Further analysis of stat. error calculation for emittance calculation

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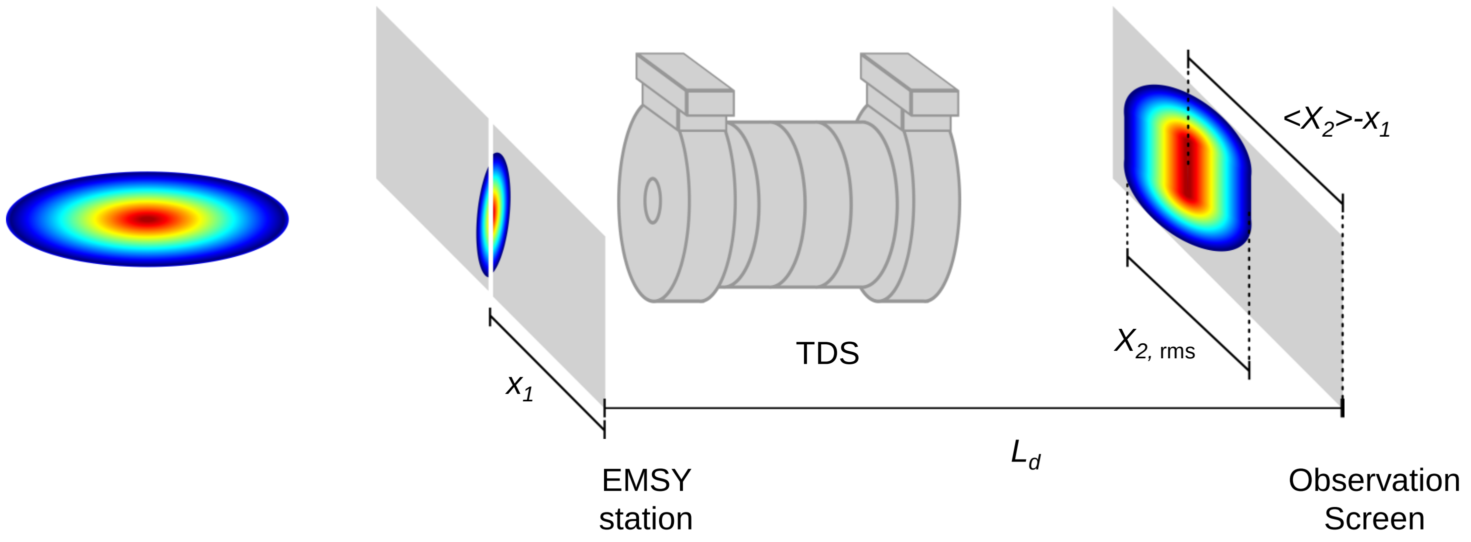
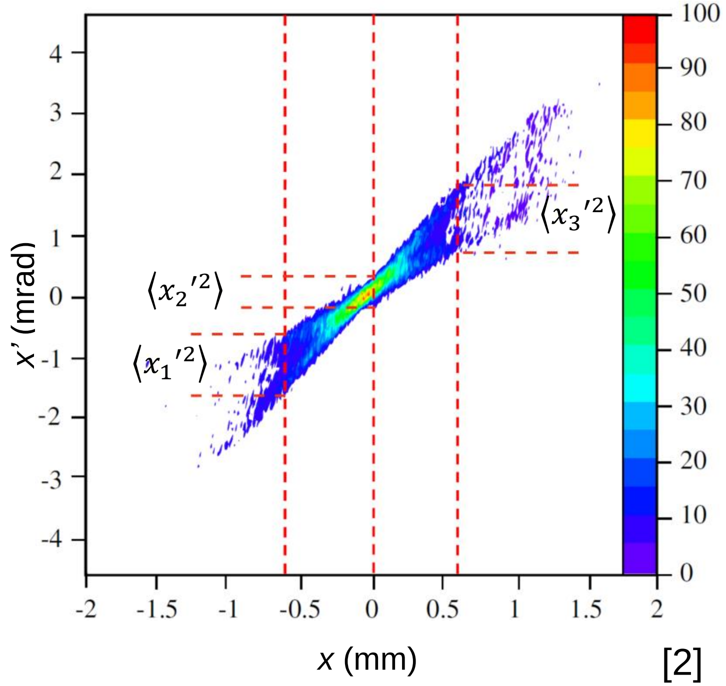
Recap: Slit Scan Method

Slit-Scan-based slice emittance measurements





[2]

- Cut out **emittance-dominated beamlets** from **space charge-dominated beam** with a slit
  - Measure the **size**, **position** and **intensity** of each beamlet on screen
- Reconstruct the phase space at slit position
  - Emittance via $\epsilon = \beta\gamma \frac{\sigma_x}{\sqrt{<x^2>}} \sqrt{\langle x_0{}^2\rangle\langle x_0{}'^2\rangle - \langle x_0 x_0{}'\rangle^2}$

[2] S. Rimjaem et al., Nucl. Instr. Meth. Phys. Res. A **671**, 62 – 75 (2012).

