Measured charge error: + or -?

Error bar treatment by bunch charge measurements

M. Krasilnikov, G. Vashchenko
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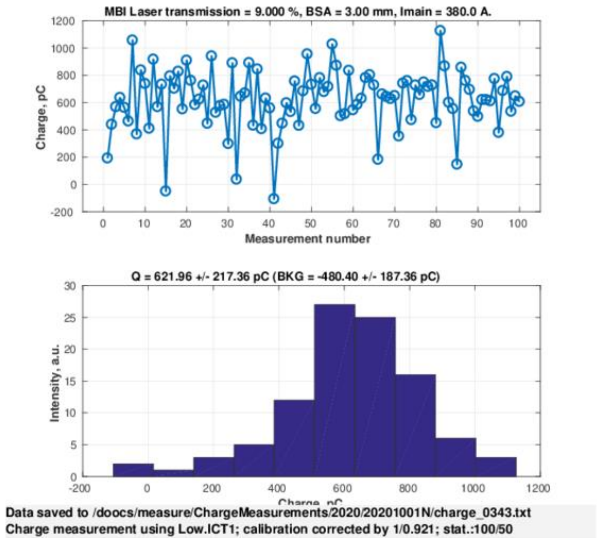
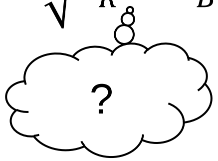


Measured charge: $Q = \langle Q \rangle \pm \sigma_Q$

Background: $B = \langle B \rangle \pm \sigma_B$
Raw signal: $R = \langle R \rangle \pm \sigma_R$

$$Q = R - B$$
$$\langle Q \rangle = \langle R \rangle - \langle B \rangle$$

$$\sigma_Q = \sqrt{\sigma_R^2 + \sigma_B^2}$$



A Summary of Error Propagation

Suppose you measure some quantities $a, b, c, \ldots$ with uncertainties $\delta a, \delta b, \delta c, \ldots$. Now you want to calculate some other quantity Q which depends on a and b and so forth. What is the uncertainty in Q? The answer can get a little complicated, but it should be no surprise that the uncertainties $\delta a, \delta b$, etc. "propagate" to the uncertainty of Q. Here are some rules which you will occasionally need; all of them assume that the quantities a, b, etc. have errors which are *uncorrelated* and *random*. (These rules can all be derived from the Gaussian equation for normally-distributed errors, but you are not expected to be able to derive them, merely to be able to use them.)

1 Addition or Subtraction

If Q is some combination of sums and differences, i.e.

$$Q = a + b + \cdots + c - (x + y + \cdots + z), \tag{1}$$

then

$$\delta Q = \sqrt{(\delta a)^2 + (\delta b)^2 + \cdots + (\delta c)^2 + (\delta x)^2 + (\delta y)^2 + \cdots + (\delta z)^2}. \tag{2}$$

In words, this means that the uncertainties add *in quadrature* (that's the fancy math word for the square root of the sum of squares). In particular, if $Q = a + b$ or $a - b$, then

