FEL simulation code: GENESIS 1.3

FEL concept

GENESIS 1.3

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PITZ physics seminar
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HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

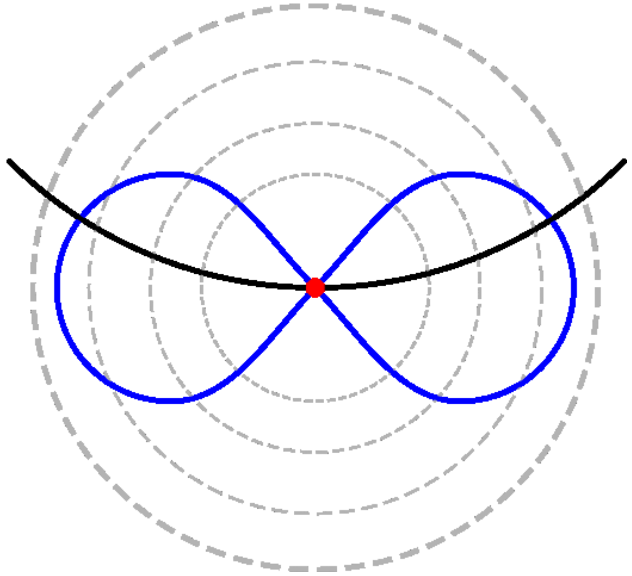
Contents

- Synchrotron radiation
- Free electron laser (FEL) concept
- FEL codes
- GENESIS 1.3 overview
- GENESIS updates

Synchrotron radiation

- Accelerated charge
  - Larmor formula

$$P = \frac{q}{6\pi\epsilon_0 m_0^2 c^3}\left(\frac{\mathrm{d}p}{\mathrm{d}t}\right)^2$$



Synchrotron radiation

- Accelerated charge
  - Larmor formula
- Bending magnets (1st and 2nd gen.)
  - Relativistic Doppler
  - Relativistic contraction
  - Photon energy increase, $E_{ph} \propto \gamma^2$
  - Opening angle, $\theta \sim 1/\gamma$

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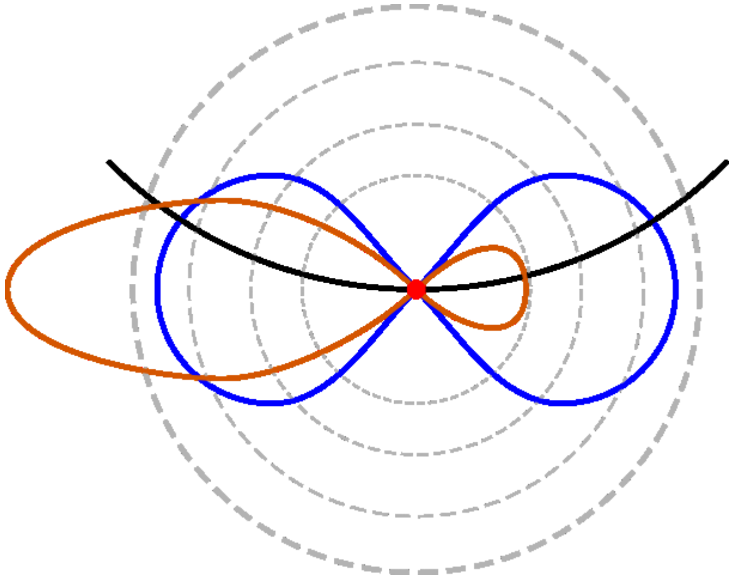

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  - Radiation power
  - Constructive interference

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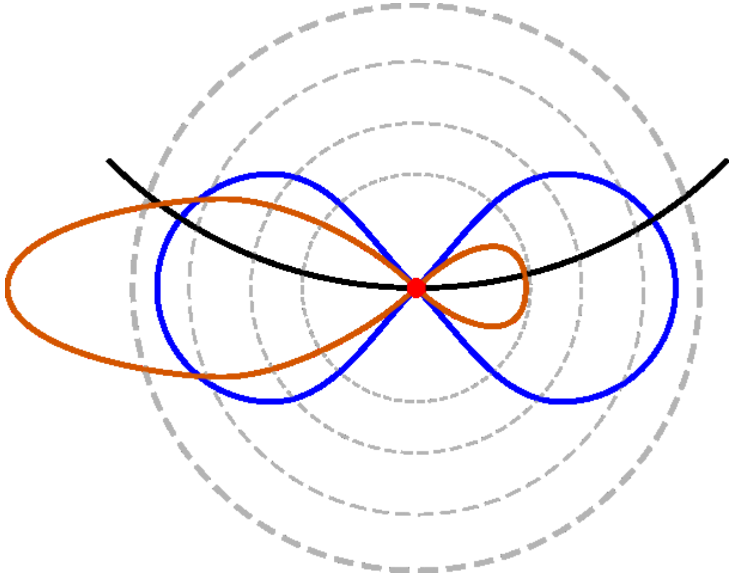

