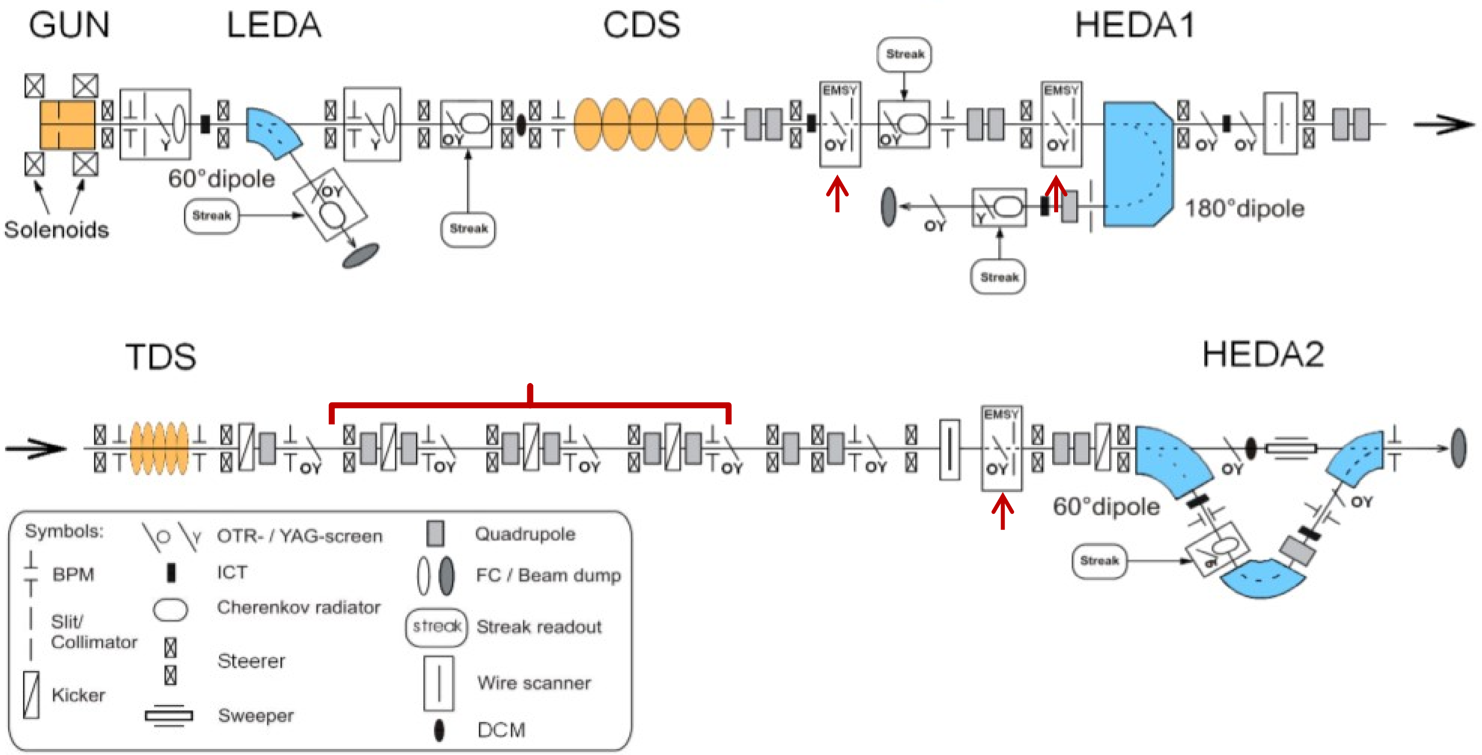
Thesis Outline, Initial Simulations for Transverse Phase Space

Namra Aftab

PITZ Physics Seminar, DESY

PITZ beamline layout



Thesis Outline

Motivation

- Characterize 6D phase space of the beam
- Investigate coupling between transverse/longitudinal dimensions
- Analyze and correct asymmetries to further optimize the photo injector.

