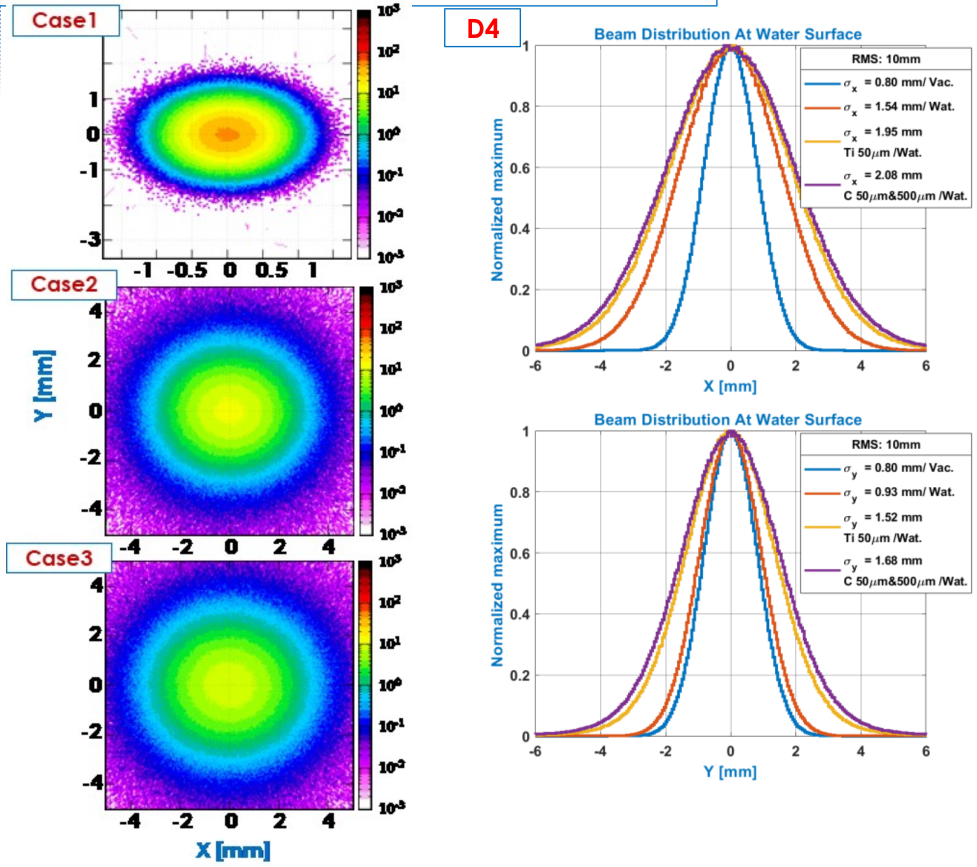
BEAM PROFILE ON WATER SURFACE

Beam Profile on Water Surface

Focused center of water

Beam Profile on Water Surface





Beam profiles before entering to the water phantom area in **XY** plane for **rms 10mm**. The focusing point **rms** $\sigma_x = \sigma_y = 0.05mm$ **[D1].**

Beam profiles before entering to the water phantom area in **XY** plane for **rms 10mm**. The focusing point **rms** $\sigma_x = \sigma_y = 0.8mm$ **[D4].**

BEAM PROFILE ON WATER SURFACE

Incident Beam RMS 10mm / Focused Center of Water			
Vacuum (mm)	Water(mm)	Ti $50\mu m$ & Water (mm)	C $50/500\mu m$& Water (mm)
[D1] $\sigma_x = \sigma_y = 0.05$	$\sigma_x = 0.62$ $\sigma_y = 0.26$	$\sigma_x = 1.36$ $\sigma_y = 1.24$	$\sigma_x = 1.53$ $\sigma_y = 1.43$
[D2] $\sigma_x = \sigma_y = 0.15$	$\sigma_x = 0.71$ $\sigma_y = 0.36$	$\sigma_x = 1.40$ $\sigma_y = 1.27$	$\sigma_x = 1.57$ $\sigma_y = 1.45$
[D3] $\sigma_x = \sigma_y = 0.27$	$\sigma_x = 0.82$ $\sigma_y = 0.49$	$\sigma_x = 1.46$ $\sigma_y = 1.30$	$\sigma_x = 1.63$ $\sigma_y = 1.48$
[D4] $\sigma_x = \sigma_y = 0.80$	$\sigma_x = 1.54$ $\sigma_y = 0.93$	$\sigma_x = 1.95$ $\sigma_y = 1.52$	$\sigma_x = 2.08$ $\sigma_y = 1.68$

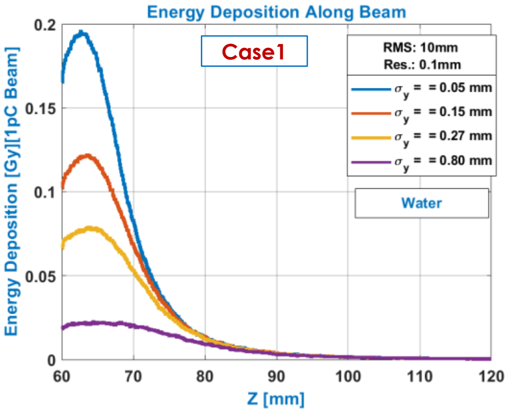
The table shows that by increasing the size of the beam at the focal point from **D1** to **D4** (by means of changing the magnetic fields), the electron beam size **RMS** in the horizontal and vertical directions **at the entrance to the water surface** increases by **2.5** and **3.6** times, respectively.

