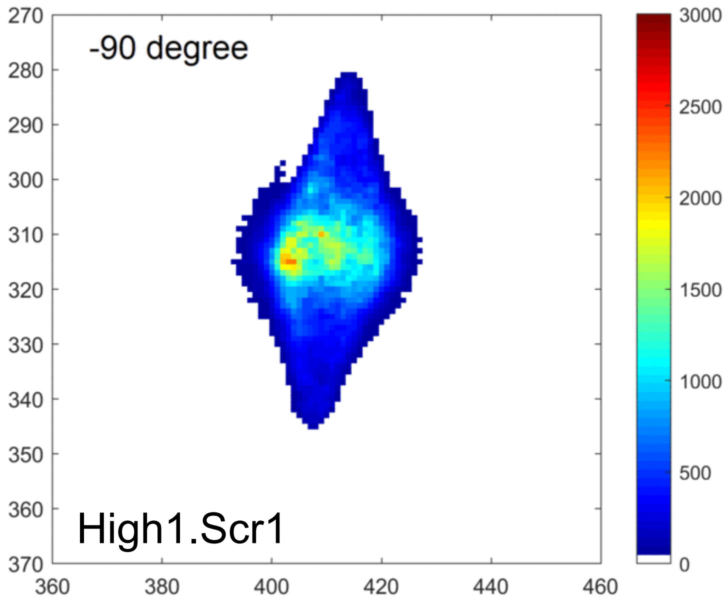
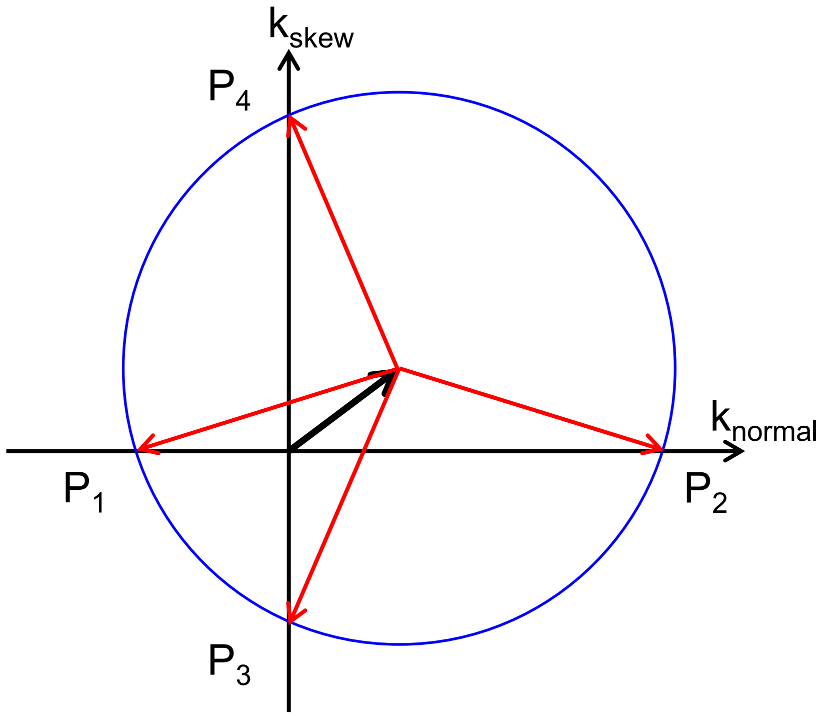
Transverse phase space coupling due to quadrupole field error and cathode laser asymmetry





26.03.2020 @ PPS
H. Qian

Outline

- Introduction
- Transverse coupling by quadrupole field error w/o space charge
- Transverse coupling by quadrupole field error w/ space charge
- Transverse coupling by cathode laser asymmetry
- Simulations & experiments
- Summary

Introduction to phase space coupling

4D phase space

- 4D beam phase space $X_{4D} = [x\ x'\ y\ y']^T$
- 4D beam covariance matrix
  - $C_{4D} = \langle X_{4D} X_{4D}^T \rangle = \begin{bmatrix} C_{XX} & C_{XY} \\ C_{XY}^T & C_{YY} \end{bmatrix}$
  - $C_{XX} = \begin{bmatrix} \langle xx \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle xx' \rangle \end{bmatrix}$
  - $C_{YY} = \begin{bmatrix} \langle yy \rangle & \langle yy' \rangle \\ \langle yy' \rangle & \langle y'y' \rangle \end{bmatrix}$

    } Projected phase space beam matrix
  - $C_{XY} = \begin{bmatrix} \langle xy \rangle & \langle xy' \rangle \\ \langle x'y \rangle & \langle x'y' \rangle \end{bmatrix}$
- Emittance
  - $\varepsilon_{4D}^4 = \det C_{4D}$
  - $\varepsilon_x^2 = \det C_{XX}$
  - $\varepsilon_y^2 = \det C_{YY}$

    } Projected emittance
  - Coupling factor: $\frac{\sqrt{\varepsilon_x \varepsilon_y}}{\varepsilon_{4D}} - 1$

- No X/Y coupling $C_{XY} = 0,\ \varepsilon_x \varepsilon_y = \varepsilon_{4D}^2$
- X/Y Coupling $C_{XY} \neq 0,\ \varepsilon_x \varepsilon_y > \varepsilon_{4D}^2$
- Most FEL main linac lattice design assumes no transverse coupling
  - Effective beam brightness ($\frac{Q}{\varepsilon_x \varepsilon_y}$)
  - Injector needs to decouple X/Y phase space to make full use of 4D beam brightness ($\frac{Q}{\varepsilon_x \varepsilon_y} = \frac{Q}{\varepsilon_{4D}^2}$)
- Concept can be extended to 6D phase space
  - $X_{6D} = \left[x\ x'\ y\ y'\ t\ \frac{dp}{p}\right]^T$
  - X(Y)/Z coupling → degrade effective beam brightness
    - Coupler kick (x' & t coupling)
    - Chromatic effect (x' & $\frac{dp}{p}$ coupling)

Introduction to phase space coupling

What can cause a X/Y coupling

- Skew quadrupole (thin lens model)
  - $x' = x'_0 + k_s y, y' = y'_0 + k_s x$
- Cathode residual B field
  - $p_x = p_{x0} + k_c y,\ \ p_y = p_{y0} - k_c x,\ k_c = \frac{eB_c}{2}$
- Solenoid Larmor rotation
  - Larmor rotation angle, $\theta_{Larmor} = \int \frac{B_z}{2p/e} dz$
  - Assuming a non-coupled beam before rotation

$$C_0 = \begin{bmatrix} C_{XX} & 0 \\ 0 & C_{YY} \end{bmatrix}$$

  - After rotation (by 4D rotation matrix R_{rot})

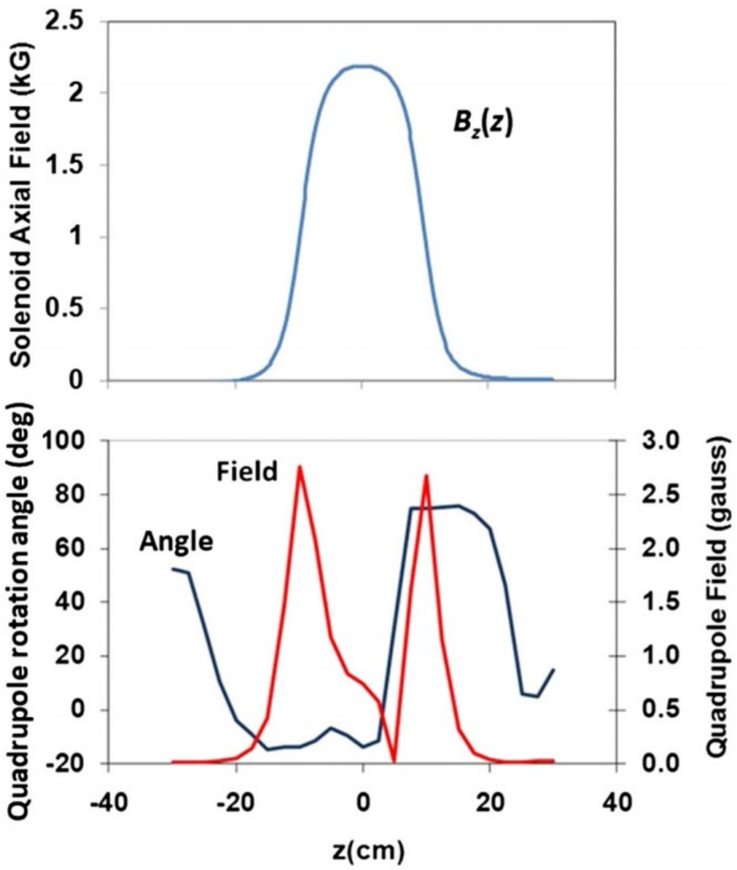
$$C_{rot} = R_{rot} C_0 R_{rot}^T$$
$$= \begin{bmatrix} \cos^2\theta\, C_{XX} + \sin^2\theta\, C_{YY} & \sin\theta\cos\theta\,(C_{YY} - C_{XX}) \\ \sin\theta\cos\theta\,(C_{YY} - C_{XX}) & \sin^2\theta\, C_{XX} + \cos^2\theta\, C_{YY} \end{bmatrix}$$

- Solenoid Larmor rotation (cont'd)
  - $C_{YY} = C_{XX}$, i.e. a symmetric beam, $C_{rot} = C_0$
  - $C_{YY} \neq C_{XX}$, X/Y coupling forms after rotation
  - Beam asymmetry causes
    - Cathode laser shape asymmetry
    - Magnetic quadrupole field error (solenoid)
    - Gun coupler kick
    - Cathode/laser misalignment
    - Solenoid/beam misalignment
  - Beam asymmetry corrections
    - A solenoid with zero Larmor rotation
    - Cathode laser profile optimization @virtual cathode
    - Gun quads corrector
    - Symmetric gun coupler
    - Laser BBA
    - Solenoid alignment with beam