Topics

1. Transverse Phase Space
   I. Virtual Pepper Pot
   II. Tomography 4D

2. Longitudinal Phase Space
   I. TDS + HEDA2
   II. Tomography with Booster

3. Transverse to Longitudinal Coupling

Transverse Phase Space

- 2D $x - x'$ and $y - y'$ sub phase spaces: non coupling elements (normal quadrupole, dipole, accelerating gap)
- 4D phase space: coupling elements (solenoid, skew quadrupole and RF kicker)
- 4D beam matrix that describes the transverse statistical properties of the beam

$$\sigma^{4D} = \begin{pmatrix} <x^2> & <xx'> & <xy> & <xy'> \\ <xx'> & <x'^2> & <x'y> & <x'y'> \\ <xy> & <x'y> & <y^2> & <yy'> \\ <xy'> & <x'y'> & <yy'> & <y'^2> \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ {\sigma_{xy}}^T & \sigma_{yy} \end{pmatrix}$$

$$U^T \sigma^{4D} U = \begin{pmatrix} \epsilon_1 & 0 & 0 & 0 \\ 0 & \epsilon_1 & 0 & 0 \\ 0 & 0 & \epsilon_2 & 0 \\ 0 & 0 & 0 & \epsilon_2 \end{pmatrix}$$

$$\epsilon_{4D} = \epsilon_1 \epsilon_2 \leq \epsilon_x \epsilon_y$$

$$C = \sqrt{\frac{\epsilon_x \epsilon_y}{\epsilon_1 \epsilon_2}} - 1 \geq 0$$

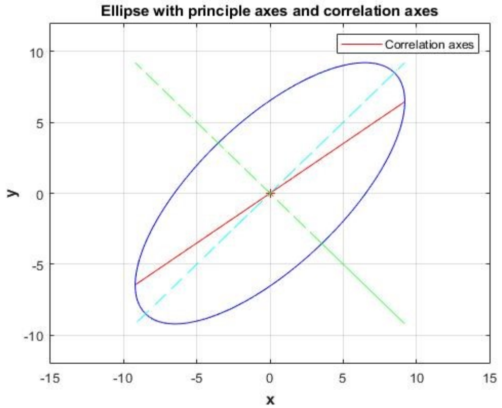
- PS density and emittance inferred from beam properties(position, profile, momenta) which are available from experimental setup
- Slit Scan Technique for high-current low-energy beams
- Three emittance measuring setups at PITZ: EMSY1, EMSY2 and EMSY3

Slit Scan Technique(2D) to Virtual Pepper Pot(4D)

1) Slit scan technique
   i. Main idea
   ii. Algorithm for 2D phase space reconstruction
   iii. Systematic limitations and error analysis
2) Pepper pot technique
   i. Main Idea
   ii. Limitations
3) Virtual pepper-pot technique
   i. Main idea
   ii. Algorithm for 4D phase space reconstruction
   iii. Astra simulations (ideal case, simulations with various couplings) - reconstruction using VPP algorithm
   iv. Algorithm to be applied to the experimental data at PITZ, systematic limitation and error sources
   v. Simulation of measurements (ASTRA results on top of real noise frames from experiments
   vi. Treatment and interpretation of experimental data

Coupling Analysis

Introduction

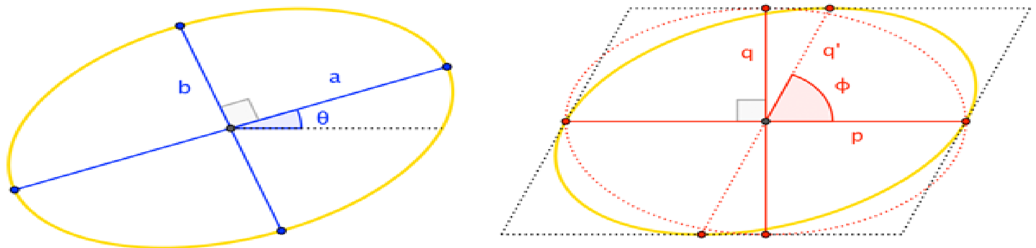


- 2D emittance calculation: linearly correlated transverse momentum removed from the distribution by a straight momentum becomes $p'_{x,i} = p_{x,i} - m.x$, $m = \frac{<xp_x>}{<x^2>}$
- Emittance is just given by the product of the rms values of both coordinates as

$$\epsilon_{x,n,rms} = \frac{1}{m_o c}\sqrt{< x^2 >< p'^2_x >} \qquad \epsilon_{x,n,rms} = \frac{1}{m_o c}\sqrt{< xx >< x'^2 > - < xx' >^2}$$