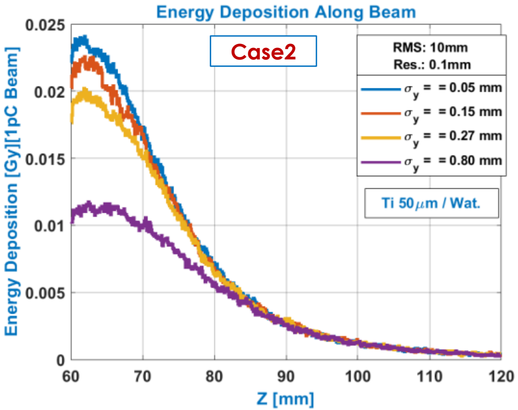
In the case of **exit window (Ti $50\mu m$)**, the beam size at the entrance water surface increases by **1.43** and **1.22** times.

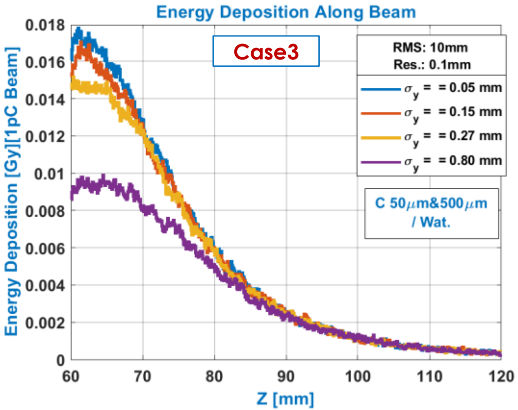
RMS: 10 mm

DOSE DISTRIBUTIONS

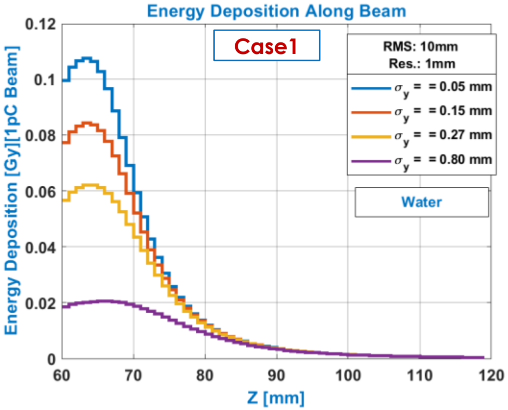
Focused center of water

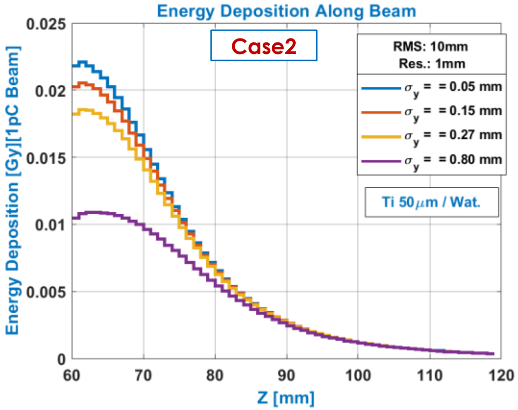


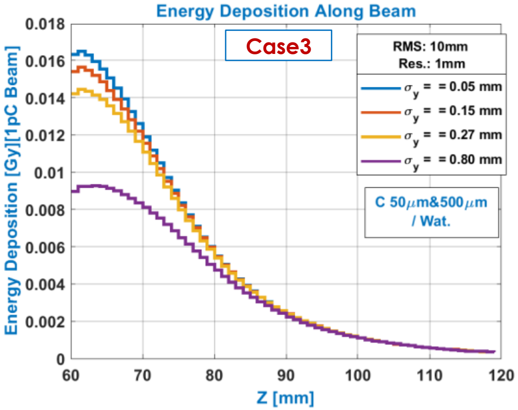




Energy deposition along the beam path where two transverse dimensions are integrated for central $\mathbf{0.1 \times 0.1 mm^2}$ space.







Energy deposition along the beam path where two transverse dimensions are integrated for central $\mathbf{1 \times 1 mm^2}$ space. The incident beam **RMS is 10mm.**
Line colors correspond for different **beam sizes at the focal point [D1,D2,D3 and D4].**

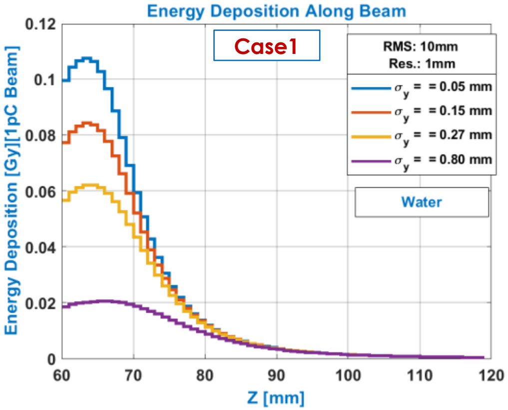
RMS: 10 mm

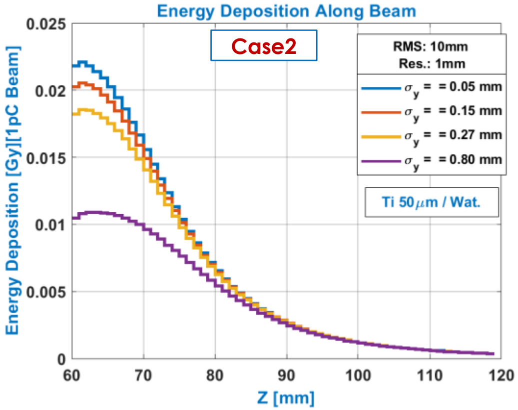
DOSE DISTRIBUTIONS

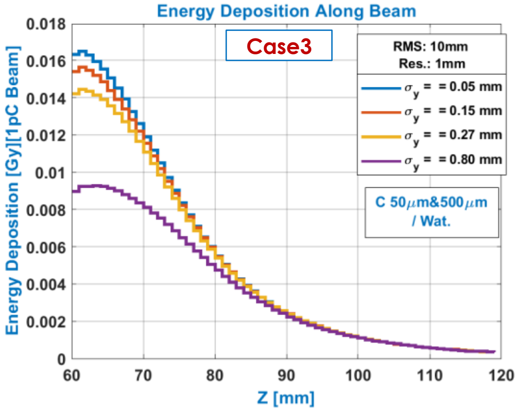
Focused center of water

Focused initial beam RMS 10 mm:

- **In case on exit window FLUKA Monte Carlo simulations demonstrate that**
  - ➢ **the energy deposition along the path of the beam peaks at a depth of 5 mm in water.**
  - ➢ **The energy deposition, depending on the size of the beam at the focal point, leads to a change in the peak value, and a slight change in position with depth of water..**
- **In case exit window (on both)**
  - ➢ **the energy deposition along the path of the beam no have peaks.**
  - ➢ **the energy deposition behaves like the case parallel beam.**





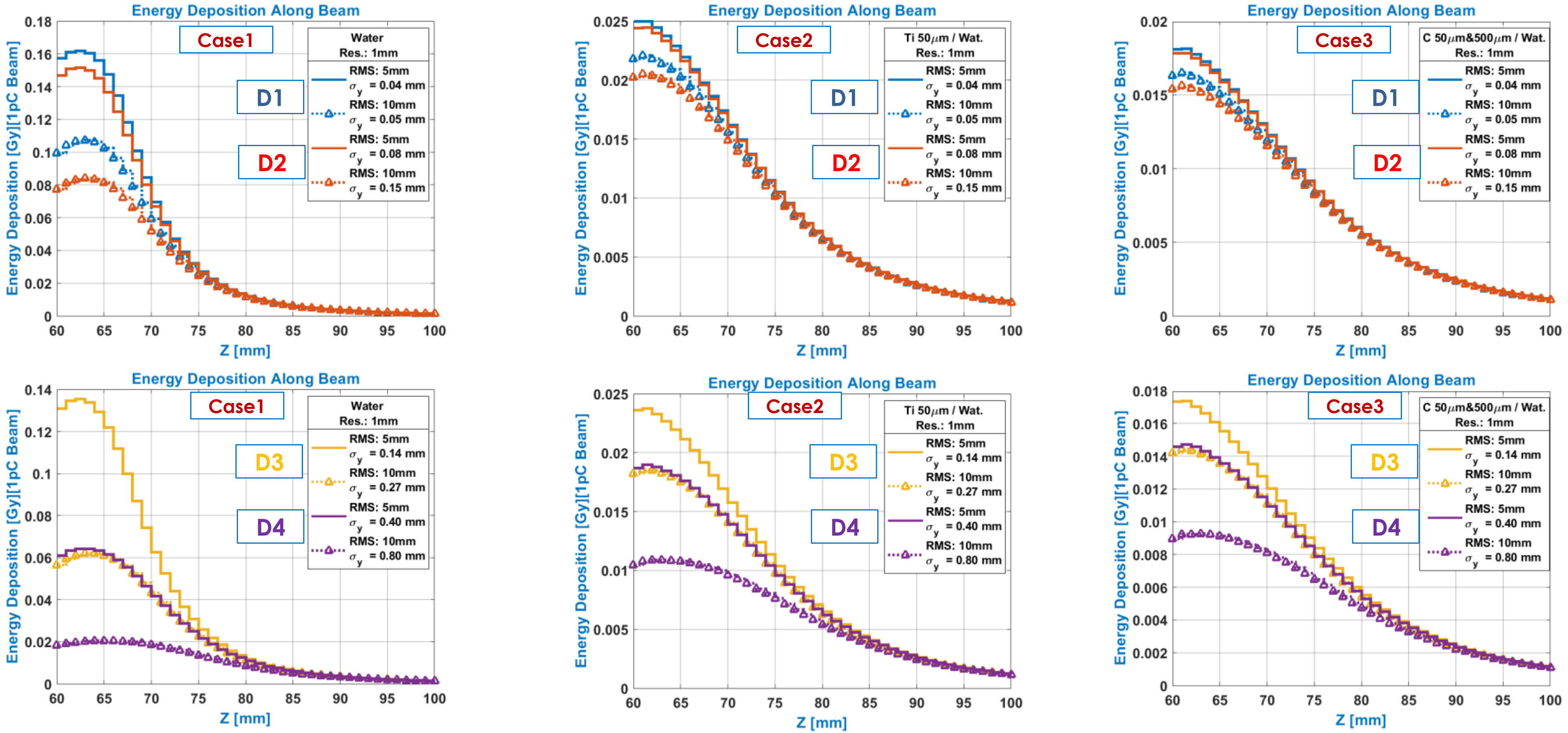


Energy deposition along the beam path where two transverse dimensions are integrated for central $\mathbf{1} \times \mathbf{1}\boldsymbol{mm}^{\mathbf{2}}$ space. The incident beam **RMS is 10mm.**
Line colors correspond for different **beam sizes at the focal point [D1,D2,D3 and D4].**