$$\delta Q = \sqrt{(\delta a)^2 + (\delta b)^2}. \tag{3}$$

Charge measurements at PITZ

Background (noise) subtraction

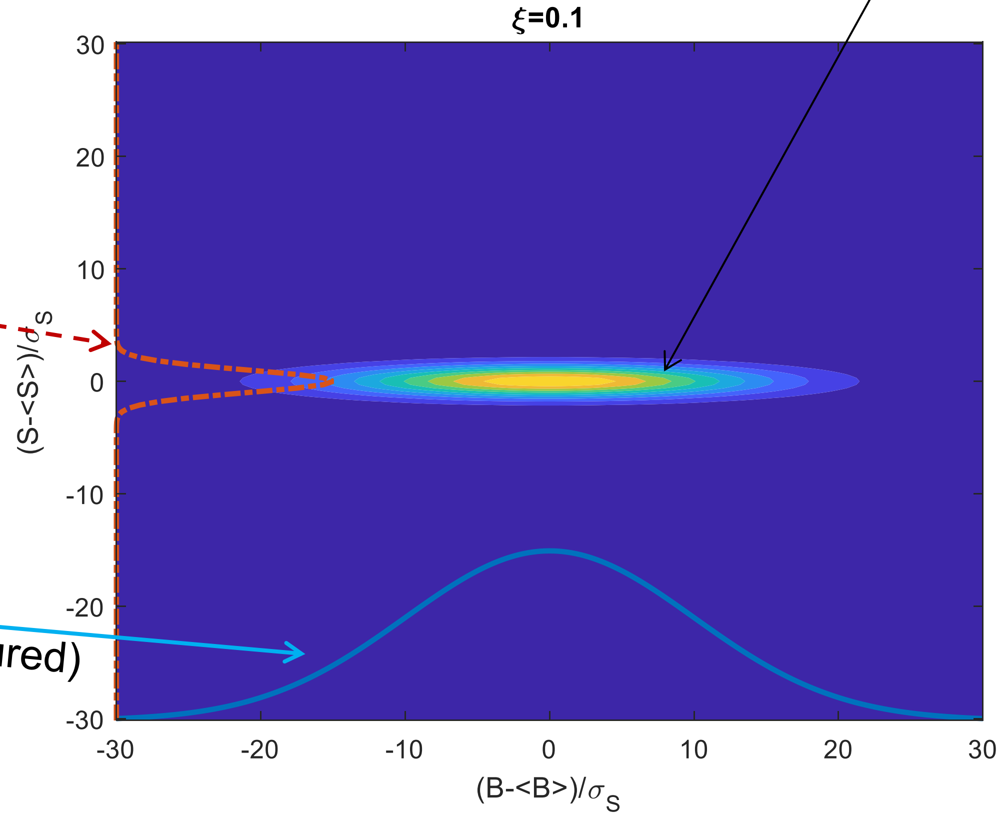
- Assumption – normal (Gaussian) distributions for signals:
  - B = Noise →OK
  - $Q = S$ = Signal ("pure") → generally notGaussian, but for illustration OK
  - B and S are independent

$$P_{s+n}(S,B) = \frac{1}{2\pi\sigma_S\sigma_B}\exp\left[-\frac{(S-\langle S\rangle)^2}{2\sigma_S^2}-\frac{(B-\langle B\rangle)^2}{2\sigma_B^2}\right]$$

Probability to have signal S and background B

$$P_{signal}(S) = \frac{1}{\sqrt{2\pi}\sigma_S}\exp\left[-\frac{(S-\langle S\rangle)^2}{2\sigma_S^2}\right]$$

unknown



$$\xi = \frac{\sigma_S}{\sigma_B}$$

$$P_{noise}(B) = \frac{1}{\sqrt{2\pi}\sigma_B}\exp\left[-\frac{(B-\langle B\rangle)^2}{2\sigma_B^2}\right]$$

known (measured)