- 4D emittance calculation and phase space manipulation: rms value of a two dimensional distribution develops if it is rotated by some angle
- The original distribution $\rho(x_o, y_o)$ can be rotated by an angle φ using the rotation matrix
- $R = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$
- $x = x_o \cos\varphi - y_o \sin\varphi$
  $y = x_o \sin\varphi + y_o \cos\varphi$
- $x = x_o \cos\theta \cos\varphi - y_o \sin\theta \sin\varphi$
  $y = x_o \cos\theta \sin\varphi + y_o \sin\theta \cos\varphi$
- $\tan\theta_{max} = -\frac{y_o}{x_o}\tan\varphi$

$$\tan\varphi_{corr} = -\frac{1}{2}\tan 2\varphi \, \frac{y^2_{max} - x^2_{max}}{x^2_{max}}$$

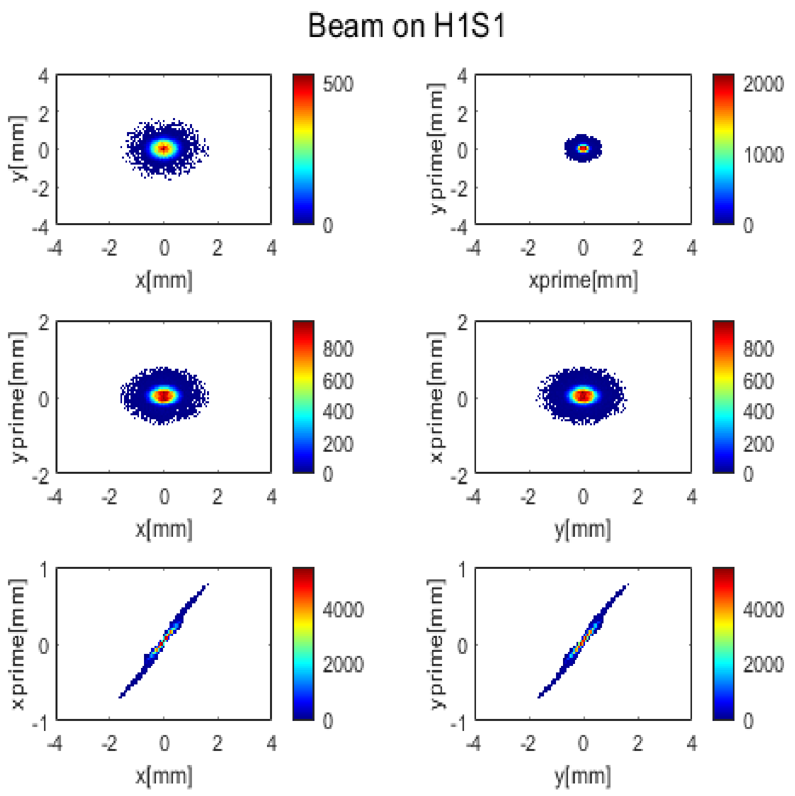


Rotated ellipse = horizontally sheared ellipse

Coupling Analysis

- ASTRA beam: 0.2 million particles
- geometric emittance = 0.0113 mm mrad
- normalized emittance = 0.3981 mm mrad