Recap: What we’ve had until now

Result from K’n’K seminar on Friday, January 18th

> Emittance uncertainty, calculated via Gaussian error propagation (neglecting covariance) gives high stat. error

$$\Delta\epsilon = \sqrt{\left(\frac{\mathrm{d}\epsilon}{\mathrm{d}Vx}\Delta Vx\right)^2 + \left(\frac{\mathrm{d}\epsilon}{\mathrm{d}Vp}\Delta Vp\right)^2 + \left(\frac{\mathrm{d}\epsilon}{\mathrm{d}Cov}\Delta Cov\right)^2}$$

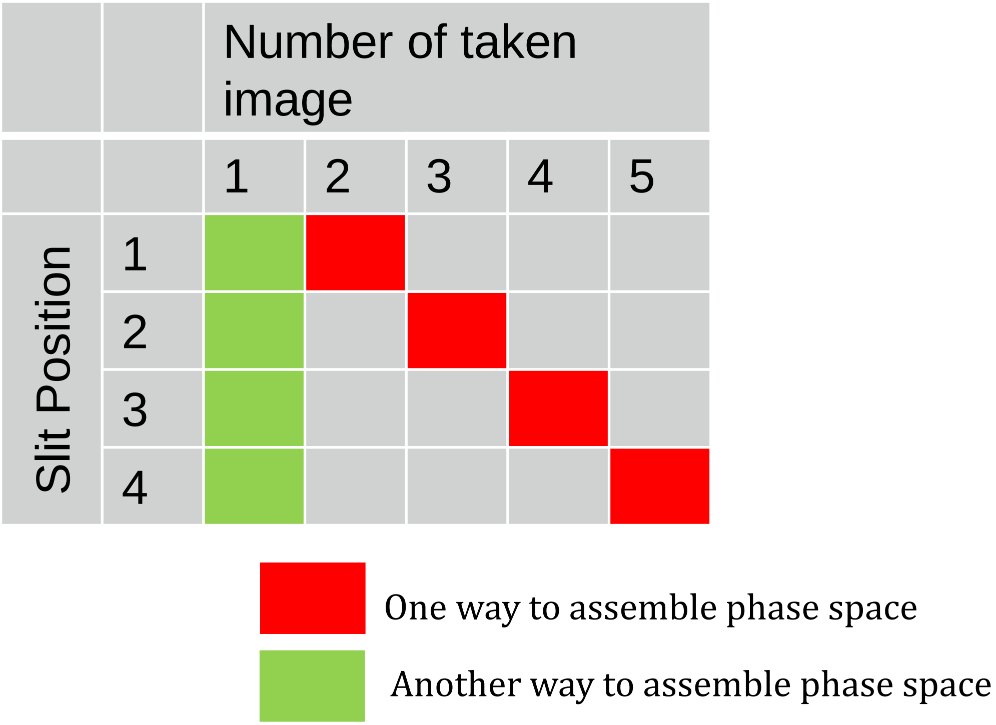
Gives ~ 34 % error for data looked at

> RMS ‘error’ from several-times calculated emittance is small and reasonable
  > Reconstructing ten phase spaces, first from the first image at every slit position, second from second image etc.
  > Calculating the geom. emittance ten times
  > Emittance = mean(ten emittances), Emittance error = rms(ten emittances)
  > Gives ~ 3% error for data looked at

Recap: What was suggested

Result from K’n’K seminar on Friday, January 18th

1. Fourth-order moment is uncertainty of variance, see [1]
2. Include correlation terms in Gaussian error propagation
3. Calculate rms value of ten emittances as error, but put phase spaces together differently