RMS: 5mm & 10 mm

DOSE COMPARISON

Focused center of water

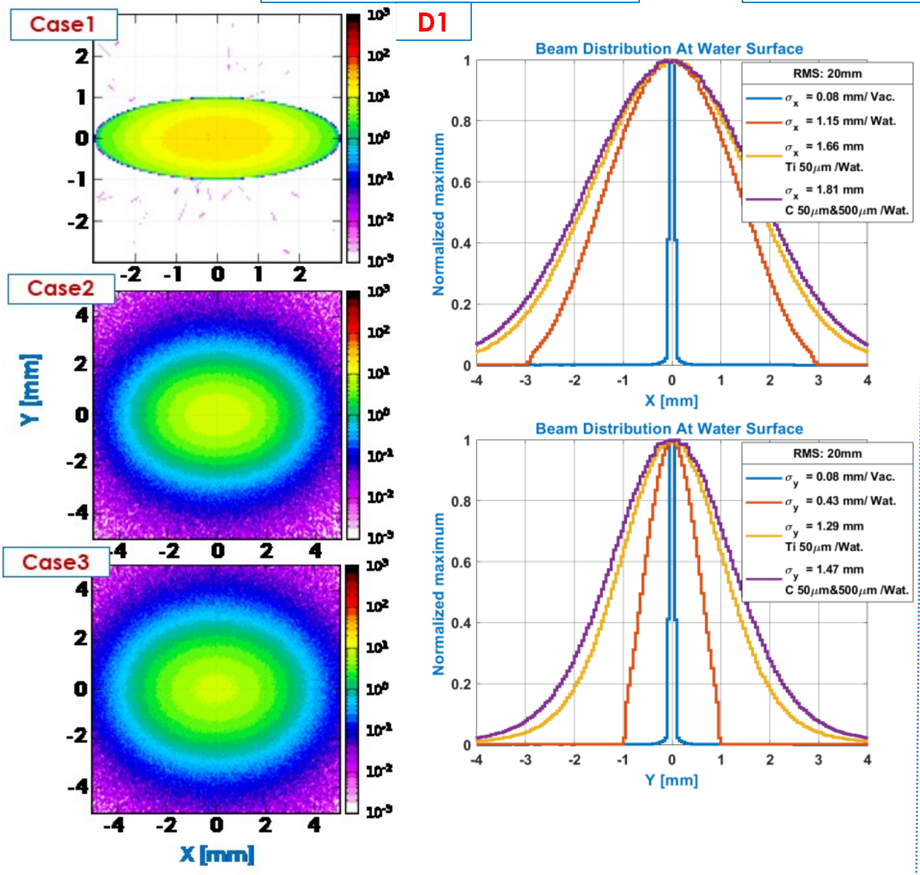


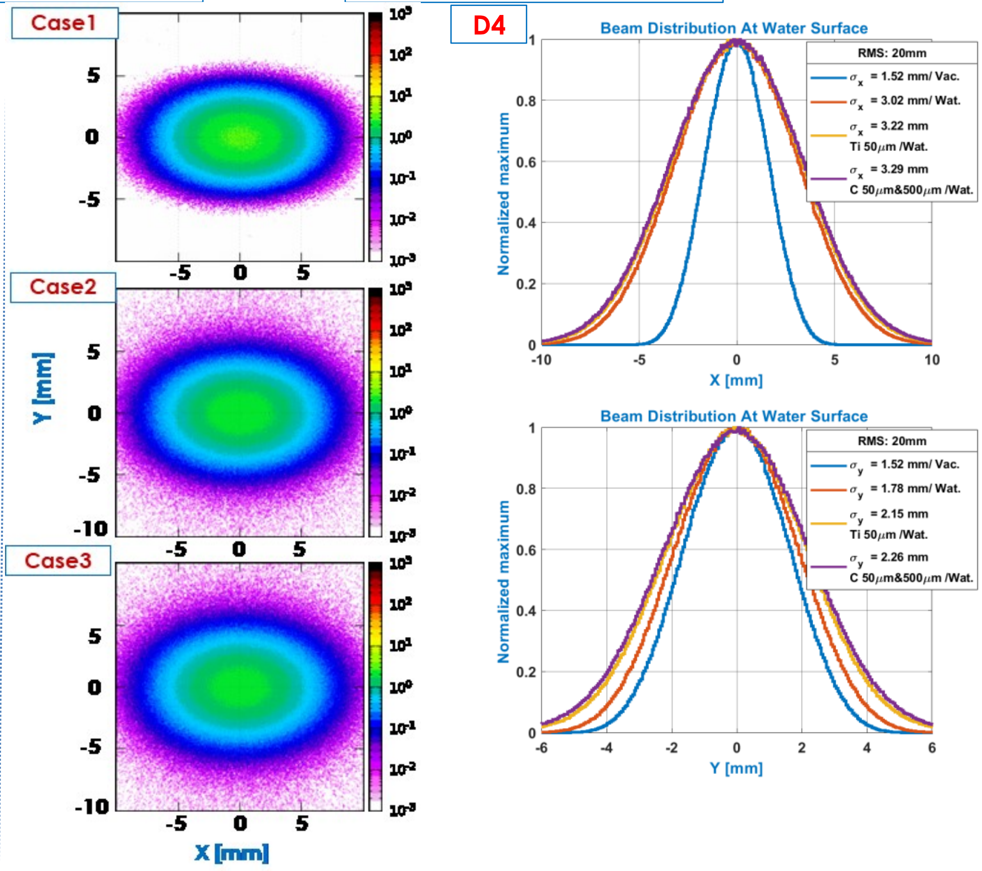
Comparison of energy deposition in water with a resolution of $1 \times 1 \times 1mm^3$ from a beam of **rms 5mm** and **10mm** sizes of different focus sizes. The left graph correspond to the **case 1**, the center graph correspond to the **case 2** and the right graph **case 3**. The deposition energy unit is **Gray** for 1pC beam.

RMS: 20 mm

BEAM PROFILE ON WATER SURFACE

Beam Profile on Water Surface | Focused center of water | Beam Profile on Water Surface





Beam profiles before entering to the water phantom area in **XY** plane for **rms 20mm**. The focusing point **rms** $\sigma_x = \sigma_y = 0.08mm$ [D1].

Beam profiles before entering to the water phantom area in **XY** plane for **rms 20mm**. The focusing point **rms** $\sigma_x = \sigma_y = 1.52mm$ [D4].