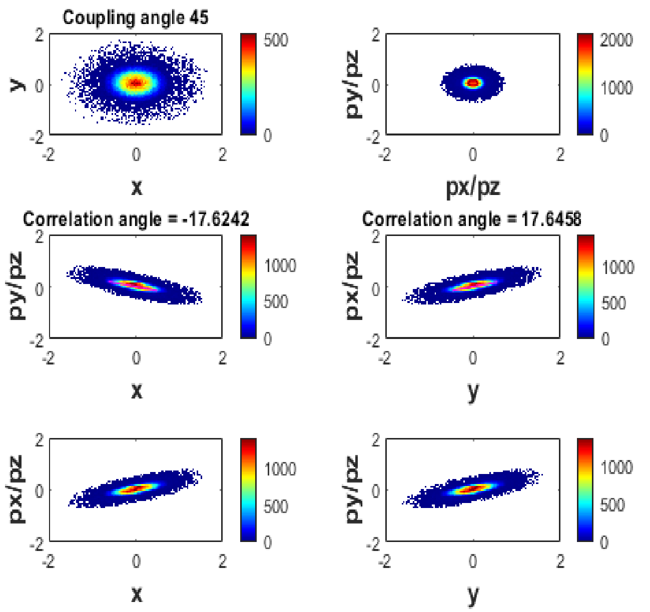
No coupling



Coupling Analysis

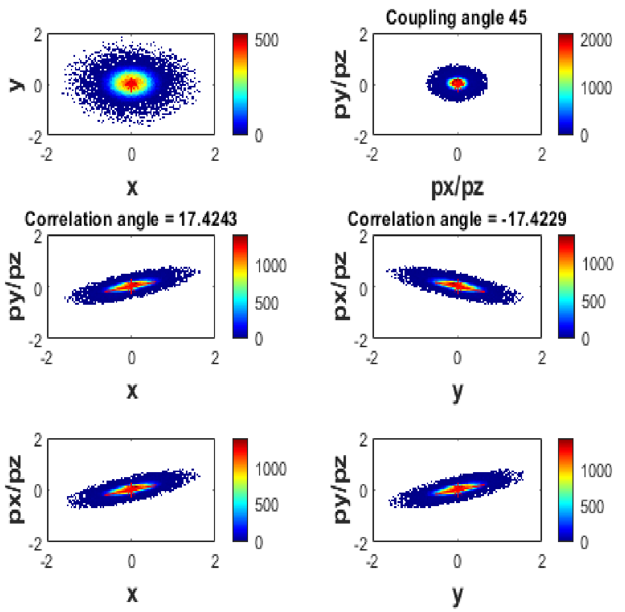
xy coupling

Beam on H1S1



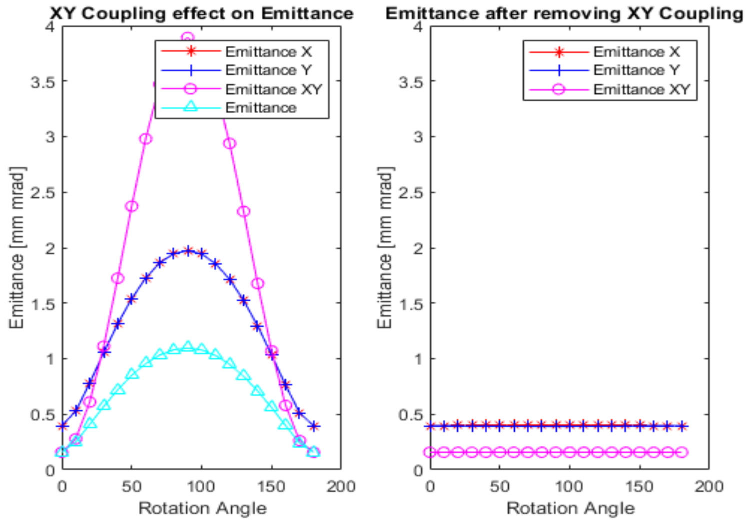
x’y’ coupling

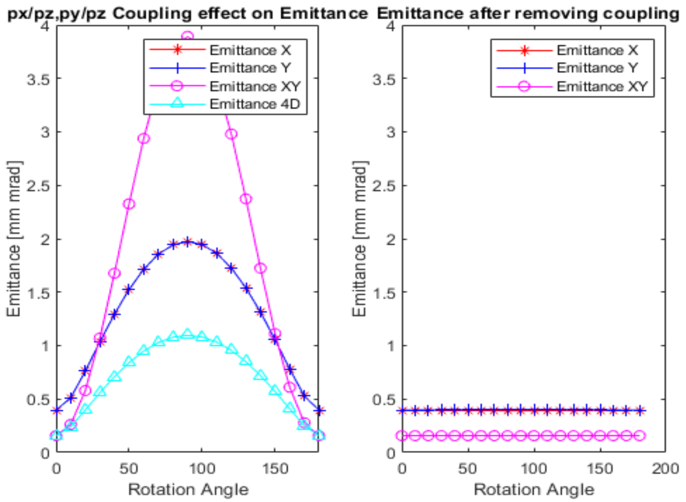
Beam on H1S1



$$\epsilon_{x,n,rms} = \frac{1}{m_o c}\sqrt{< xx >< x'^2 > - < xx' >^2 - < xy' >^2}$$