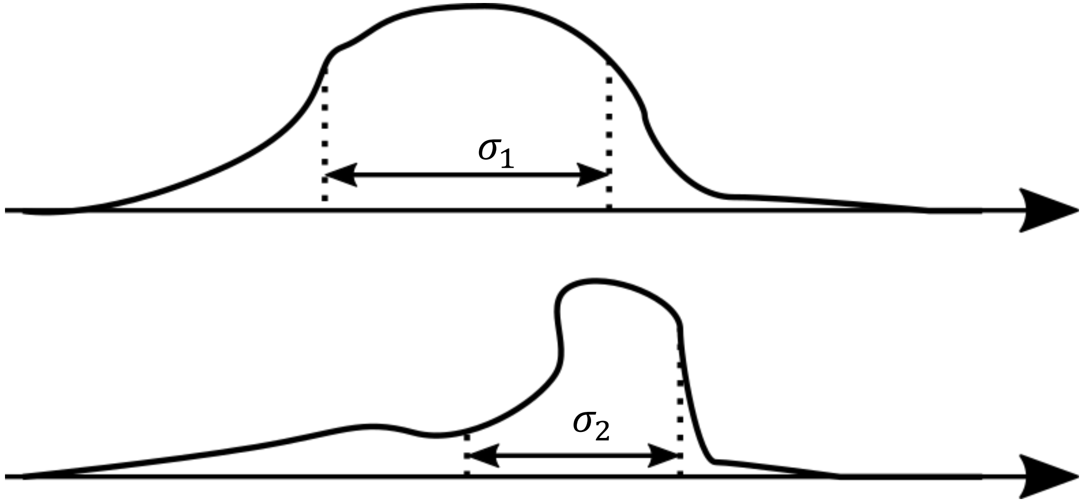


[1] M. G. Kendall, *The advanced theory of statistics*, Vol. 1, 4th edition (1997)

Fourth-order as uncertainty:

See M. G. Kendall *The advanced theory of statistics,* Vol. 1, 4th edition

- We are interested in variance, i.e. 2nd-order moment
  - Variance as angular spread and beam size (of the phase space)
- It's uncertainty is 4th-oder moment, hence we use this as uncertainty
- But: This uses only **one** measurement, not all distributions
  - This gives you impossibly **statistical** error



Side note: Relative statistical errors of beam moments were small, hence another calculation of these errors wouldn't reduce the stat. error of the emittance

[1] M. G. Kendall, *The advanced theory of statistics,* Vol. 1, 4th edition (1997)

Gaussian error propagation

Adding correlation terms to the Gaussian error propagation formula

- The ‘standard’ Gaussian error propagation (EP) goes like

$$\Delta\epsilon = \sqrt{\sum_i \left(\frac{\partial\epsilon}{\partial x_i}\sigma_i\right)^2} = \sqrt{\left(\frac{\mathrm{d}\epsilon}{\mathrm{d}Vx}\Delta Vx\right)^2 + \left(\frac{\mathrm{d}\epsilon}{\mathrm{d}Vp}\Delta Vp\right)^2 + \left(\frac{\mathrm{d}\epsilon}{\mathrm{d}Cov}\Delta Cov\right)^2}$$

- This however ignores **correlation** between different terms

$$\epsilon_{\mathrm{geom}} = \sqrt{Vx * Vp - Cov^2}$$

- General EP is $\Delta\epsilon = \sqrt{\sum_i \left(\frac{\partial\epsilon}{\partial x_i}\sigma_i\right)^2 + \sum\sum_{i\neq j}\frac{\partial\epsilon}{\partial x_i}\frac{\partial\epsilon}{\partial x_j}\sigma_{ij}}$

Correlation between inputs, not the same as covariance!!

- This yields in a smaller statistical error, 24.2 % instead of 34.0 % (neglecting correlation)
- Still a quite high statistical error

[1] K. O. Arras, *An Introduction To Error Propagation […],* Technical Report http://srl.informatik.uni-freiburg.de/papers/arrasTR98.pdf (visited 2019-04-02)

Reassembling the phase spaces

Do different reconstructions of the phase space and compare the results

- Five different ways to reconstruct the phase space, see right
- Calculate statistical error as

$$\Delta\epsilon = \mathrm{rms}(\epsilon_i - \mathrm{mean}(\epsilon_i))$$

		Emittance	Error
Reconstruction	1	3.96 µm	0.14 µm
	2	3.96 µm	0.15 µm
	3	3.96 µm	0.15 µm
	4	3.96 µm	0.15 µm
	5	3.96 µm	0.15 µm