Charge measurements

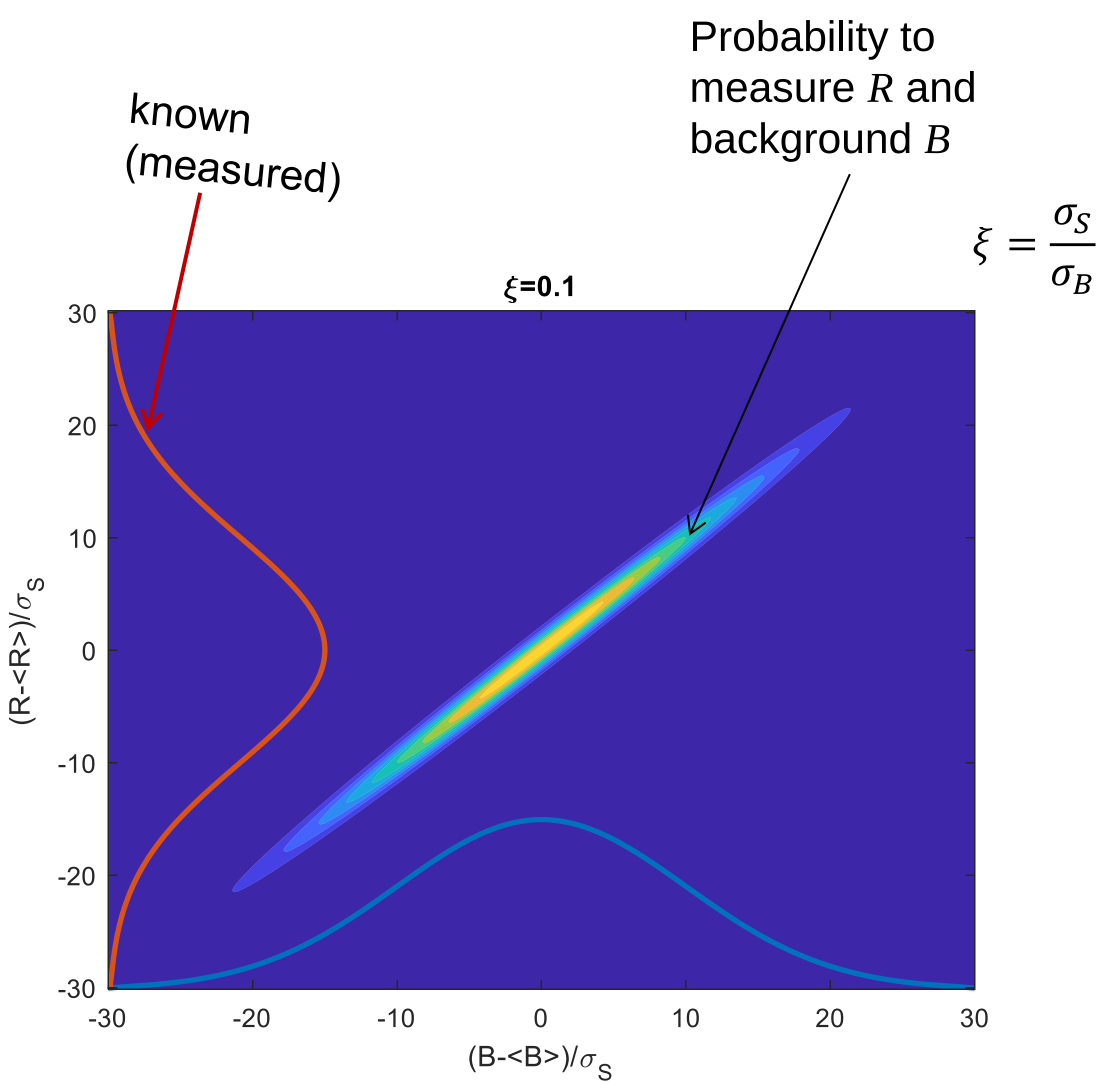
From “pure”signal-noise to raw signal-noise

$$P_{s+n}(S,B) = \frac{1}{2\pi\sigma_S\sigma_B}\exp\left[-\frac{(S-\langle S\rangle)^2}{2\sigma_S^2}-\frac{(B-\langle B\rangle)^2}{2\sigma_B^2}\right]$$

$$R = S + B$$

$$\langle R\rangle = \langle S\rangle + \langle B\rangle$$

$$P_{r+n}(R,B) = \frac{1}{2\pi\sigma_S\sigma_B}\exp\left[-\frac{(R-B-\langle R\rangle+\langle B\rangle)^2}{2\sigma_S^2}-\frac{(B-\langle B\rangle)^2}{2\sigma_B^2}\right]$$