Transverse coupling by quadrupole field error

No space charge case

- PRAB 21, 010101, by D. Dowell
  - SLAC gun solenoid quad field distortion



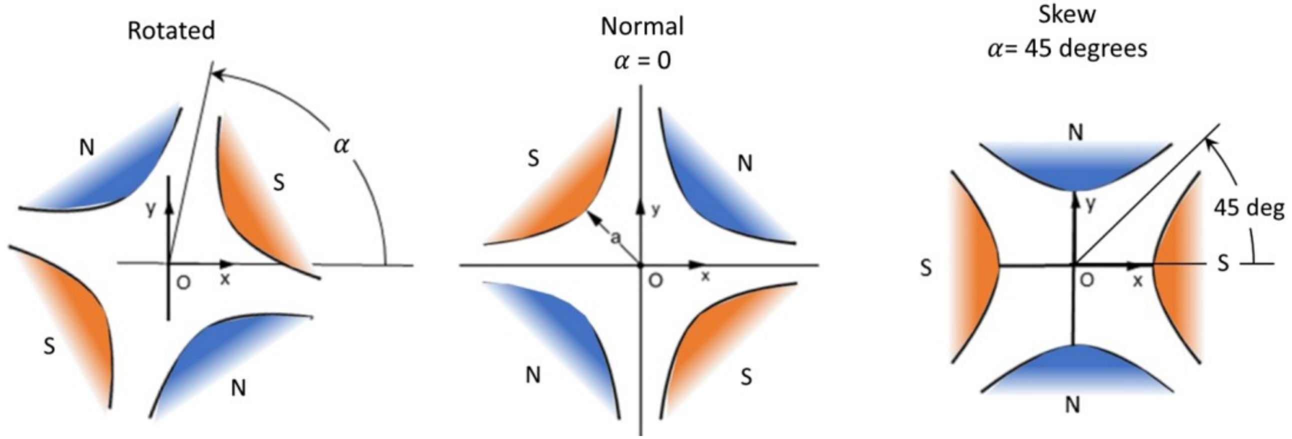
- Rotated quadrupole transfer matrix

$$R_{rotQuad} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -k_N & 1 & -k_s & 0 \\ 0 & 0 & 1 & 0 \\ -k_s & 0 & k_N & 1 \end{bmatrix}$$

$k_{quad} = \frac{1}{f}$ (integrated quad strength)

$$k_N = k \cos 2\alpha$$
$$k_s = k \sin 2\alpha$$

A normal/skew quad pair is a rotated qudrupole.



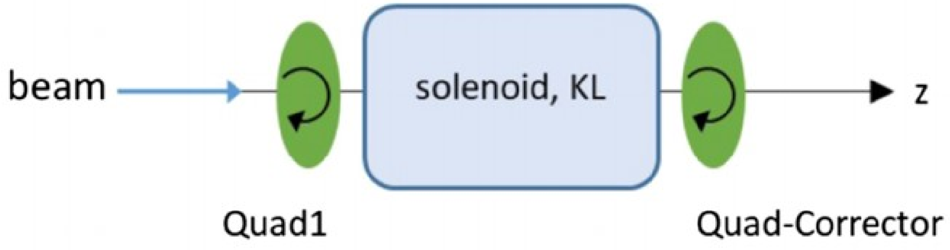
- Total transfer matrix of solenoid + quad error field
  - Simplification: integrated thin lens quad model
  - $R_{sol}R_{rotQuad}$

Transverse coupling by quadrupole field error

No space charge case

- PRAB 21, 010101, by D. Dowell
  - Emittance growth
    - Assuming symmetric beam before quad
      - $C_{YY} = C_{XX}$
    - By beam matrix transportation
      - $\mathrm{C} = (\mathrm{R}_{sol}\mathrm{R}_{rotQuad})C_0(\mathrm{R}_{sol}\mathrm{R}_{rotQuad})^T$
      - $\varepsilon^2 = \varepsilon_0^2 + \Delta\varepsilon^2$,
        $\Delta\varepsilon_x = \Delta\varepsilon_y = \sigma_{beam}^2 \left| k_{quad} \sin 2(\theta_{Larmor} + \alpha) \right|$
        ⇩
        Skew quad component
      - Emittance growth is only related to the skew quad error strength
      - Emittance growth is square dependent on beam size → high charge case suffers more
      - Slice emittance grows

- Quadrupole corrector model
  - Place a rotated quad corrector at solenoid exit



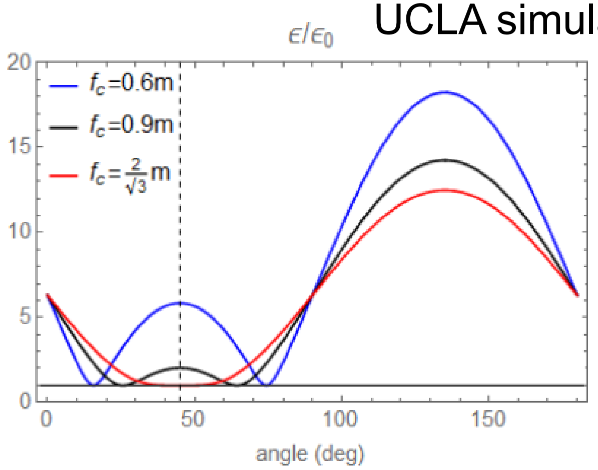
  - $\Delta\varepsilon = \left| \sigma_1^2 k_{quad1} \sin 2(\theta_{Larmor} + \alpha_1) + \sigma_c^2 k_c \sin 2\alpha_c \right|$
  - Tune k_c and α_c to zero emittance growth
    - Tune skew quad corrector to make $\Delta\varepsilon$ zero
    - Normal quad component is not related
      - Infinite solutions to make $\Delta\varepsilon$ zero
      - X and Y emittance still equals
      - Beam is not round downstream
        - Not critical for no space charge case

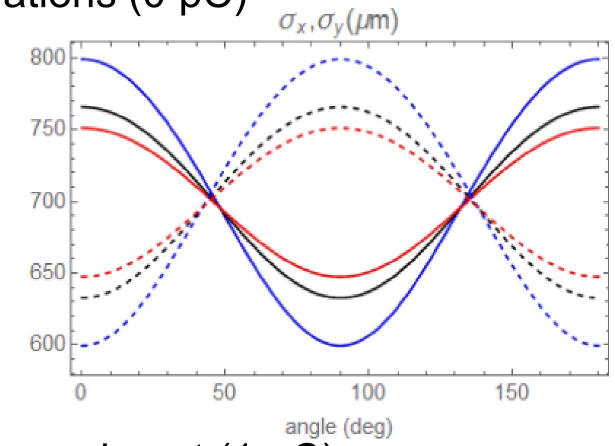
Transverse coupling by quadrupole field error

Experiments & simulations in no (low) space charge case

- Simulations & experiments reports
  - SLAC/Cornell/DESY/ANL/UCLA …

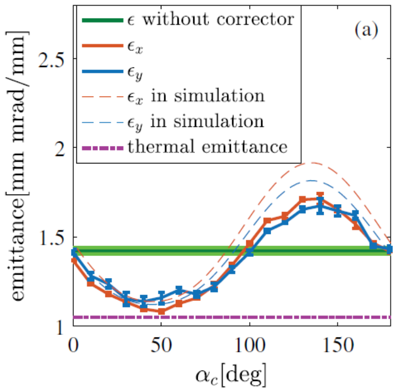
UCLA simulations (0 pC)



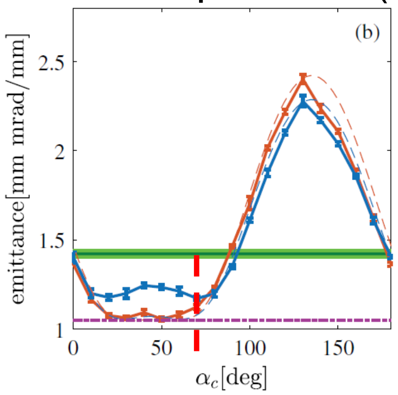


- Quadrupole corrector tuning in these studies
  - Simulation to fit the integrated quad error according to measured beam profile distortions
    - Get roughly amplitude and angle
    - Fix amplitude, scan angle to minize <xy> or emittance
- For low charge case, simulation is not necessary, fix amplitude, scan angle can zero $\Delta\varepsilon$

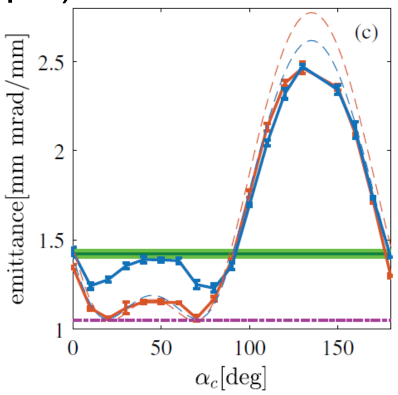
ANL experiment (1 pC)



36 G/m

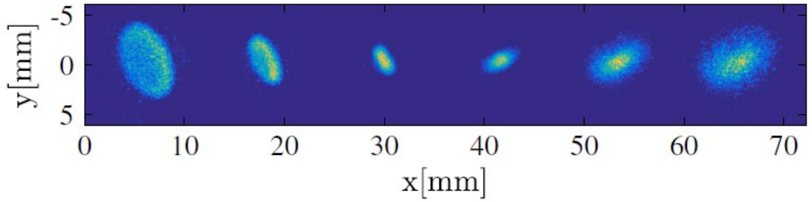


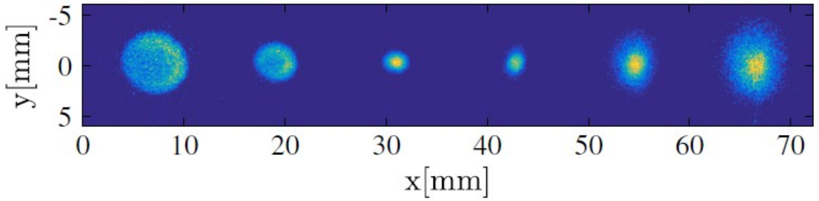
72 G/m



96 G/m

ANL experiment (1 pC)