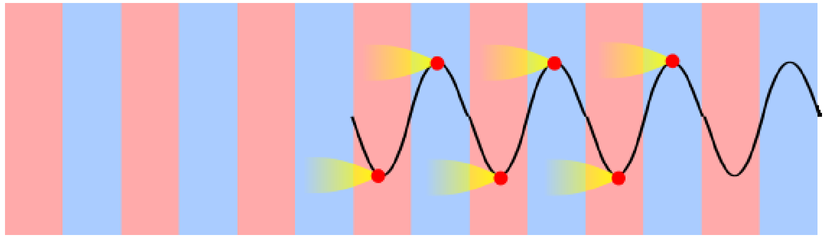

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- FELs
  - Electron motion in SR
  - Coherence
  - Peak brilliance

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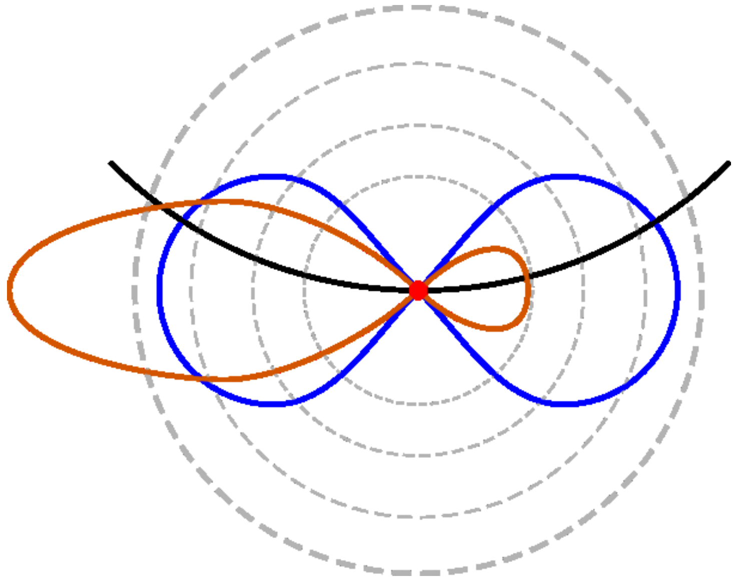

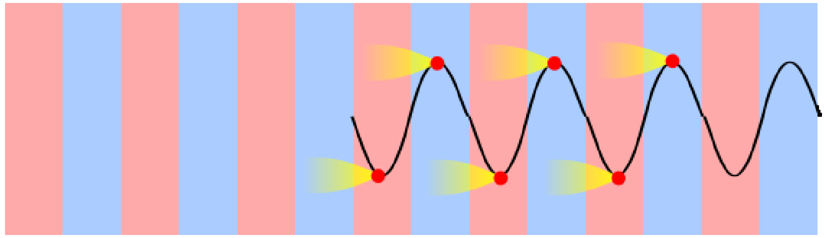

$$\text{Brilliance} = \frac{\text{Flux}}{\text{area} \times \text{solid angle} \times \text{bandwidth}}$$

Device	Peak. Br.
Dipole	~10^{13}
Wiggler	~10^{15}
Undulator	~10^{21}
FEL	~10^{33}

Key concepts I

- Ponderomotive force
  - Electron motion in inhomogeneous EM wave
  - Ponderomotive phase