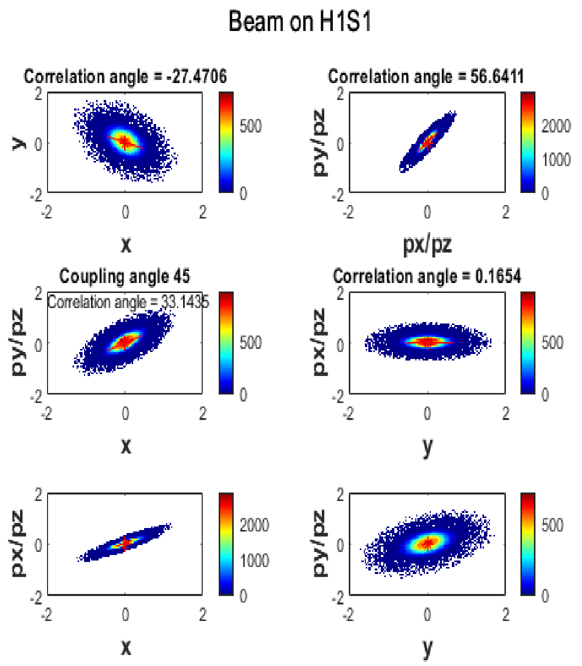
$$\epsilon_{y,n,rms} = \frac{1}{m_o c}\sqrt{< yy >< y'^2 > - < yy' >^2 - < yx' >^2}$$