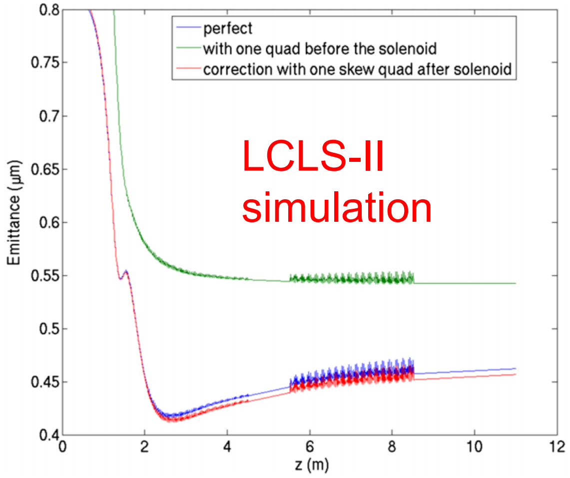


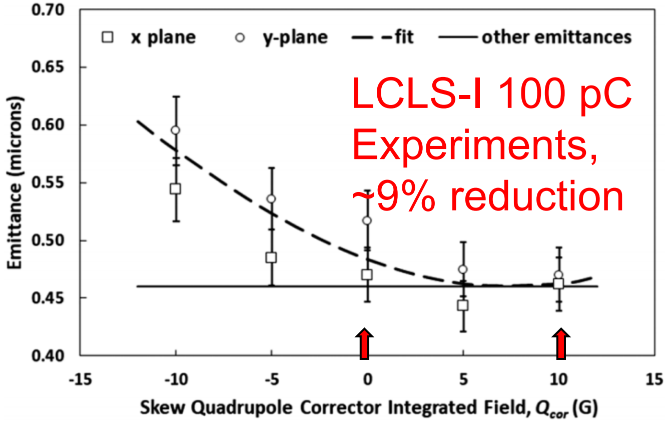


Transverse coupling by quadrupole field error

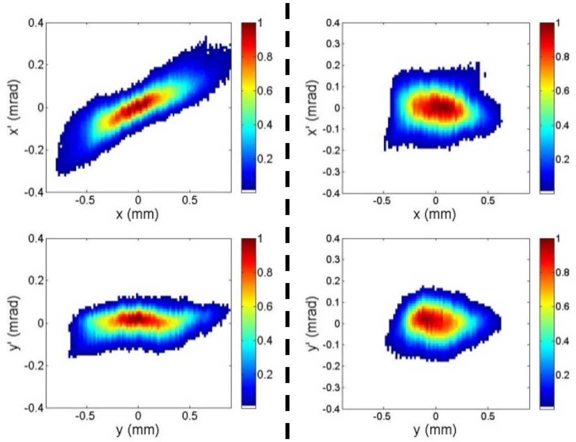
With space charge

- SLAC injector optimization by skew gun quads scan





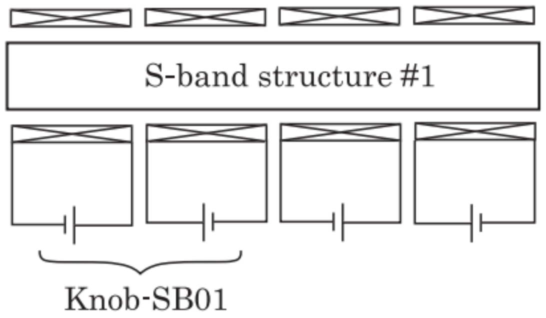
- PITZ injector optimization by gun quads optimizer



Symmetrize beam after booster by optimizer iterations before emittance measurement.

~9% emit reduction for 500 pC.

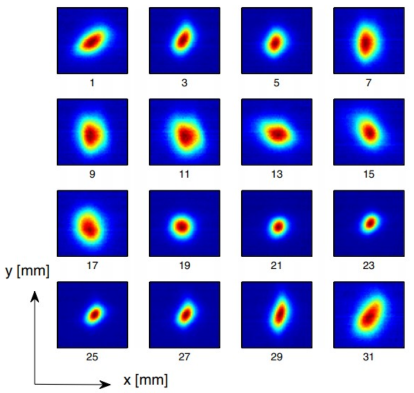
- PSI injector decoupling
  - By gun quads & booster solenoid



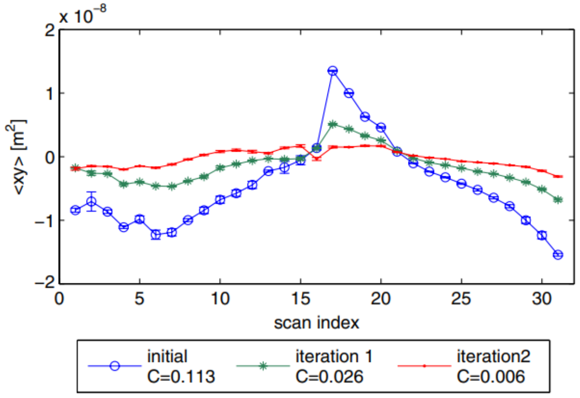
Change booster solenoid current

change Larmor angle

- Measure 4D phase space with multi-quad scan
  - Measure C_{XY} response matrix to gun quads & booster solenoid
  - Iterations, coupling factor 11% → 0.6%



Coupling 2.6%

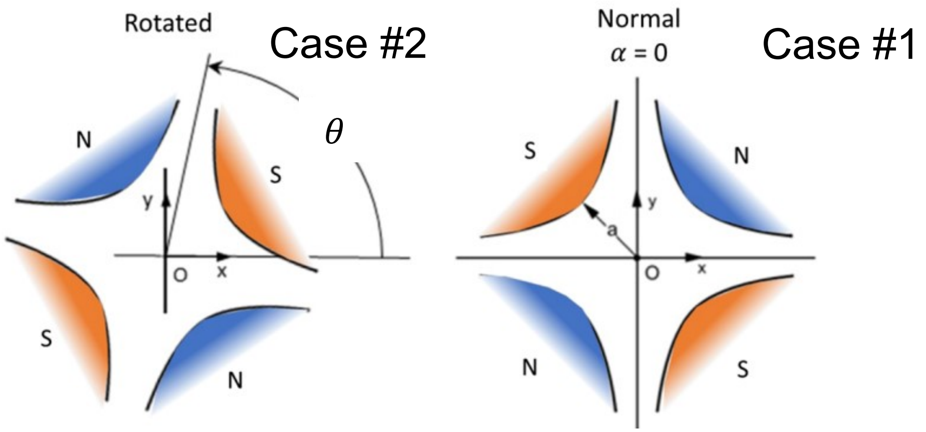


DESY.

How to tune gun quads under space charge case

A new method

- Current methods
  - Simulation vs experiments to fit quad errors (ANL, UCLA, PITZ)
  - Scan skew quad, measure emittance (SLAC)
  - Corrector iterations by measured sensitivity matrix of C_{XY} vs correctors, measure 4D beam matrix (PSI)
  - Iterations by optimizer to symmetrize beam profile (PITZ)
- A new method
  - Case #1: only a normal quad error in solenoid, no skew quad error, all other elements are symmetric
    - Beam after booster, no coupling, but X/Y asymmetry
    - $C_0 = \begin{bmatrix} C_{XX} & 0 \\ 0 & C_{YY} \end{bmatrix}$
    - $C_{YY} \neq C_{XX}, \varepsilon_x \neq \varepsilon_y$
- A new method (cont'd)
  - Case #2: a rotated quad error in solenoid, rotation angle θ, amplitude is same as case #1



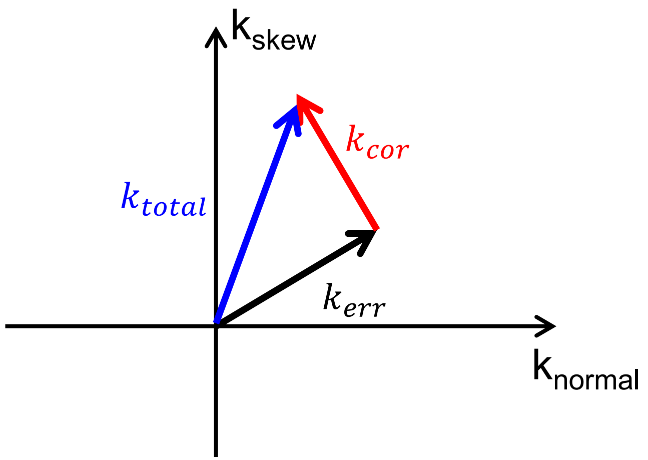
  - Case #2 is equivalent to injector #1 rotated by θ
    - So beam after injector of case #2 is
    - $C_\theta = R_{rot} C_0 R_{rot}^T = \begin{bmatrix} \cos^2\theta\, C_{XX} + \sin^2\theta\, C_{YY} & \sin\theta\cos\theta\,(C_{YY} - C_{XX}) \\ \sin\theta\cos\theta\,(C_{YY} - C_{XX}) & \sin^2\theta\, C_{XX} + \cos^2\theta\, C_{YY} \end{bmatrix}$