BEAM PROFILE ON WATER SURFACE

Incident Beam RMS 20mm / Focused Center of Water			
Vacuum (mm)	Water(mm)	$Ti\ 50\mu m$ & Water (mm)	$C\ 50/500\mu m$& Water (mm)
$\sigma_x = \sigma_y = 0.08$	$\sigma_x = 1.15$ $\sigma_y = 0.43$	$\sigma_x = 1.66$ $\sigma_y = 1.29$	$\sigma_x = 1.81$ $\sigma_y = 1.47$
$\sigma_x = \sigma_y = 0.23$	$\sigma_x = 1.34$ $\sigma_y = 0.65$	$\sigma_x = 1.80$ $\sigma_y = 1.37$	$\sigma_x = 1.93$ $\sigma_y = 1.55$
$\sigma_x = \sigma_y = 0.46$	$\sigma_x = 1.57$ $\sigma_y = 0.89$	$\sigma_x = 1.97$ $\sigma_y = 1.50$	$\sigma_x = 2.10$ $\sigma_y = 1.66$
$\sigma_x = \sigma_y = 1.52$	$\sigma_x = 3.02$ $\sigma_y = 1.78$	$\sigma_x = 3.22$ $\sigma_y = 2.15$	$\sigma_x = 3.29$ $\sigma_y = 2.26$

The table shows that by increasing the size of the beam at the focal point from **D1** to **D4** (by means of changing the magnetic fields), the electron beam size **RMS** in the horizontal and vertical directions **at the entrance to the water surface** increases by **2.6** and **4.1** times, respectively.

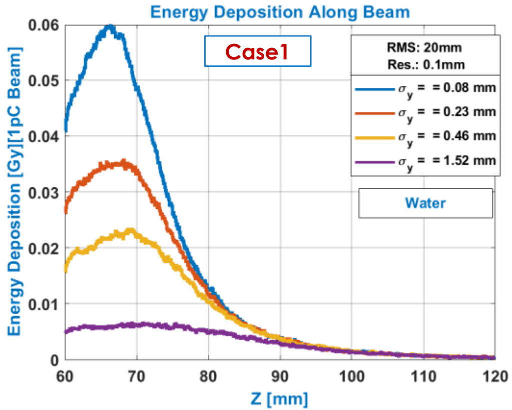
In the case of **exit window (Ti $50\mu m$)**, the beam size at the entrance water surface increases by **2.93** and **1.7** times.

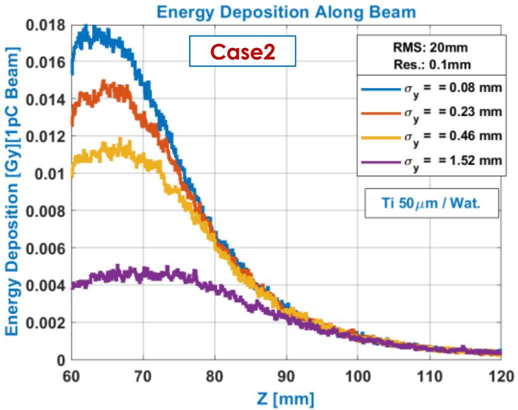
In the case of **exit window ($C\ 50/500\mu m$)**, the beam size at the entrance water surface increases by **1.81** and **1.53** times.

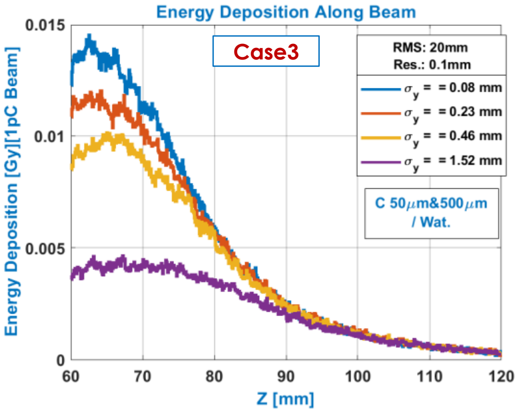
RMS: 20 mm

DOSE DISTRIBUTIONS

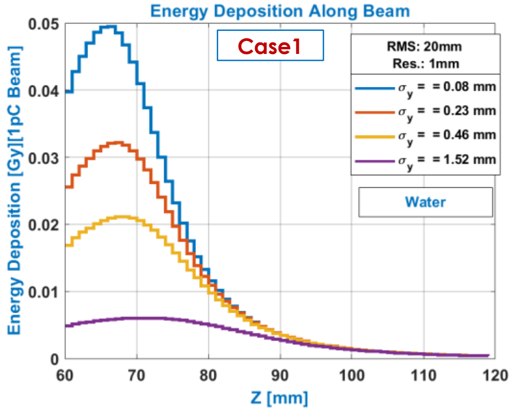
Focused center of water

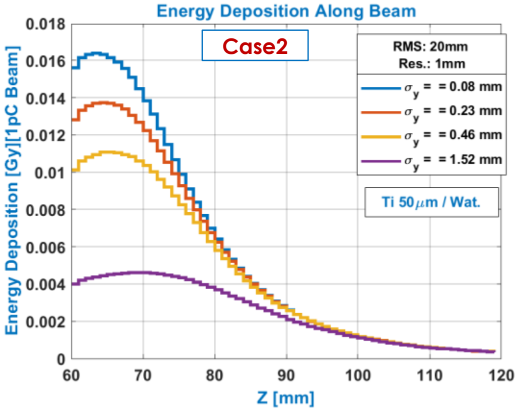


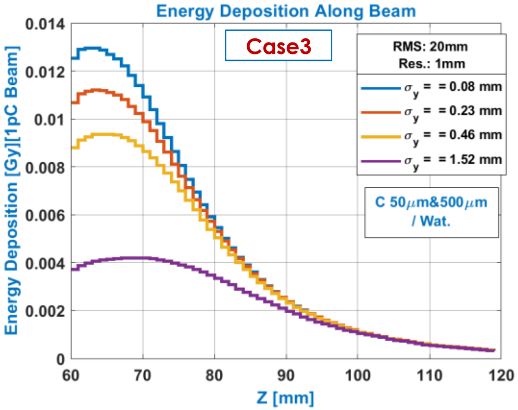




Energy deposition along the beam path where two transverse dimensions are integrated for central $\mathbf{0.1 \times 0.1 mm^2}$ space.