$$\theta = (k_{und} + k_{rad})z + \omega_{rad}t$$

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- Separatrix
  - Longitudinal phase space
    - "Bucket" of the ponderomotive wave
  - Pendulum motion - oscillation
  - Untrapped motion – desynchronized
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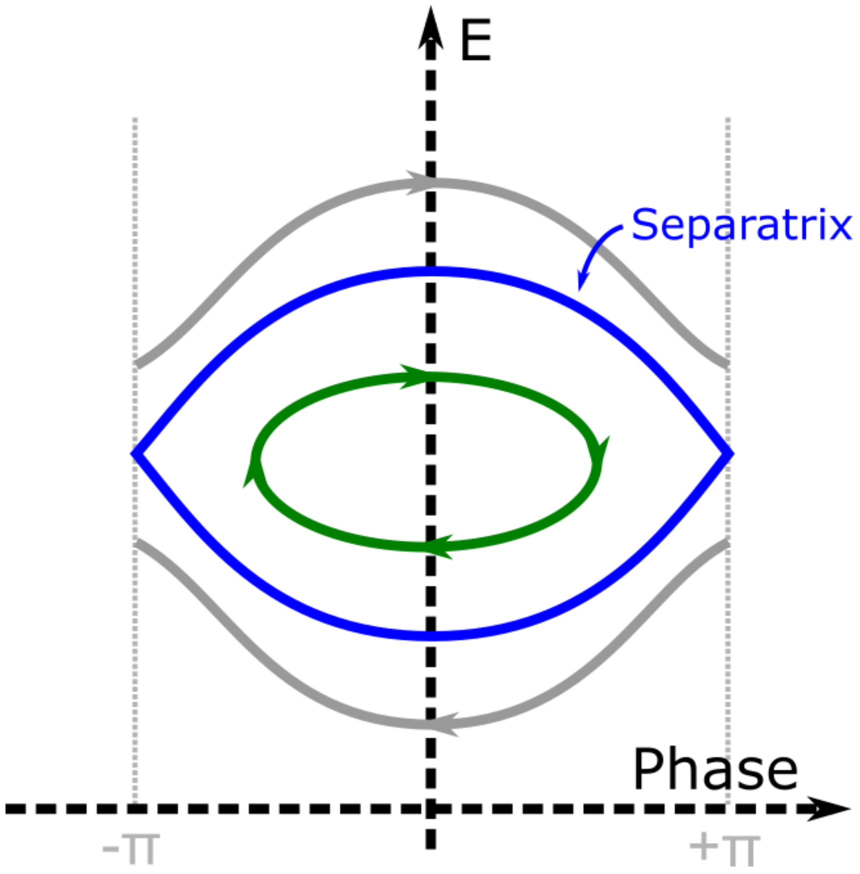

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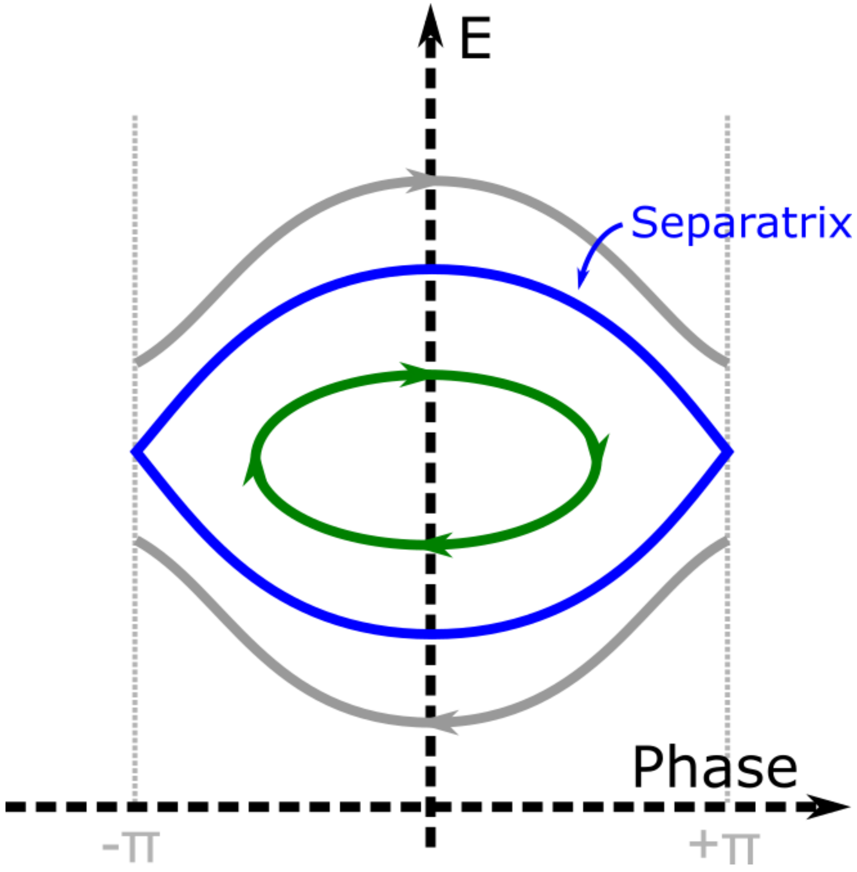
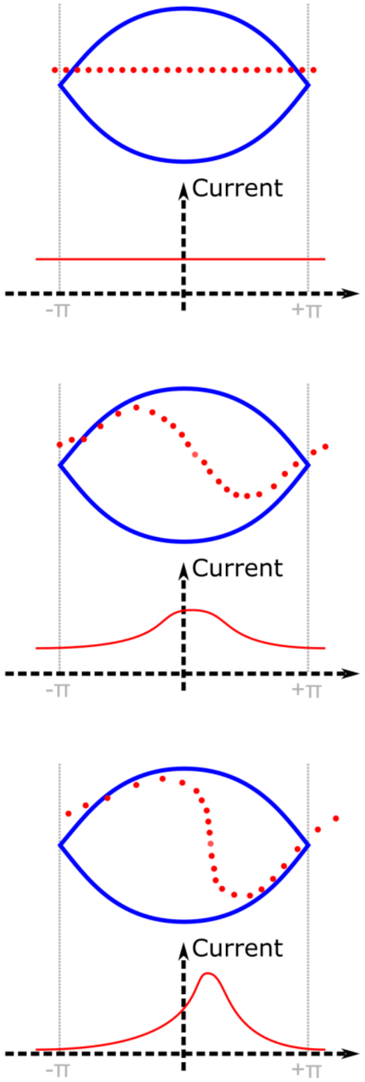
  - Pendulum motion - oscillation
  - Untrapped motion – desynchronized
  - Bucket width - undulator and radiation fields
- Electrons & ponderomotive phase synchronized
  - Continuous energy exchange