How to tune gun quads under space charge case

A new method

- A new method (cont'd)
  - Case #2: $C_{YY} \neq C_{XX}$, so beam is X/Y coupled
    - One way to decouple the beam w/o gun quad is to design a rotation lattice line after booster, but beam will loose symmetry
  - $C_\theta = \begin{bmatrix} \cos^2\theta\, C_{XX} + \sin^2\theta\, C_{YY} & \sin\theta\cos\theta\,(C_{YY} - C_{XX}) \\ \sin\theta\cos\theta\,(C_{YY} - C_{XX}) & \sin^2\theta\, C_{XX} + \cos^2\theta\, C_{YY} \end{bmatrix}$
    - $\langle xx \rangle = \cos^2\theta\,\langle x_0x_0 \rangle + \sin^2\theta\,\langle y_0y_0 \rangle$
    - $\langle yy \rangle = \sin^2\theta\,\langle x_0x_0 \rangle + \cos^2\theta\,\langle y_0y_0 \rangle$
    - $\langle xy \rangle = \frac{1}{2} sin\,2\theta\,(\langle y_0y_0 \rangle - \langle x_0x_0 \rangle)$
  - $\langle xx \rangle = \langle yy \rangle$ → $\theta = \pi/4$, i.e. a pure skew quad
    - $\sigma_x = \sigma_y,\ \varepsilon_x = \varepsilon_y$
  - $\langle xy \rangle = 0$ → $\theta = 0$, i.e. a pure normal quad
    - Largest asymmetry for X/Y planes, but no coupling

- A new method (cont'd)
  - An integrated quad error at solenoid exit + a rotated quad corrector at solenoid exit
    - Quad error $k_{err} = k_0 e^{i2\theta} = k_0(cos2\theta + i * sin2\theta)$
    - Quad corrector $k_{cor} = k_c e^{i2\alpha}$
    - Total quad $k_{total} = k_0 e^{i2\theta} + k_c e^{i2\alpha} = k_t e^{i2\phi}$
      - Quad error is fixed
      - Quad corrector is variable
  - Vector plot of quad correction

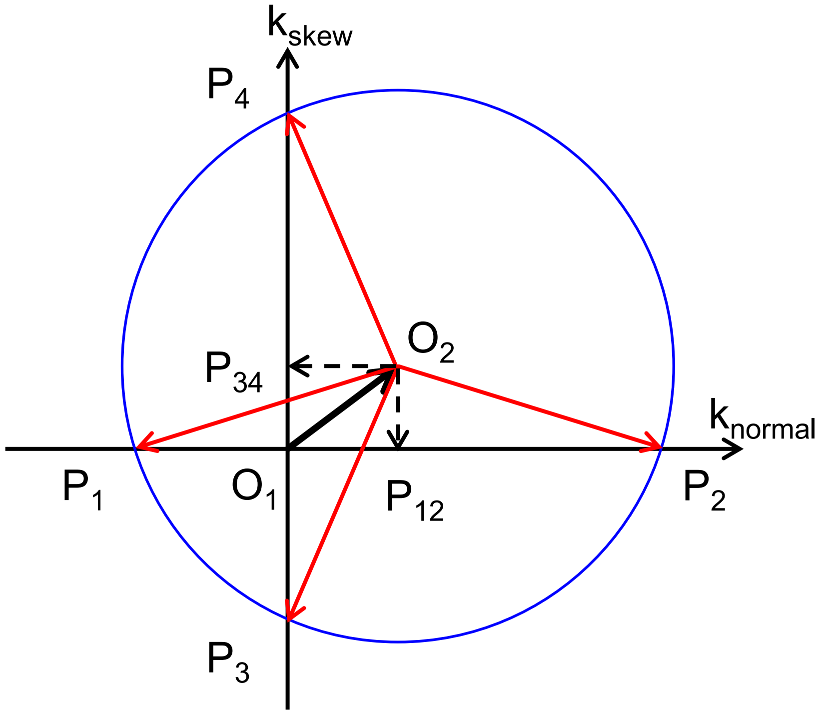


How to tune gun quads under space charge case

A new method

- A new method (cont'd)
  - Vector plot of quad correction
    - Angle scan with arbitrary corrector amplitude
    - O_2P_1 & O_2P_2 correction
      - Pure normal quad error left, $\langle xy \rangle = 0$
    - O_2P_3 & O_2P_4 correction
      - Pure skew quad error left, $\sigma_x = \sigma_y$
  - Normal/skew quad correction
    - $(O_2P_1 + O_2P_2)/2 = O_2P_{12}$ → skew quad corrector
      - O_2P_{12} is also the skew quad axis
    - $(O_2P_3 + O_2P_4)/2 = O_2P_{34}$ → normal quad corrector
      - O_2P_{34} is also the normal quad axis
    - $O_2P_{12} + O_2P_{34} = O_2O_1$, correction of both normal & skew quad errors

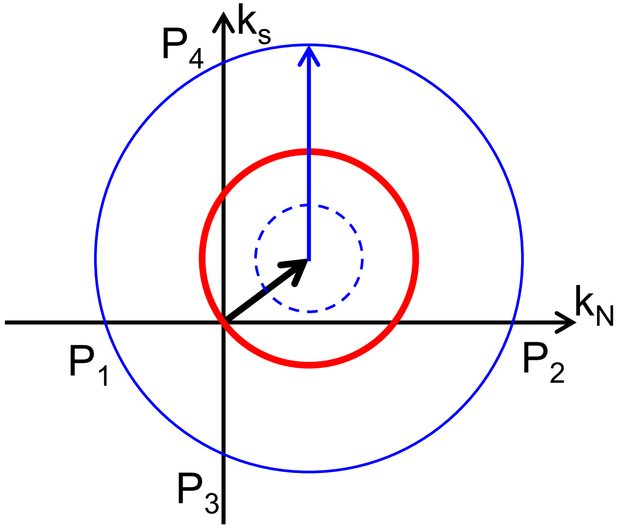
- Advantages of such a method
  - No prior knowledge of quad error angle or amplitude needed, no simulation fit needed
  - No emittance measurement required
  - In ideal case, no iterations



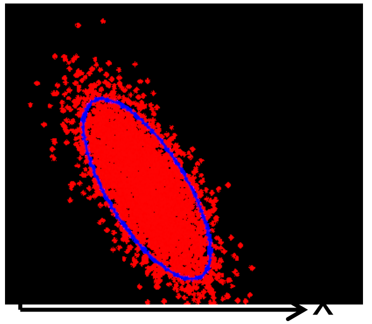
How to tune gun quads under space charge case

A new method

- A new method (cont'd)
  - Special cases
    - When corrector amplitude much smaller than quad error amplitude
      - no P1/2/34
    - When corrector amplitude equals error amplitude
      - P1/P3 become one point



- How to find P1/2/3/4
  - Take one corrector amplitude I_0
  - Measure $\langle xy \rangle$, $\langle xx \rangle$, $\langle yy \rangle$, vs corrector angle
  - Find the 4 angles for $\langle xy \rangle$=0 & $\sigma_x = \sigma_y$
  - The corrector solution is $I_c = \frac{1}{2} I_0 \sum_{n=1}^{4} e^{i2\alpha_n}$
    - Gun Q1: $re\ (I_c)$
    - Gun Q2: $im\ (I_c)$
- Normalized beam profile properties
  - RMS ellipse $gx^2 + 2axy + by^2 = e$
    - $e = \sqrt{\langle xx \rangle \langle yy \rangle - \langle xy \rangle^2}$
    - $b = \langle xx \rangle / e$
    - $g = \langle yy \rangle / e$ ⇨ b=g, a=0
    - $a = -\langle xy \rangle / e$



How to tune gun quads under space charge case

A new method

- A new method (cont'd)
  - Special cases like FLASH/XFEL injector
    - 1st screen after booster is already after a quadrupole based lattice
    - The transport matrix from before the 1st high energy quad to the 1st screen
      - $R = \begin{bmatrix} R_X & 0 \\ 0 & R_Y \end{bmatrix}, R_X \neq R_Y$
    - In case #2, the beam on the 1st screen is
    - $C_{scr} = R C_\theta R^T$

$$\begin{bmatrix} R_X(\cos^2\theta\, C_{XX} + \sin^2\theta\, C_{YY}))R_X^T & \sin\theta\cos\theta\, R_X(C_{YY} - C_{XX})R_Y^T \\ \sin\theta\cos\theta\, R_X(C_{YY} - C_{XX})R_Y^T & R_Y(\sin^2\theta\, C_{XX} + \cos^2\theta\, C_{YY})R_Y^T \end{bmatrix}$$

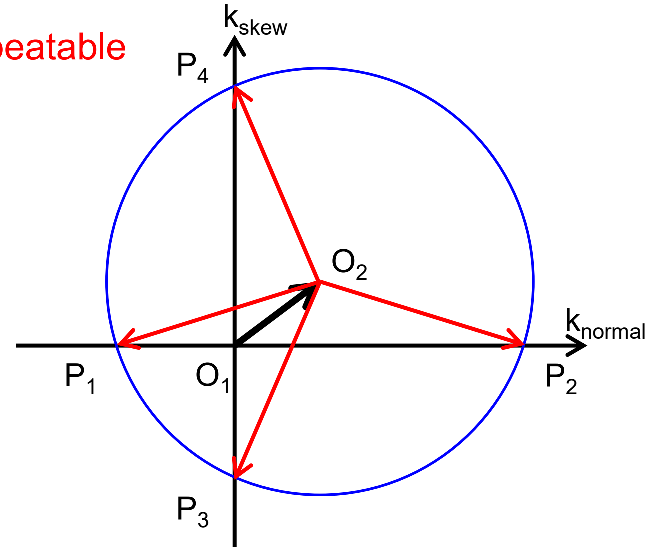
- A new method (cont'd)
  - Special cases like FLASH/XFEL injector
    - $\theta = 0$, i.e. a pure normal quad → $\langle xy \rangle = 0$
    - $\theta = \pi/4$, i.e. a pure skew quad

$$\frac{1}{2}\begin{bmatrix} R_X(C_{XX}+C_{YY})R_X^T & R_X(C_{YY} - C_{XX})R_Y^T \\ R_X(C_{YY} - C_{XX})R_Y^T & R_Y(C_{XX}+C_{YY})R_Y^T \end{bmatrix}$$

$$\sigma_x \neq \sigma_y$$

- P1/P2 can be found
  - Skew quad correction repeatable
- P3/P4 can not be found

Normal quad correction not repeatable, unless do emittance measurement, otherwise variable normal quad component leads to burden on injector matching.