Energy deposition along the beam path where two transverse dimensions are integrated for central $\mathbf{1 \times 1 mm^2}$ space. The incident beam **RMS is 20mm.**
Line colors correspond for different **beam sizes at the focal point [D1,D2,D3 and D4].**

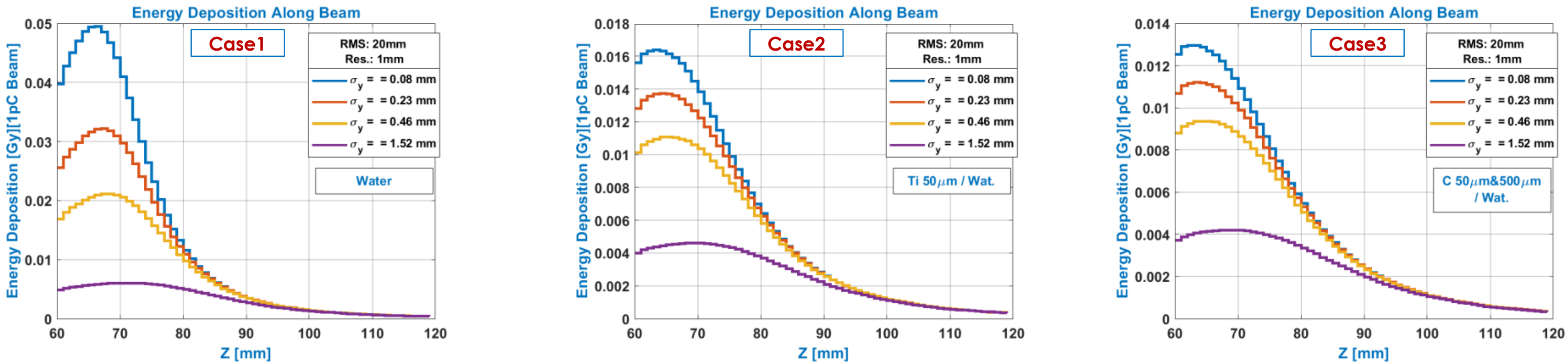
RMS: 20 mm

DOSE DISTRIBUTIONS

Focused center of water

Focused initial beam RMS 20 mm:

- **In case on exit window FLUKA Monte Carlo simulations demonstrate that**
  - ➢ **the energy deposition along the path of the beam peaks at a depth of in water.**
  - ➢ **The energy deposition, depending on the size of the beam at the focal point, leads to a change in the peak value, and a slight change in position with depth of water from 6 mm to 11mm .**
- **In case exit window (on both)**
  - ➢ **The energy deposition along the path of the beam reaches its maximum at a depth of 3 mm to 10 mm in water, depending on the size of the beam in focus.**

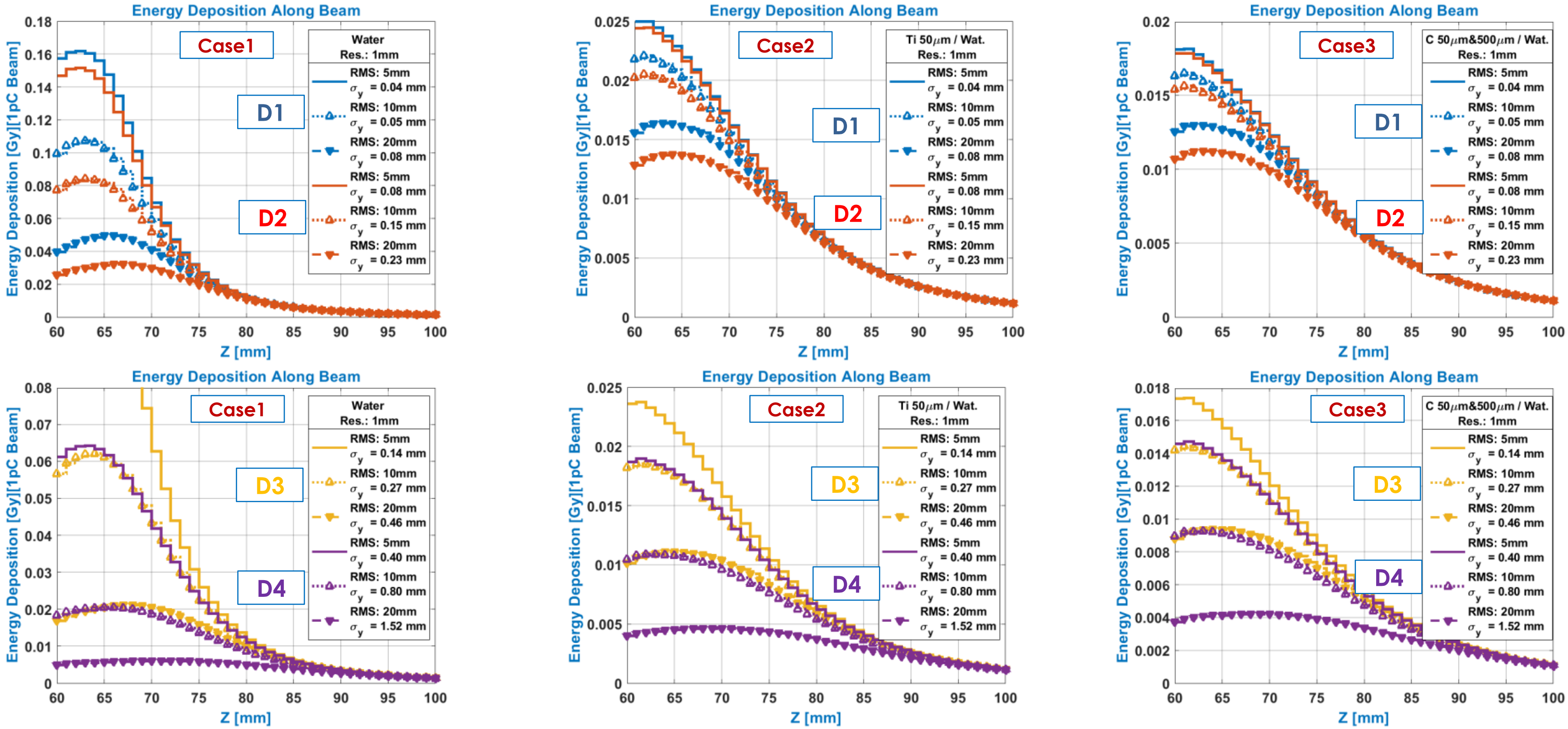


Energy deposition along the beam path where two transverse dimensions are integrated for central $\mathbf{1} \times \mathbf{1}\boldsymbol{mm}^{\mathbf{2}}$ space. The incident beam **RMS is 20mm.**
Line colors correspond for different **beam sizes at the focal point [D1,D2,D3 and D4].**