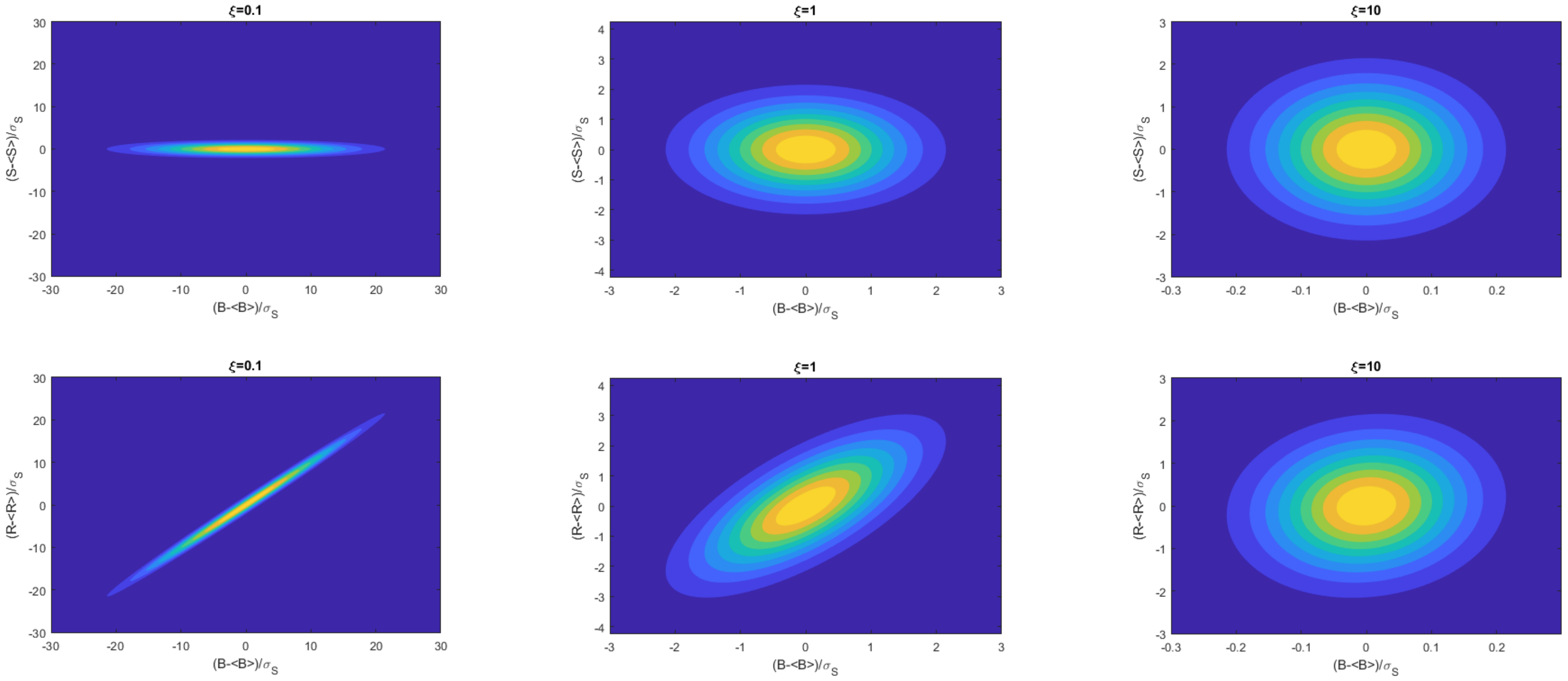


$$\sigma_R^2 = \iint_{-\infty}^{\infty}(R-\langle R\rangle)^2 P_{r+n}(R,B)\,dR\,dB = \sigma_S^2+\sigma_B^2 \quad \Rightarrow \quad \sigma_S^2 = \sigma_R^2 - \sigma_B^2$$