Transverse coupling by cathode laser asymmetry

- No space charge case
  - Assumption
    - Small cathode laser asymmetry between x and y, <xy>=0
    - Twiss parameter similar between x and y planes inside Larmor coordinate
    - No quad errors
  - Beam matrix after solenoid rotation

$$C_{rot} = \begin{bmatrix} \cos^2\theta\, C_{XX} + \sin^2\theta\, C_{YY} & \sin\theta\cos\theta\,(C_{YY} - C_{XX}) \\ \sin\theta\cos\theta\,(C_{YY} - C_{XX}) & \sin^2\theta\, C_{XX} + \cos^2\theta\, C_{YY} \end{bmatrix}$$

  - Projected emittance

$$\varepsilon_x = \cos^2\theta\,\varepsilon_{x0} + \sin^2\theta\,\varepsilon_{y0}$$

$$\varepsilon_y = \sin^2\theta\,\varepsilon_{x0} + \cos^2\theta\,\varepsilon_{y0}$$

  - $\varepsilon_x\varepsilon_y \geq \varepsilon_{4D}^2 = \varepsilon_{x0}\varepsilon_{y0}$

- Additional effects with space charge
  - Assumption
    - 3D ellipsoidal case with a small non-equal x and y semi axis
      - $r_x = r_0(1+\delta),\ r_y = r_0(1-\delta)$
  - Space charge force asymmetry

$$E_{sx} = coef * \frac{x}{r_x} = k_0(1-\delta)x$$

$$E_{sy} = coef * \frac{y}{r_y} = k_0(1+\delta)y$$

    - Equivalent to a symmetric case plus a normal quadrupole
  - The final quad error angle will be decided by the solenoid rotation and initial laser asymmetry axis

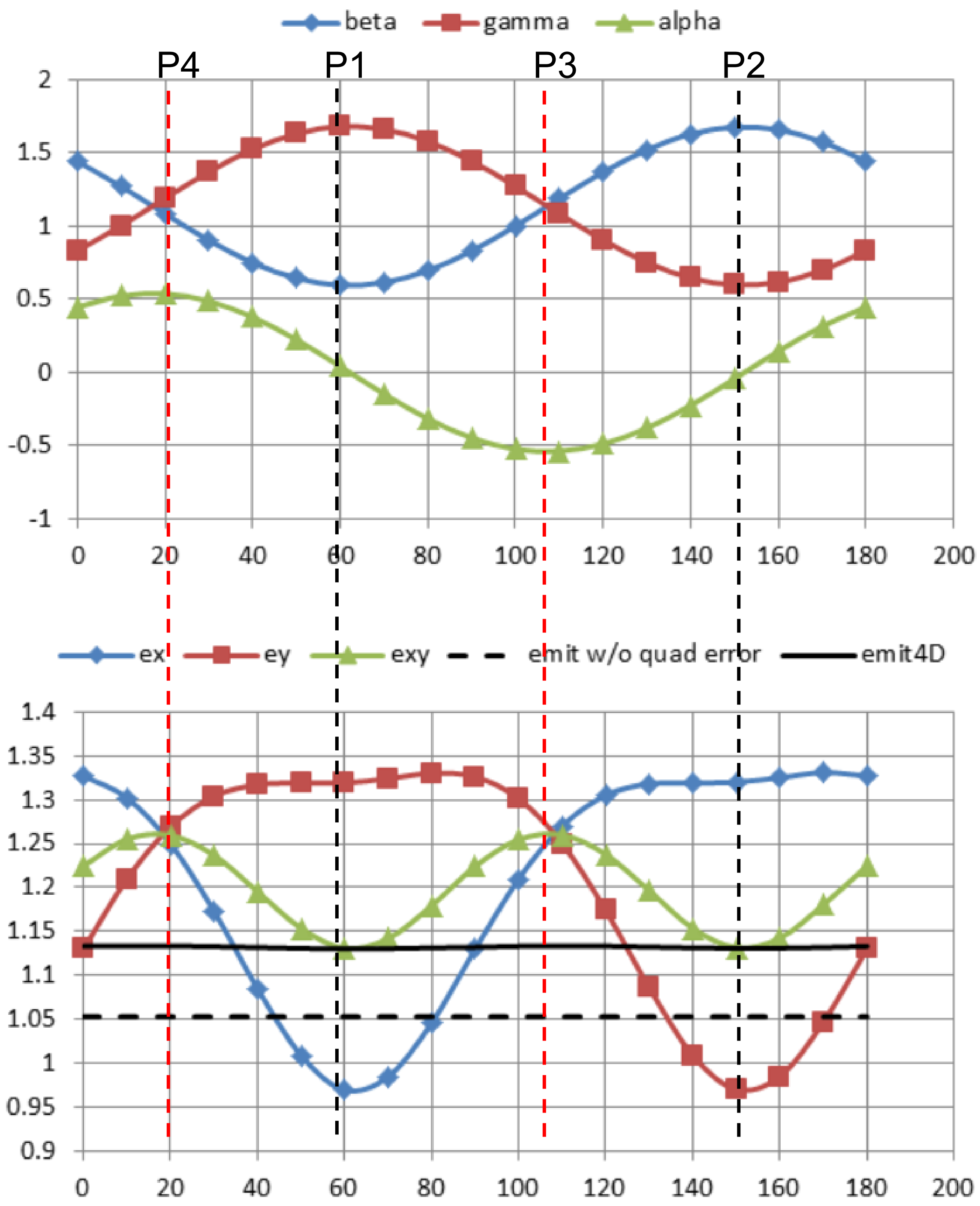
Simulations

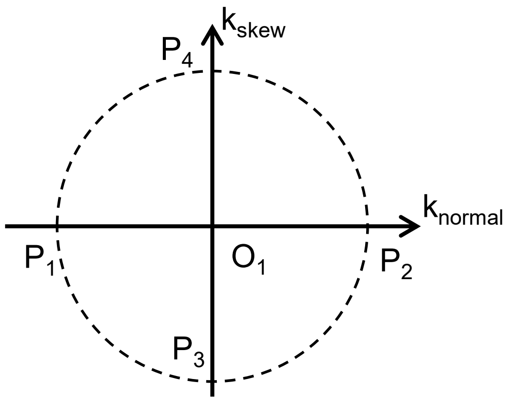
- Simulation setup
  - Laser, 6 ps FWHM, BSA1.3 mm, 500 pC (PITZ experiment optimizations)
  - PITZ setup, gun 6.3 MeV/c + booster
  - A distributed quadrupole error field like SLAC measurement
    - Using gun solenoid field map as a quad field map, inspired by the ANL study, quad angle can be configured in ASTRA
    - Quadrupole error strength assumption in simulation
      - Typical solenoid focusing strength k is ~3.5 m^{-1}
      - Integrated focusing strength k is ~0.01 m^{-1}, focal length ~100 m (typical range 50–100 m), roughly ~0.3% compared to solenoid strength
- Simulation setup (cont'd)
  - Rotated quad corrector model
    - Around current Gun Q1/Q2 position
    - 5 cm effective length
    - Angle and gradient variable
- Case studies
  - Quadrupole error study
    - Quad error only, angle and amplitude scan
    - Quad error + quad corrector
  - Laser asymmetry
    - No corrector
    - With quad corrector

Quadrupole error study

Quad error only, angle scan with step 10 degree

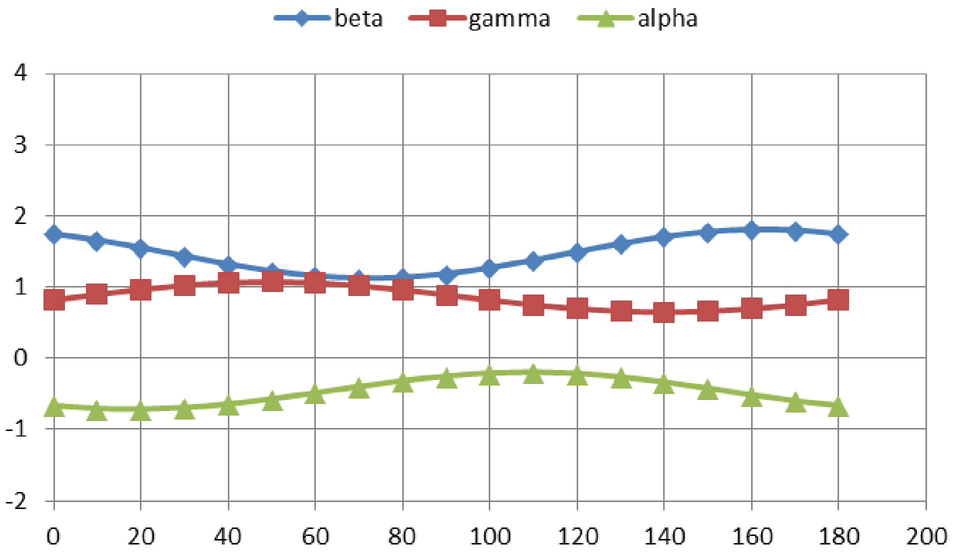
- ASTRA simulations w/ 3D space charge
- Observations:
  - Regular oscillations of parameters vs quad error angle
  - In contrast to no space charge case, emitX and emitY are not equal
  - 4D emittance is constant w.r.t. angle
  - 4D emittance with quad error larger than 4D emittance without quad error, in contrast to no space charge case
  - Best emittance, no coupling, worst X/Y symmetry
  - Simulation consistent with beam matrix rotation