		Number of taken image				
		1	2	3	4	5
Slit Position	1					
	2					
	3					
	4					

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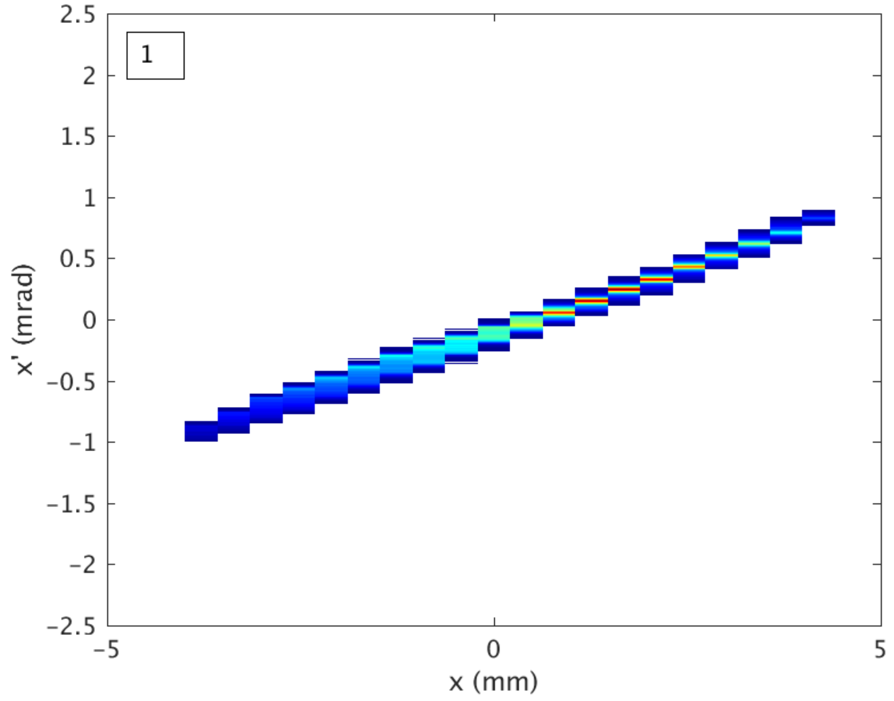
Let's look on the data

How does the phase spaces in this example look like?

- Data was taken on 2018-11-24T0639_XFEL_beam_studies
- Ten reconstructed phase spaces shown right
- Only $\Delta\epsilon = \mathrm{rms}(\epsilon_i - \mathrm{mean}(\epsilon_i))$ gives reasonable error
- Different reconstruction methods don't change error significantly
- Other errors are far too big



Gaussian error propagation neglecting correlations: $\frac{\Delta\epsilon}{\epsilon} = 34.0\ \%$

Gaussian error propagation considering correlations: $\frac{\Delta\epsilon}{\epsilon} = 24.2\ \%$

RMS of ten emittances as error: $\frac{\Delta\epsilon}{\epsilon} = 3.7\ \%$

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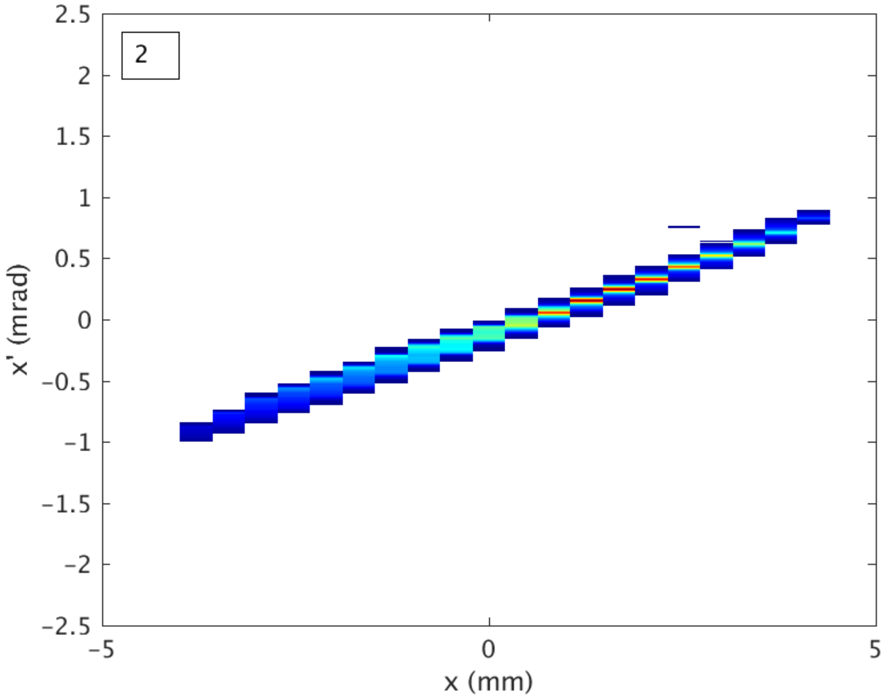