$$\theta = (k_{und} + k_{rad})z + \omega_{rad} t$$





Key concepts II

- Undulator parameter
- Resonant wavelength

$$K = \frac{q_e \sqrt{\langle \vec{B}^2 \rangle}}{mck_{und}} \qquad \lambda_{rad} = \frac{\lambda_{und}}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

Key concepts II

- Undulator parameter
- Resonant wavelength
- Madey theorem
- Pierce parameter (FEL parameter)

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$$G \propto -\frac{\mathrm{d}}{\mathrm{d}(\eta/2)} \frac{\sin^2(\eta/2)}{(\eta/2)^2}$$

$$\boxed{\rho_{FEL} = \left(\frac{K^2 r_e n_e \lambda_{und}^2}{32\pi\gamma^3}\right)^{1/3}}$$

$$\eta = \frac{4\pi N_{und}(\gamma - \gamma_R)}{\gamma_R}$$

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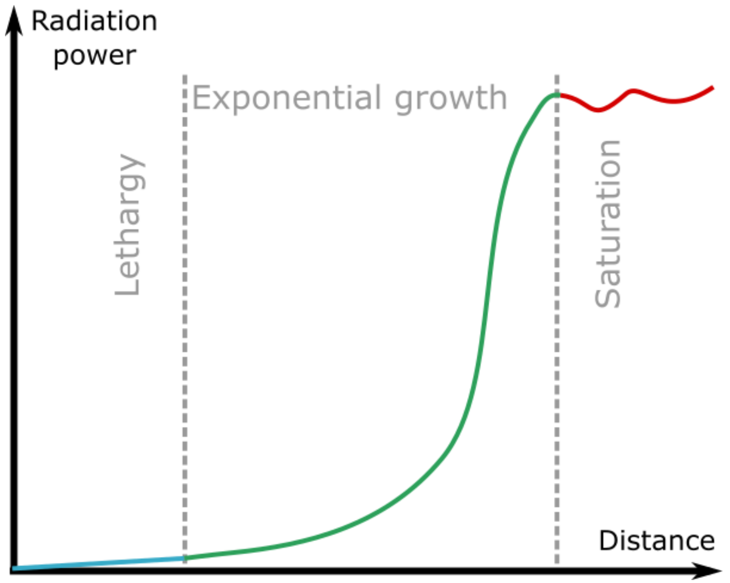
$$\boxed{\rho_{FEL} = \left(\frac{K^2 r_e n_e \lambda_{und}^2}{32\pi\gamma^3}\right)^{1/3}}$$

$$P_{sat} \approx 1.6 \times \rho_{FEL} \times P_{beam} \qquad L_{sat} \approx \lambda_{und}/\rho_{FEL}$$



FEL power is limited by slippage and overbunching.