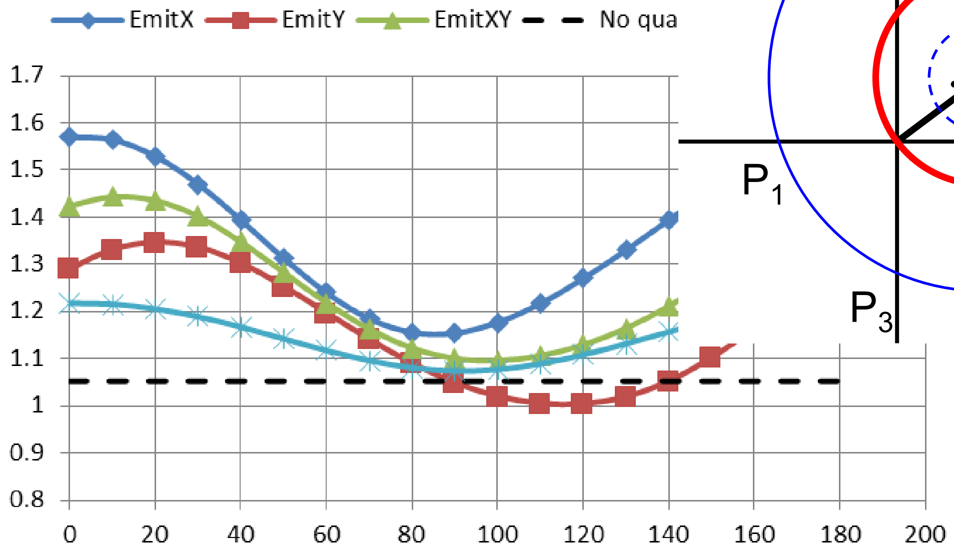


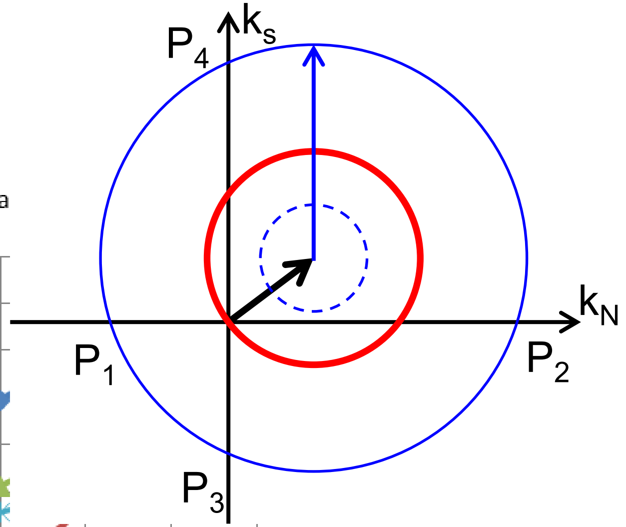


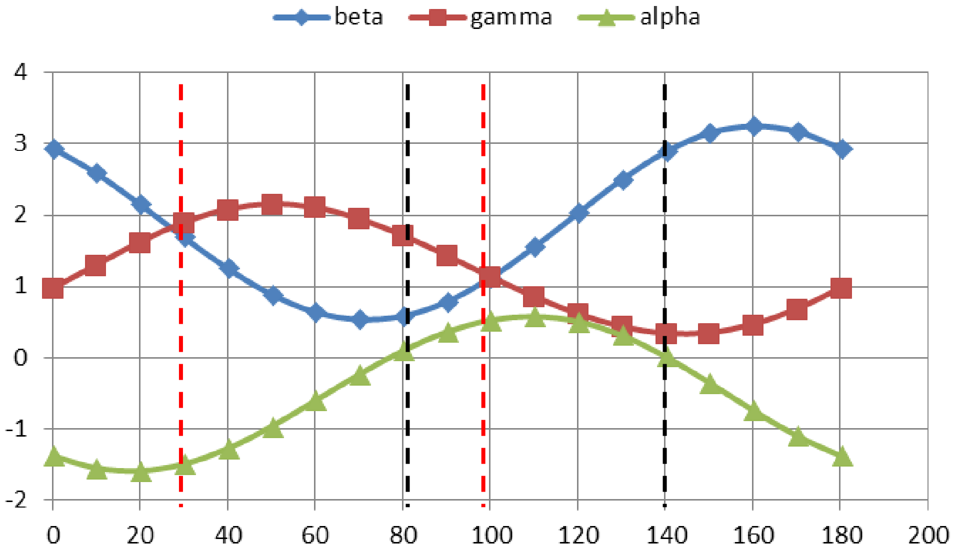
Quadrupole error study

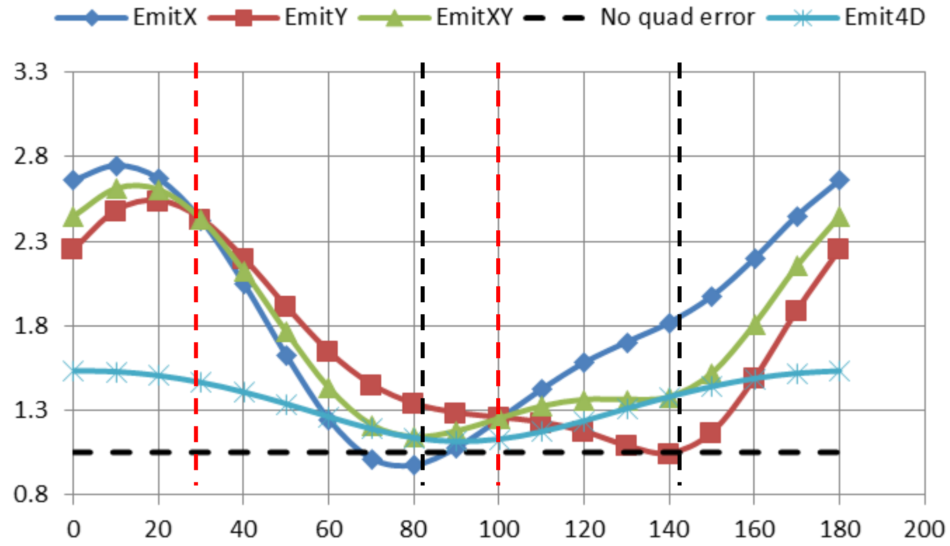
Quad error + quad corrector







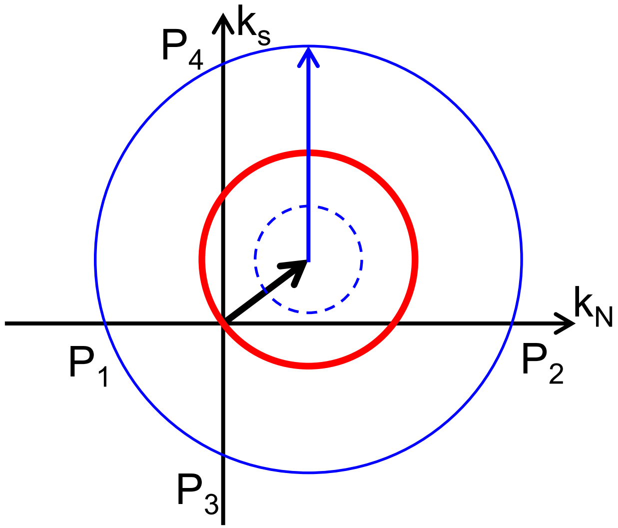


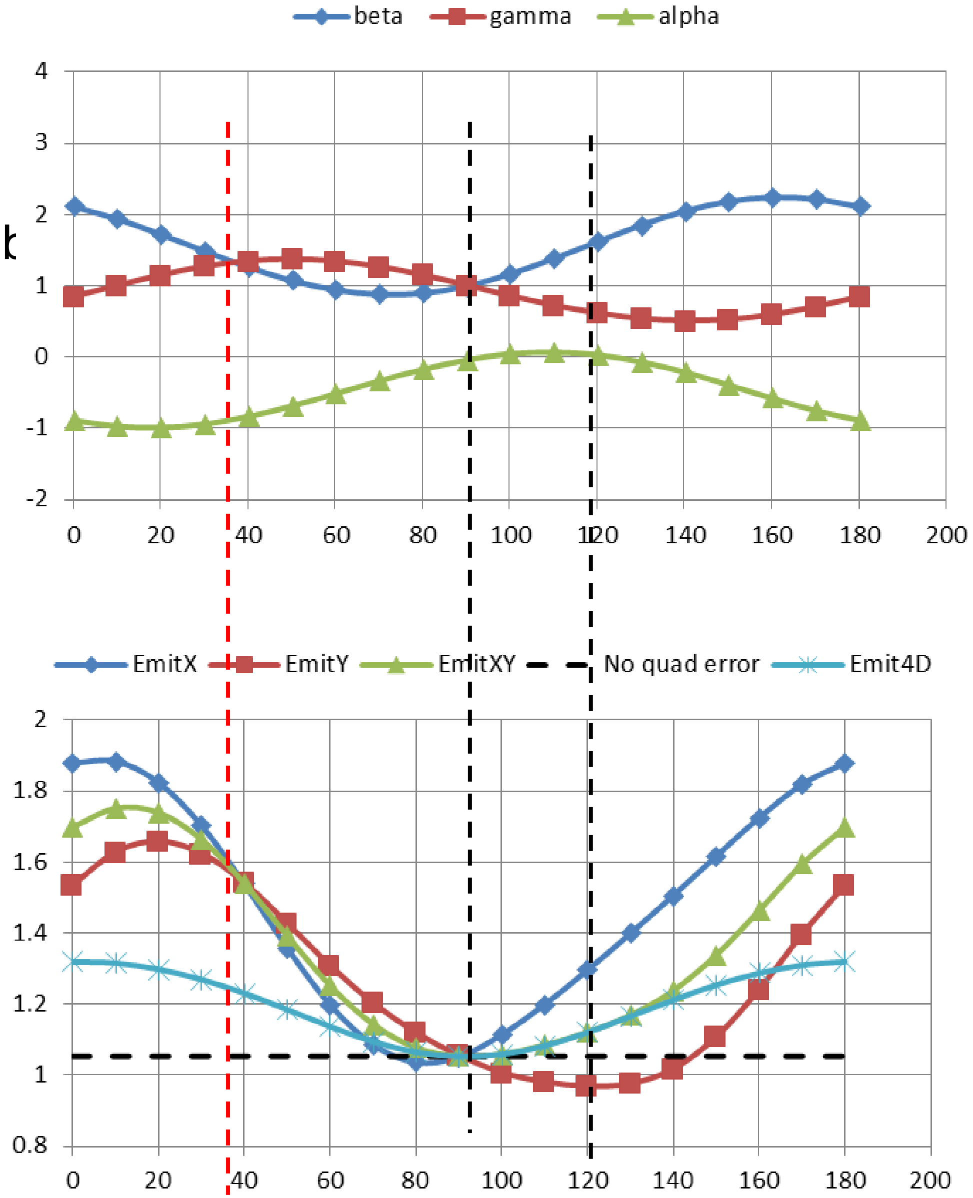


Quadrupole error study

Quad error + quad corrector

- Corrector amplitude matches error amplitude by mannual scan, both emit and beam size symmetry
  - Amp 60 G/m, angle 90 deg
  - Analytical formula prediction, $I_c = \frac{1}{2} I_0 \sum_{n=1}^{4} e^{i2\alpha_n}$
    - 63.78 G/m, angle 92.1 deg
    - Consistent results when scanning quad corrector angle with variable strength