RMS: 5mm & 10mm & 20 mm

DOSE COMPARISON

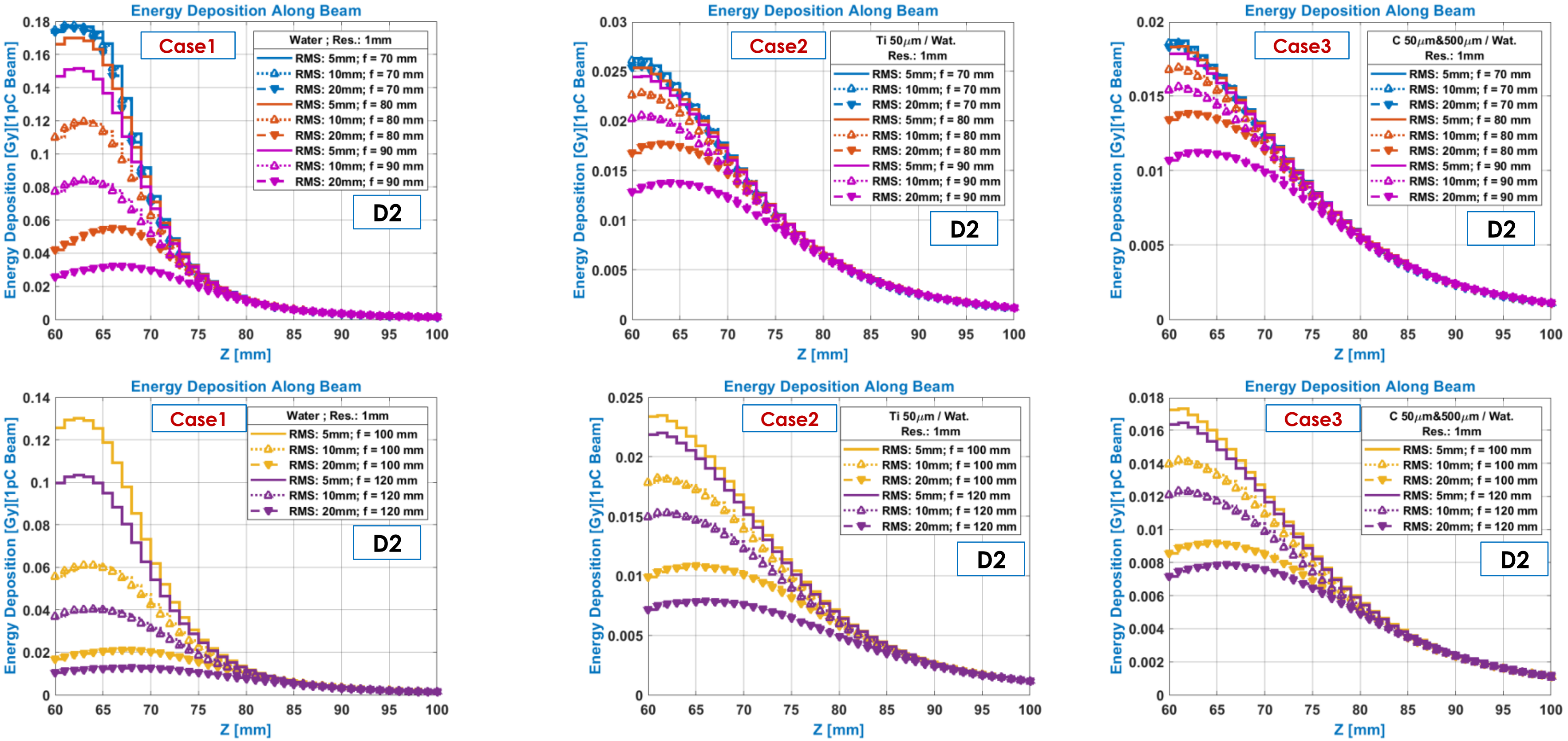
Focused center of water



Comparison of energy deposition in water with a resolution of $1 \times 1 \times 1mm^3$ from a beam of **RMS 5mm**, **10mm** and **20mm** sizes of different focus sizes. The left graph correspond to the **case 1**, the center graph correspond to the **case 2** and the right graph **case 3**. The deposition energy unit is **Gray** for 1pC beam.