Self-consistent FEL equations

GENESIS 1.3 equations

- Longitudinal motion

$$\dot{\theta}_j = ck_u - \omega\frac{1+K^2}{2\gamma_j^2} - \omega\frac{\beta_R^2}{2} - \omega\frac{|u|^2}{2\gamma^2} + \omega\frac{f_c K}{2\gamma^2}(ue^{i\theta_j} - \text{c.c.})$$

$$\dot{\gamma}_j = -\omega\frac{f_c K}{2\gamma_j}(ue^{i\theta_j} - \text{c.c.}) + \frac{q_e}{mc}\sum(E_l e^{i\theta_j} + \text{c.c.})$$

Complex amplitude:

$$u = -i\frac{q_e\sqrt{\langle\vec{E}^2\rangle}}{mc^2 k_{rad}}e^{i\Phi}$$

Transverse velocity: β_R
Coupling factor: f_c
Harmonic number: l
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$$\dot{X} = \frac{P_x}{\gamma m} \qquad \dot{Y} = \frac{P_y}{\gamma m}$$

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- Modified Maxwell's equation
  - Dispersive medium

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$$\left(\nabla_\perp^2 - \frac{l^2k_{rad}^2(1+K^2)}{\gamma_R^2}\right)\tilde{E}_l = i\frac{q_e l k_{rad}(1+K^2)}{\epsilon_0\gamma_R^2}\sum_l \delta(\Delta\vec{r}_j)e^{-il\theta_j}$$

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- relativistic electron energy;
- small transverse extension compared to the undulator period;
- **resonant interaction between radiation field and electrons**.

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$$\cdots = \frac{P_y}{\gamma m}$$

$$\cdots mc^2\frac{K^2k_y^2}{\gamma^2}Y$$

FEL code goals
- Generate e-beam
- Solve ODE for e-beam
- Solve radiation fields PDE and electrostatic fields PDE
- Store radiation field and beam parameters

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- Longitudinal dynamics
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  - Time-dependent – radiation field slippage, SASE
- Motion and radiation averaging
  - Averaging approximation (e.g. GENESIS 1.3)
  - No averaging (e.g. PUFFIN)

Slowly varying envelope (SVE) approximation:

- ρ_{FEL}<< 1
- Equations of motion are averaged over one undulator period.
- **Non-resonant interaction is excluded**..

$$\left|\frac{\mathrm{d}E}{\mathrm{d}z}\right| \ll (k_{rad} + k_{und})|E|$$

Solving FEL equations in GENESIS 1.3

Simulation steps

- Macro particle creation (quiet loading)
  - Quasi-random 4D uniform cube (Hammersley seq.)
  - Mirroring of macroparticles
  - Macro particle shot noise

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  - Beam and field offset – half step

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  - Finite difference method on Cartesian mesh
  - Dirichlet boundary condition to 0
  - Alternating direction implicit method
  - Subharmonics are suppressed

Slowly varying envelope approximation - requirement

$$\lambda_u \leq \Delta z \ll \frac{L_{sat}}{20} \approx L_G$$

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- Tail to head simulation
  - Complete slice simulation – field slips out
  - Next (forward) slice simulation – old field slips in
  - **Changed in later versions**

- Motion equations
  - Runge-Kutta integraton of 4th order
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Simulation parameters and limitations

Control parameters

- FEL lattice, field and beam parameters
- Grid size – 6 to 8 times the field RMS size
- Grid spacing – less than quarter σ_{rad}
- Above 16000 macro particles in slice
- Step size – undulator and focusing
- More slices – frequency resolution

$$\lambda_u \leq \Delta z \ll \frac{L_{sat}}{20}$$

Total number of slices: $M \geq \frac{L_u}{\Delta z}$

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- SVE approximation – strongest impact
  - FEL parameter $\ll 1$
  - Typical X-ray FEL: $\rho_{FEL} \approx 10^{-4}$ to 10^{-3}
  - PITZ SASE FEL : $\rho_{FEL} \approx 10^{-2}$
    - Gain length ~ 6 undulator periods

$$\left|\frac{\mathrm{d}E}{\mathrm{d}z}\right| \ll (k_{rad} + k_{und})|E|$$

$$L_{sat} \approx \lambda_{und}/\rho_{FEL}$$

GENESIS 1.3 updates

Current version

- GENESIS 1.3 version 4
  - Fully C++
  - MPI multiprocessing
  - HDF5 output format
  - Preprocessing to higher harmonics
  - Active development
- One4one mode
  - Full beam and field in memory
  - Self-consistent wakefield and space charge
  - Lattice file – ELEGANT/MADX inspired
  - Beam distribution – ELEGANT inspired
  - Particles can exchange slices

Recommendation: use version 4!