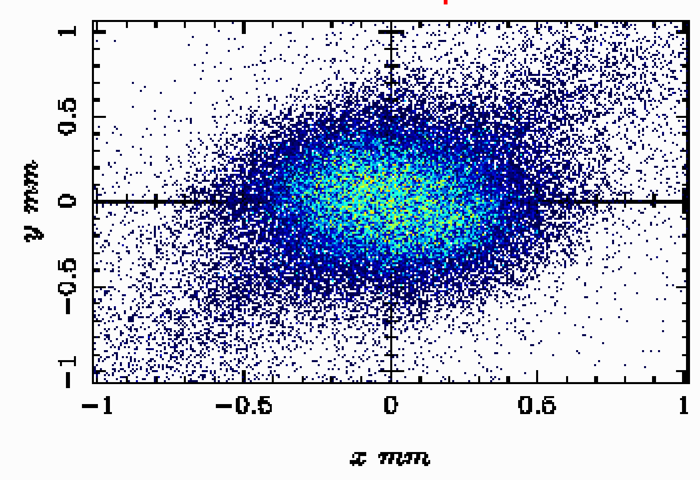


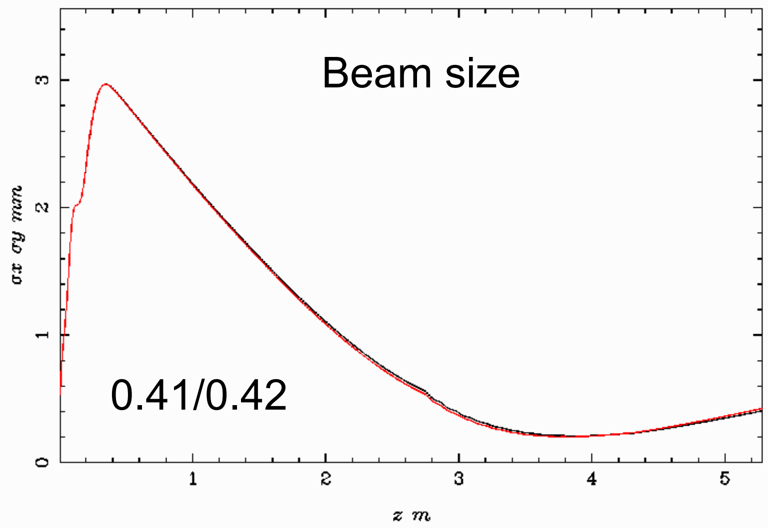


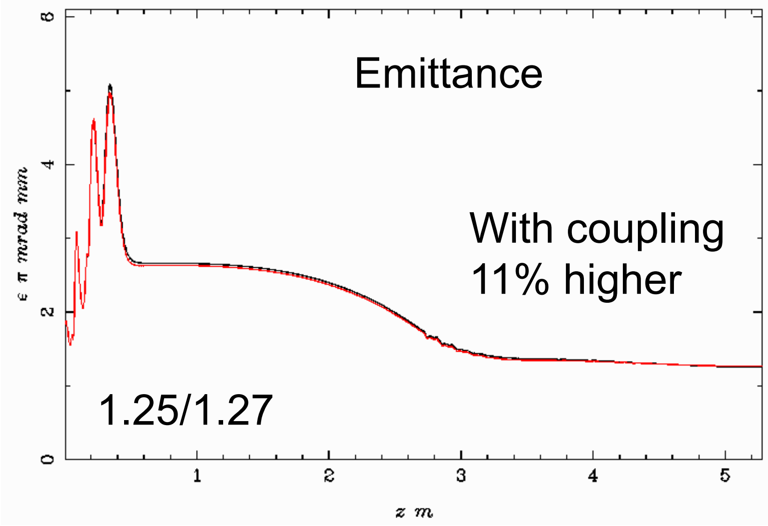
Before correction

- H1.scr1

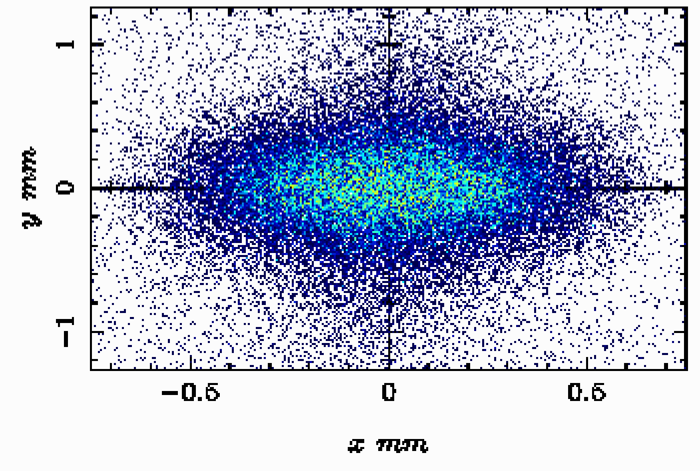
Pure skew quad error

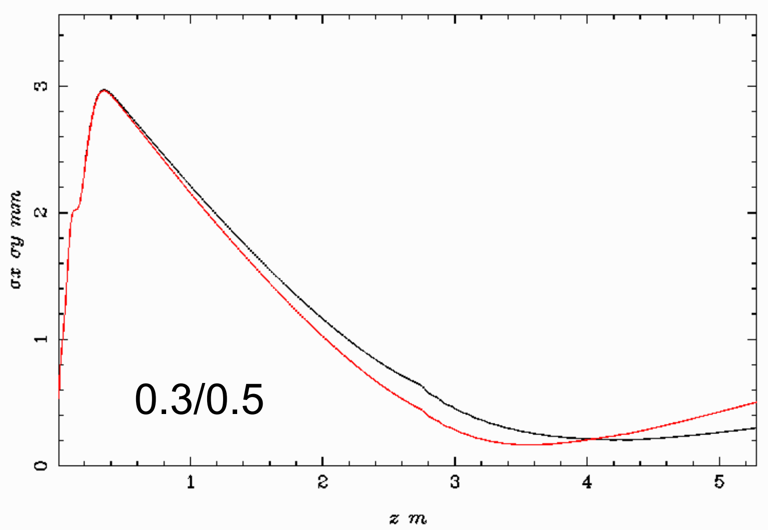


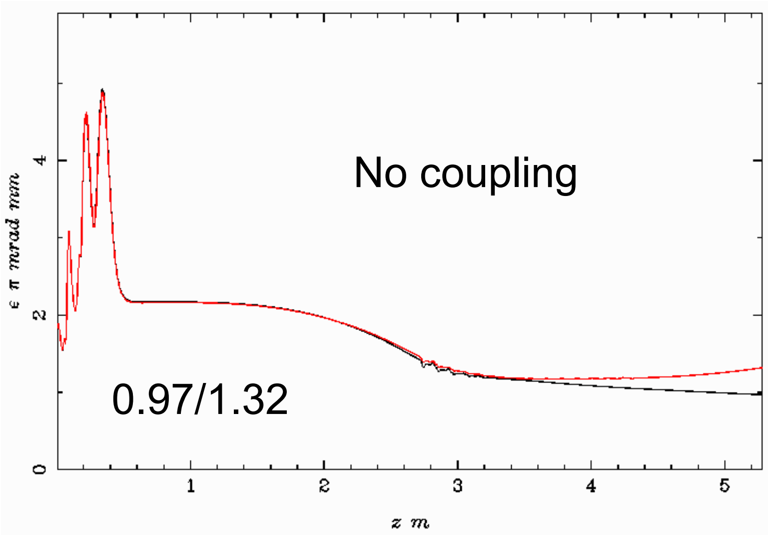




Pure normal quad error

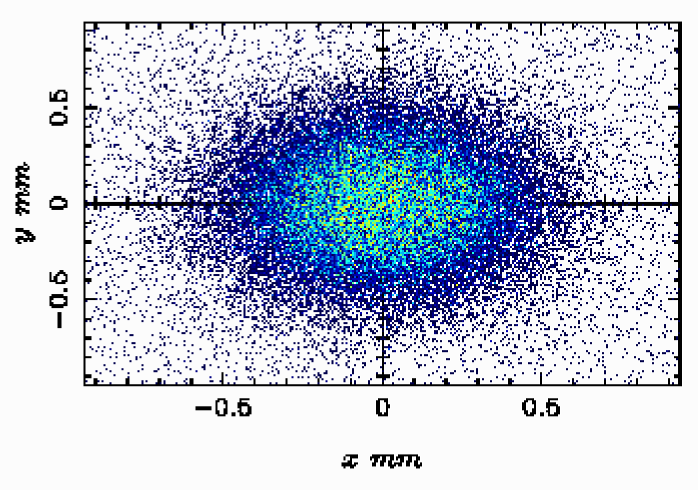


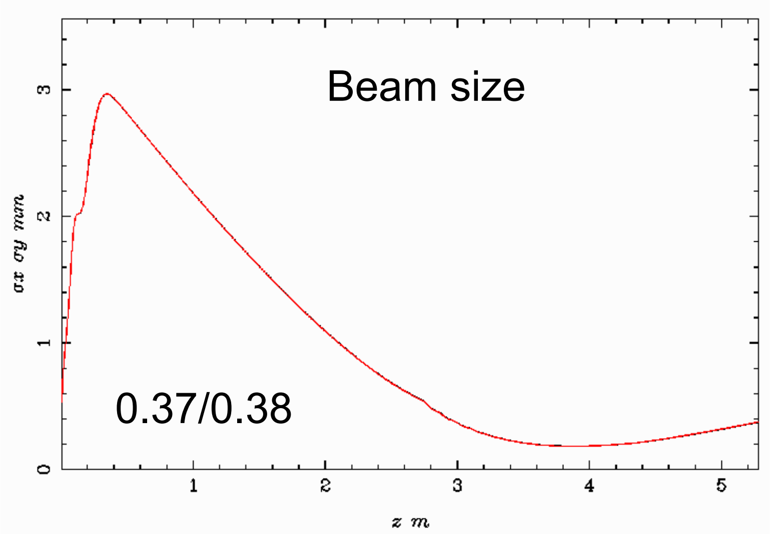


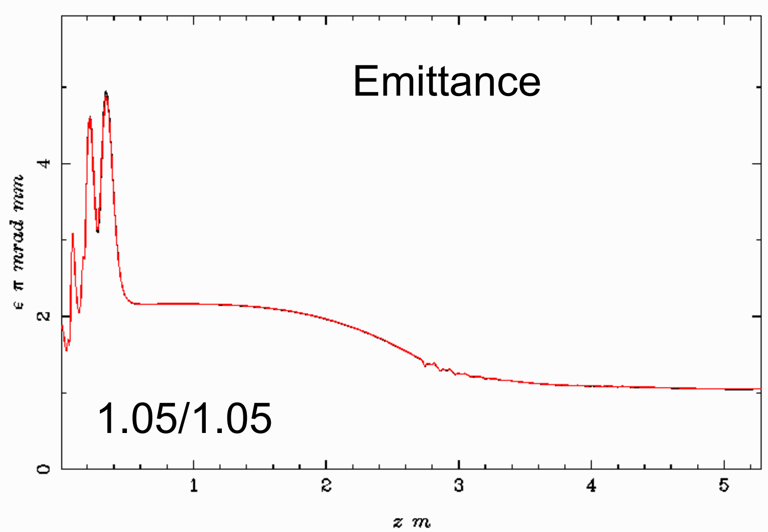


After correction

- Both beam size and emittance symmetry restored
- Emittance is same again as case without any quad error