Charge measurements

Correlations for various ξ

$$\xi = \frac{\sigma_S}{\sigma_B}$$



Discussion

Correlation calculations

$$P_{r+n}(R,B) = \frac{1}{2\pi\sigma_S\sigma_B}\exp\left[-\frac{(R-B-\langle R\rangle+\langle B\rangle)^2}{2\sigma_S^2}-\frac{(B-\langle B\rangle)^2}{2\sigma_B^2}\right]$$

$$\sigma_R^2 = \iint_{-\infty}^{\infty}(R-\langle R\rangle)^2 P_{r+n}(R,B)\,dR\,dB = \sigma_S^2+\sigma_B^2$$

$$\sigma_{RB} = \iint_{-\infty}^{\infty}(R-\langle R\rangle)(B-\langle B\rangle)P_{r+n}(R,B)\,dR\,dB = \sigma_B^2$$

e.g., https://www.asc.ohio-state.edu/gan.1/teaching/spring04/Chapter4.pdf

- If x and y are correlated, define σ_{xy} as:

$$\sigma_{xy} = \frac{1}{N}\sum_{i=1}^{N}(x_i-\mu_x)(y_i-\mu_y)$$

$$\sigma_Q^2 = \sigma_x^2\left(\frac{\partial Q}{\partial x}\right)_{\mu_x}^2 + \sigma_y^2\left(\frac{\partial Q}{\partial y}\right)_{\mu_y}^2 + 2\left(\frac{\partial Q}{\partial x}\right)_{\mu_x}\left(\frac{\partial Q}{\partial y}\right)_{\mu_y}\sigma_{xy}$$

correlated errors

Our case:

$x = R;\ \mu_x = \langle R\rangle;\ \sigma_x = \sigma_R$

y= $B;\ \mu_y = \langle B\rangle;\ \sigma_y = \sigma_B$

$Q(x,y) = Q(R,B) = R-B;\ \frac{\partial Q}{\partial x} = \frac{\partial Q}{\partial R} = 1;\ \frac{\partial Q}{\partial y} = \frac{\partial Q}{\partial B} = -1$

$\sigma_Q^2 = \sigma_R^2\cdot 1^2 + \sigma_B^2\cdot(-1)^2 + 2\cdot 1\cdot(-1)\cdot\sigma_{RB}$

$$\sigma_Q = \sigma_S = \sqrt{\sigma_R^2 + \sigma_B^2 - 2\sigma_{RB}} = \sqrt{\sigma_R^2 + \sigma_B^2 - 2\sigma_B^2} = \sqrt{\sigma_R^2 - \sigma_B^2}$$

?