Summary

- Synchrotron light sources use relativistic effects
- FEL - highest peak brilliance
- Undulator parameter has significant role
- FEL parameter defines performance
- GENESIS 1.3 is averaged time-dependent 3D FEL code
- Version 4 new features

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THANK YOU FOR YOUR ATTENTION

- Sources

  - Own notes
  - S. Reiche, Numerical Studies for a Single Pass High Gain Free-Electron Laser
  - S. Reiche, P. Musumeci, K. Goldammer, Proceedings of PAC07
  - S. Reiche, Proceedings of FEL2014
  - P. Schmüser, M. Dohlus, J. Rossbach, C. Behrens, Free-Electron Lasers in the Ultraviolet and X-Ray Regime
  - doi:10.18429/JACoW-FEL2017-MOP016
  - D. Attwood, From Undulatorsto Free Electron Lasers: The Emergence of Coherence

BACKUP

- Hammersley sequence
  - Low-discrepancy sampling methods
  - Repesent natural number in base **b**
  - $n = a_0(n)b^0 + a_1(n)b^1 + a_2(n)b^2 + \ldots$
  - Use the value
  - $v(n) = a_0(n)b^{-1} + a_1(n)b^{-2} + a_2(n)b^{-3} + \ldots$

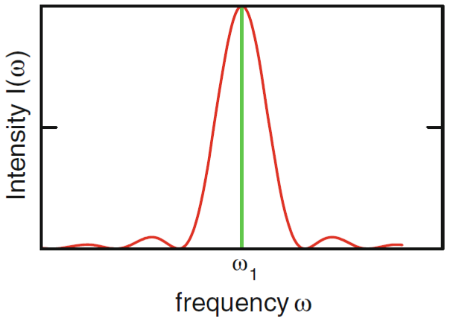
$$k_1 = f(y^*(t_0), t_0)$$

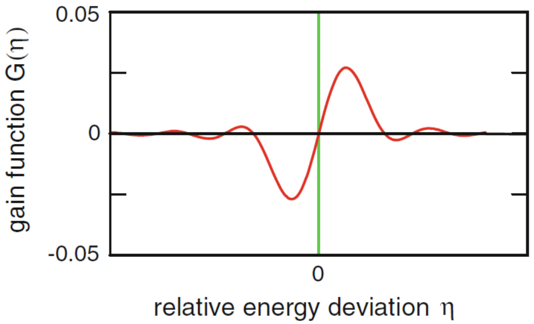
$$k_2 = f\left(y^*(t_0) + k_1\frac{h}{2}, t_0 + \frac{h}{2}\right)$$

$$k_3 = f\left(y^*(t_0) + k_2\frac{h}{2}, t_0 + \frac{h}{2}\right)$$

$$k_4 = f\left(y^*(t_0) + k_3 h, t_0 + h\right)$$

$$y^*(t_0+h) = y^*(t_0) + \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}h = y^*(t_0) + \left(\frac{1}{6}k_1 + \frac{1}{3}k_2 + \frac{1}{3}k_3 + \frac{1}{6}k_4\right)h$$





$$\nabla_\perp^2 u \rightarrow \frac{u_{i+1,j}^m + u_{i-1,j}^m + u_{i,j+1}^m + u_{i,j-1}^m - 4u_{i,j}^m}{\Delta^2}$$

$$\boldsymbol{u}^{n+1/2} = \boldsymbol{u}^n + i\frac{\Delta z}{4k}\left[\mathcal{L}_x \boldsymbol{u}^{n+1/2} + \mathcal{L}_y \boldsymbol{u}^n\right] + \boldsymbol{s}^{n+1/2}\frac{\Delta z}{2},$$

$$\boldsymbol{u}^{n+1} = \boldsymbol{u}^{n+1/2} + i\frac{\Delta z}{4k}\left[\mathcal{L}_x \boldsymbol{u}^{n+1/2} + \mathcal{L}_y \boldsymbol{u}^{n+1}\right] + \boldsymbol{s}^{n+1/2}\frac{\Delta z}{2}$$