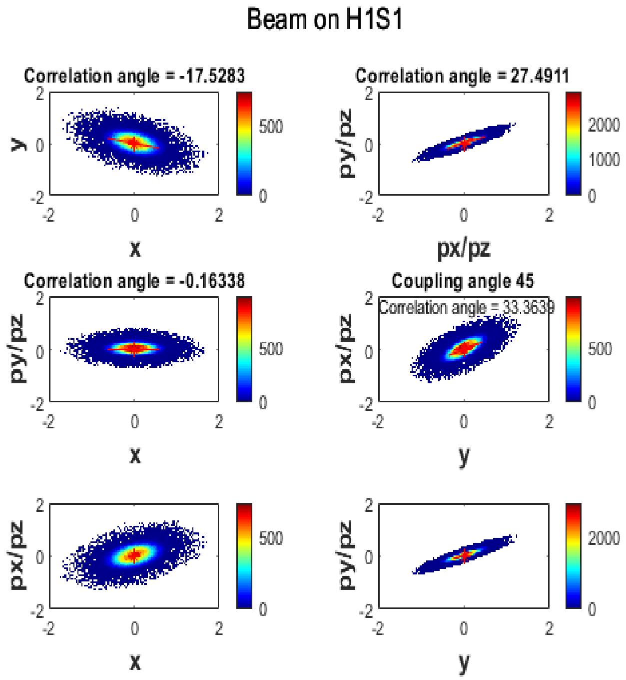


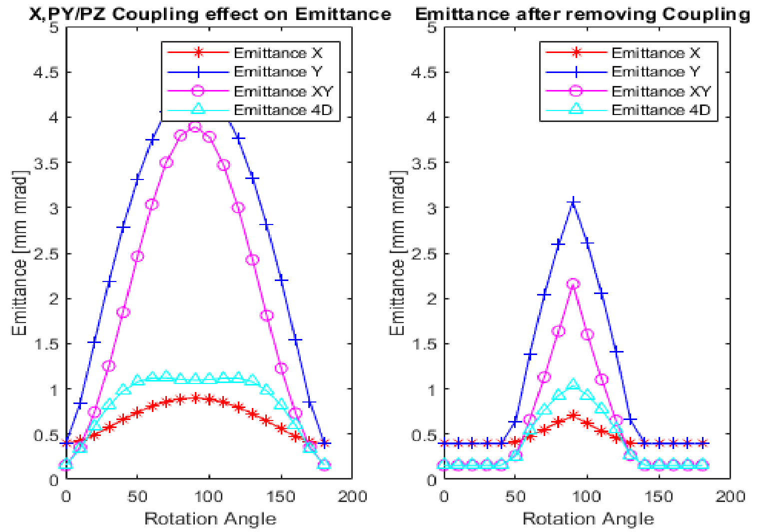
Coupling Analysis

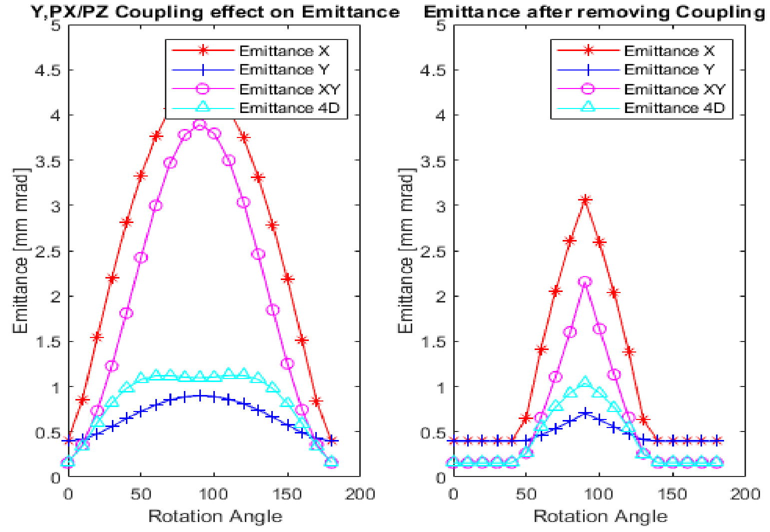
xy' coupling



yx' coupling







- There correlation angle can be obtained by the slope of xy'
- The rotation angle θ can be calculated from the correlation angle φ_{corr}

$$\tan 2\theta = -2\tan\varphi_{corr}\frac{x_{max}^2}{y_{max}^2 - x_{max}^2}$$

$$R' = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}$$

Coupling Analysis

Outlook

- Introduction of eigen-emittances (diagonalize the 4D matrix) and Invariants
- Application of 4D rotations to the uncoupled phases spaces, then recovering initial moments/x-,y- (eigen) emittances.
- After introduction of coupling, slit scan files(.imc, .bkc, .log) will be generated in Matlab(code developed by summer student) for transverse phase space analysis
- Development of tool for 4D analysis of experimental data: Virtual Pepper Pot

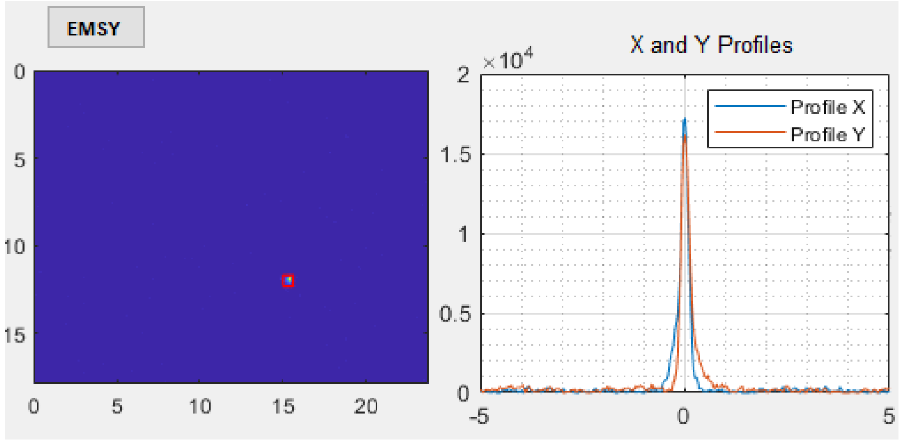
Virtual Pepper Pot Technique(VPP)

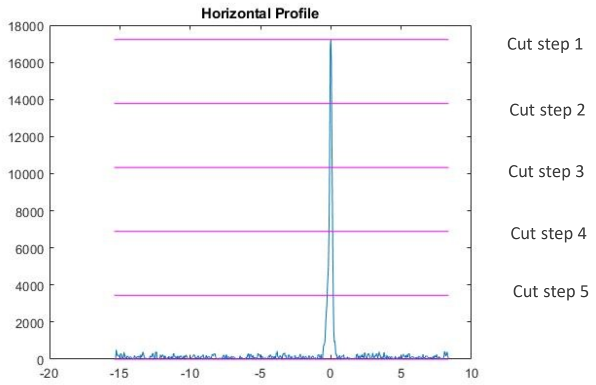
Introduction

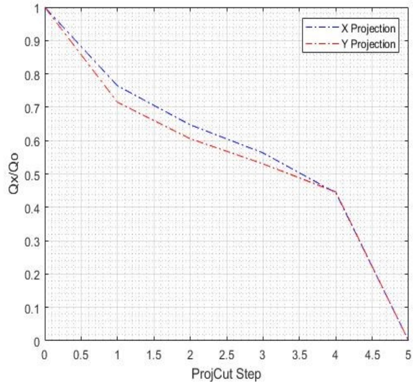
- Crossing of Horizontal and Vertical Slits
- Imitation of Pepper pot but multi-shot
- Challenge: pre processing beamlets before crossing