Results for different focused beam positions

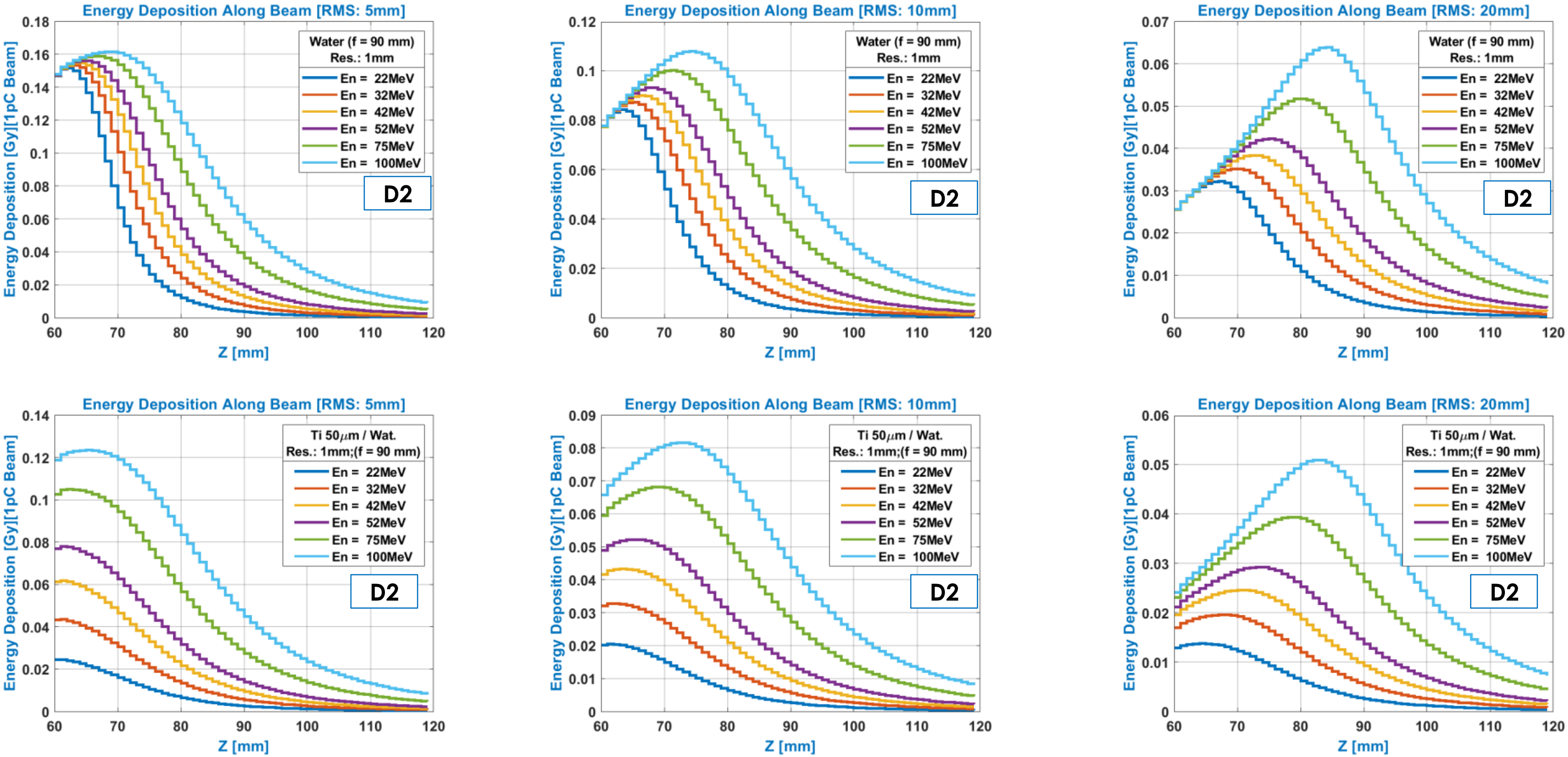
DIFFERENT FOCUSING POSITION



Comparison of energy deposition in water with a resolution of **1 mm** for different focusing position for initial beam **RMS 5mm, 10mm and 20mm.** The left graph correspond to the **case 1**, the center graph correspond to the **case 2** and the right graph **case 3**. The deposition energy unit is **Gray** for 1pC beam.

DIFFERENT ENERGY

Focused center of water



Comparison of energy deposition in water with a resolution of **1 mm** for different beam energies in the case of focusing at the center of the water and focus size [$\boldsymbol{D2}$] for the different beam **RMS** sizes **(5mm, 10mm and 20mm).**

CONCLUSION

Initial beam RMS 5 mm:

- **In case on exit window**
  - the energy deposition along the path of the beam peaks at a depth of 3 mm in water.
  - The energy deposition, depending on the size of the beam at the focal point, leads to a change in the peak value, but the position remains almost unchanged.
  - By increasing the energy of the beam, the energy deposition along the path of the peak position changes into the depth of the water from 3mm to 10mm.
- **In case exit window**
  - the energy deposition along the path of the beam no have peaks. **(case2 and case3)**
  - the energy deposition behaves like the case parallel beam. **(case2 and case3)**
  - By increasing the energy of the beam, the energy deposition along the path of the peak position changes into the depth of the water from 0.1mm to 7mm. **(case2)**

Initial beam RMS 10 mm:

- **In case on exit window**
  - the energy deposition along the path of the beam peaks at a depth of 5 mm in water.
  - The energy deposition, depending on the size of the beam at the focal point, leads to a change in the peak value, and a slight change in position with depth of water.
  - By increasing the energy of the beam, the energy deposition along the path of the peak position changes into the depth of the water from 5mm to 15mm.
- **In case exit window**
  - the energy deposition along the path of the beam no have peaks. **(case2 and case3)**
  - the energy deposition behaves like the case parallel beam. **(case2 and case3)**
  - By increasing the energy of the beam, the energy deposition along the path of the peak position changes into the depth of the water from 1mm to 14mm. **(case2)**

CONCLUSION

Initial beam RMS 20 mm:

- **In case on exit window**
  - **the energy deposition along the path of the beam peaks at a depth of in water.**
  - **The energy deposition, depending on the size of the beam at the focal point, leads to a change in the peak value, and a slight change in position with depth of water from 6 mm to 11mm .**
  - **By increasing the energy of the beam, the energy deposition along the path of the peak position changes into the depth of the water from 7mm to 24mm.**
- **In case exit window**
  - **The energy deposition along the path of the beam reaches its maximum at a depth of 3 mm to 10 mm in water, depending on the size of the beam in focus. (case2 and case3)**
  - **By increasing the energy of the beam, the energy deposition along the path of the peak position changes into the depth of the water from 5mm to 22mm. (case2)**

Thank you for your attention