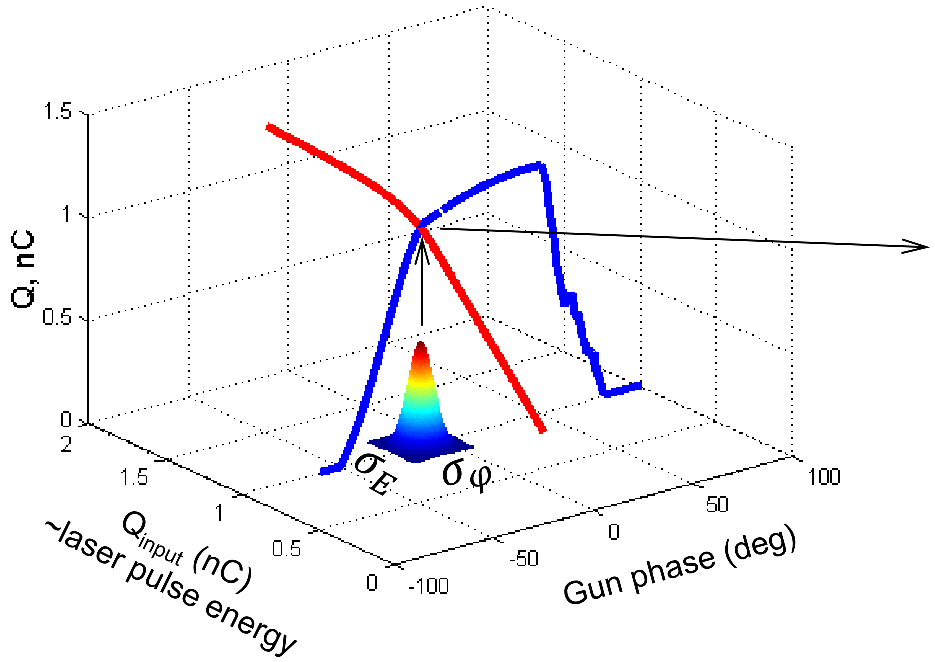
Discussion

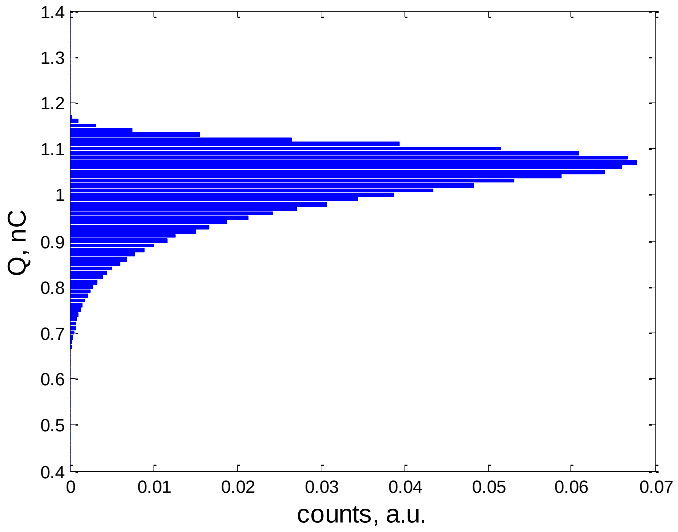
Correlation calculations

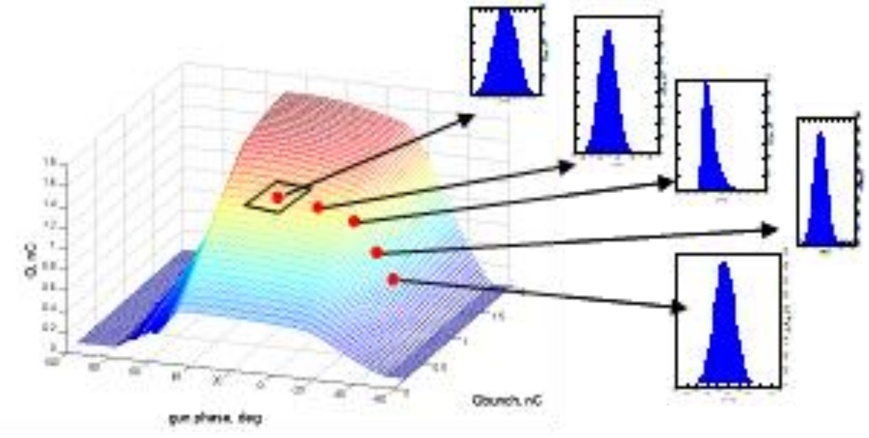
$$\sigma_Q = \sqrt{\sigma_R^2 - \sigma_B^2}$$

? Sometimes imaginary?

- Not enough statistics + very low charge (“pure” signal) error / very high noise (background) error
- Signal probability distribution is not Gaussian!







M. Krasilnikov, F. Stephan “Beam based monitoring of the RF Photo Gun stability at PITZ,”inProceedings of the14thBeam Instrumentation Workshop, BIW10, Santa Fe,New Mexico, USA, May 2010. (TUPSM104).

Conclusions and Outlook

Measured charge error

Measured charge: $Q = \langle Q \rangle \pm \sigma_Q$

Background: $B = \langle B \rangle \pm \sigma_B$
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$$\langle Q \rangle = \langle R \rangle - \langle B \rangle$$

$$\sigma_Q = \sqrt{\sigma_R^2 - \sigma_B^2}$$

- High noise jitter + non Gaussian charge probability distribution could lead to imaginary number of σ_Q

→ Analysis of the charge histogram is required (skewness, kurtosis?)

But for this one needs knowledge of the functional charge dependence on gun phase and laser pulse energy

Assumptions:

  - Gaussian distributions of gun phase and laser pulse energy jitters
  - Noise distribution is not necessarily to be a Gaussian one, but if it is the treatment might be simplified

- ?Impact onto our image analysis?