Algorithm (\\afs\ifh.de\group\pitz\doocs\measure\TransvPhSp\2020\ProjEmittance\202007\01Mstat\364A)

1. EMSY Image
   i. Process EMSY Image and produce projections
   ii. Shift projections to center of mass
   iii. EMSY projections charge cuts







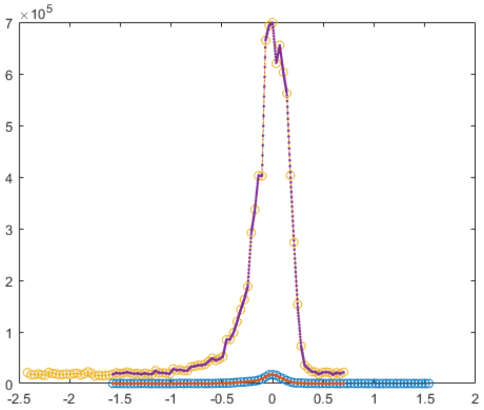
Qo: integral of full projection
Qx: integral of projection below a cut step

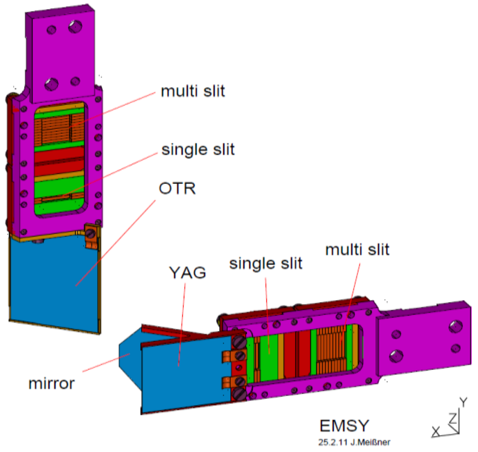
cutstep=1 →ProjCut=0 →charge(Qx/Qo)=1

VPP

Algorithm

- Beamlets
  - Sum of pixels

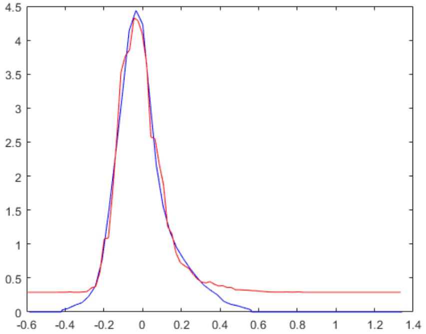


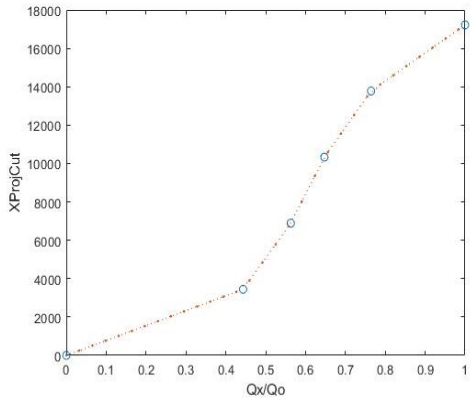


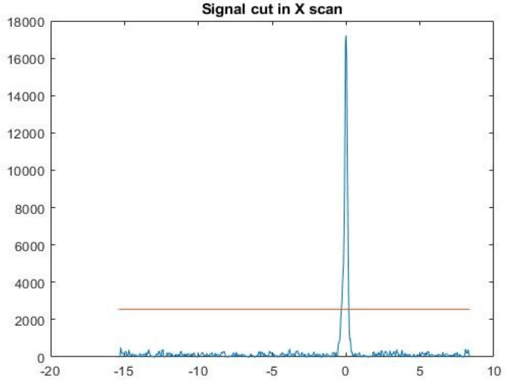
- SoP to EMSY fit
  - Least square fitting

$$\varphi(\Delta, thresh, A) = sqrt(sum(|\mathrm{ProjEmsy} - \mathrm{threshold} - \mathrm{A.SoP}|^2 * \mathrm{ProjEmsy}))$$

  - Charge cut (sum(ProjEmsy<threshold)/sum(ProjEmsy))







VPP

Algorithm: Next Steps

- Renormalizing beamlets according to XprojCut, YprojCut
- Calculating 2D cuts
- Pepper Pot beamlets
- PPemsy images
- 4D emittance, Invariants, Coupling coefficients

THANK YOU