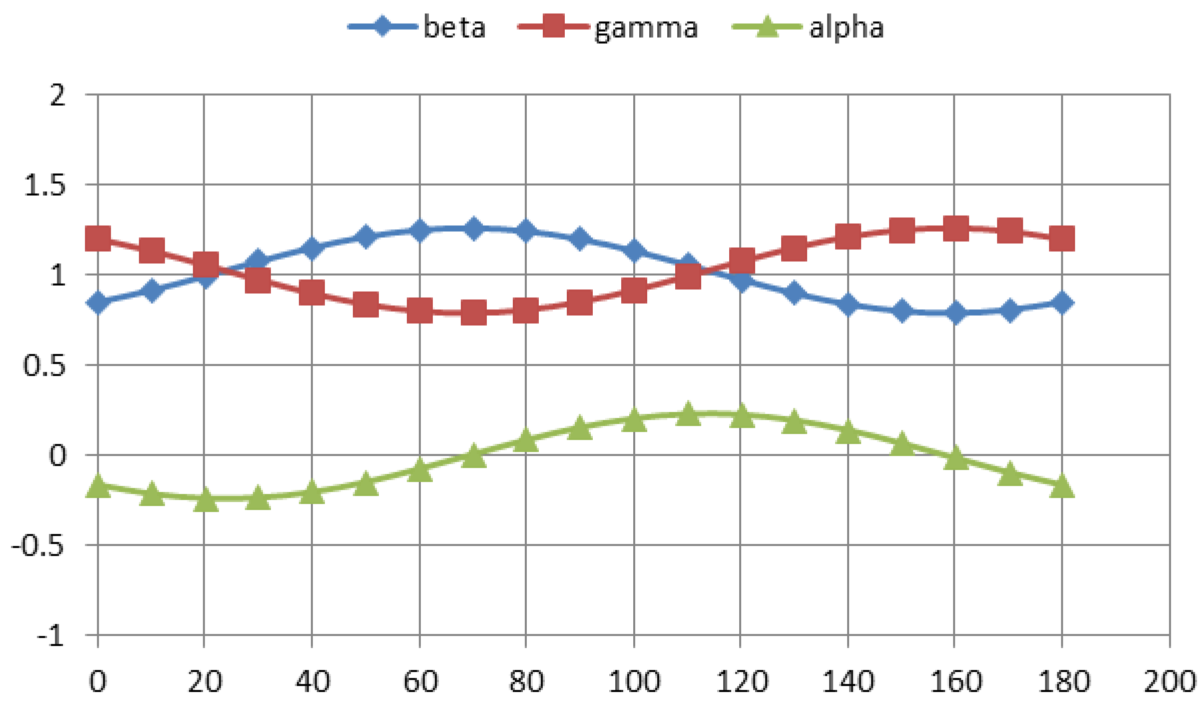




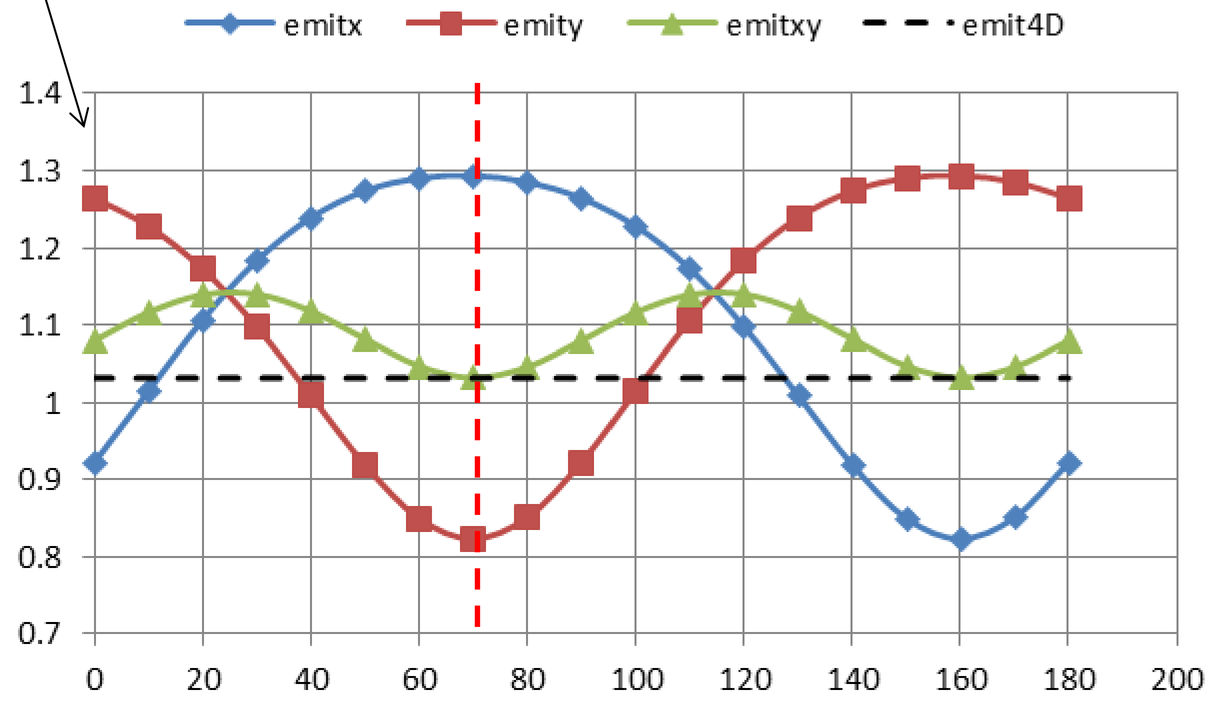
Laser asymmetry 10% p-p

w/o quad correction

- Rotation of asymmetric cathode laser on cathode
  - Smaller effect on beam size asymmetry than quad error in solenoid of strength 0.01 m^-1
  - Larger effect on beam emittance asymmetry
  - 4D emittance is same as symmetry case



Upright case



Laser asymmetry 10% p-p

w/ quad corrector

- Formula prediction
  - 35.3 G/m, -46 degree for upright laser asymmetry
- Partial correction
  - 0.36/0.36 mm
  - 0.95/1.17 mm.mrad