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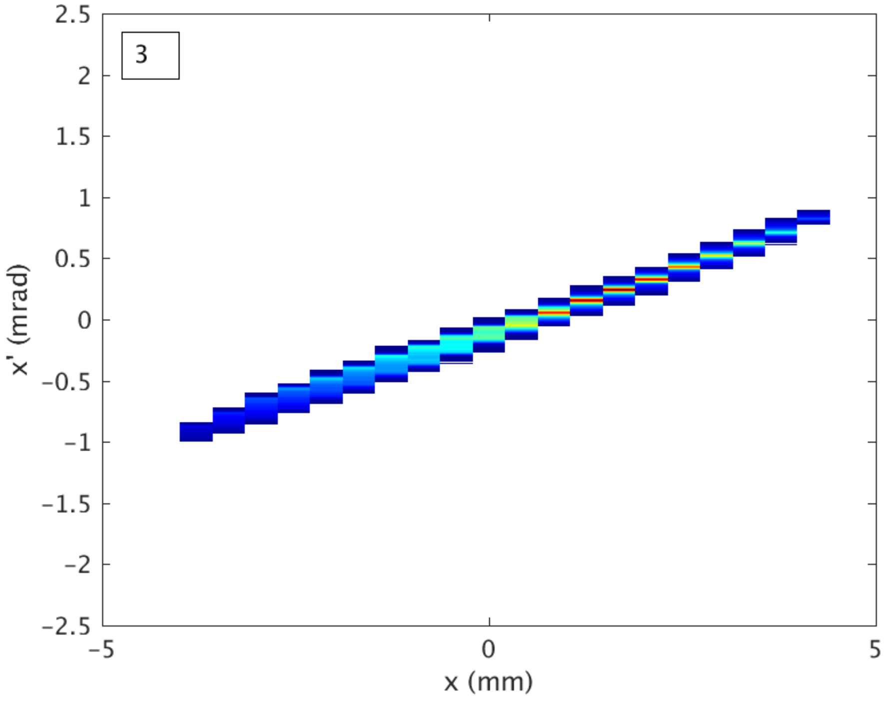

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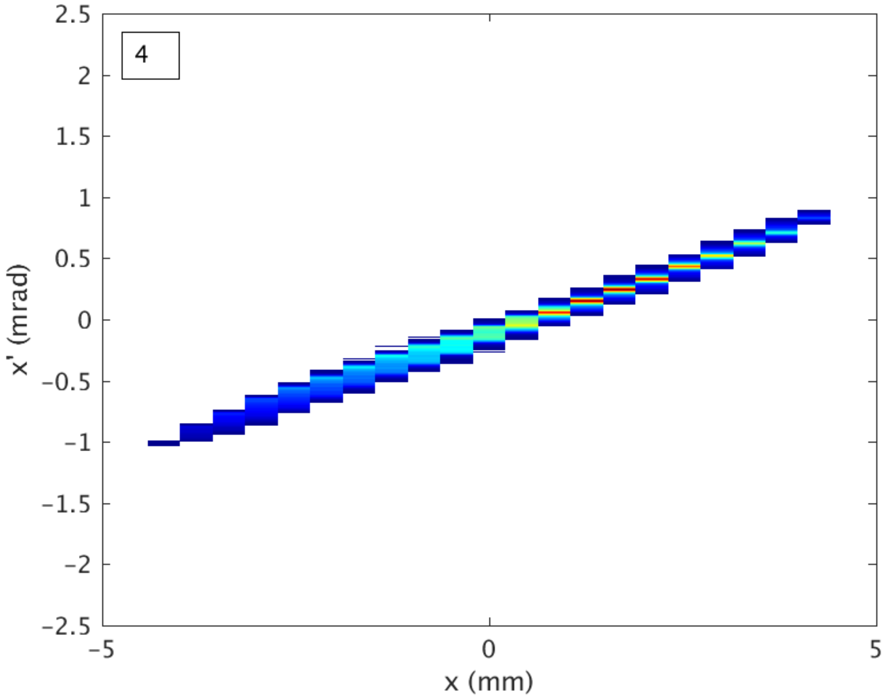

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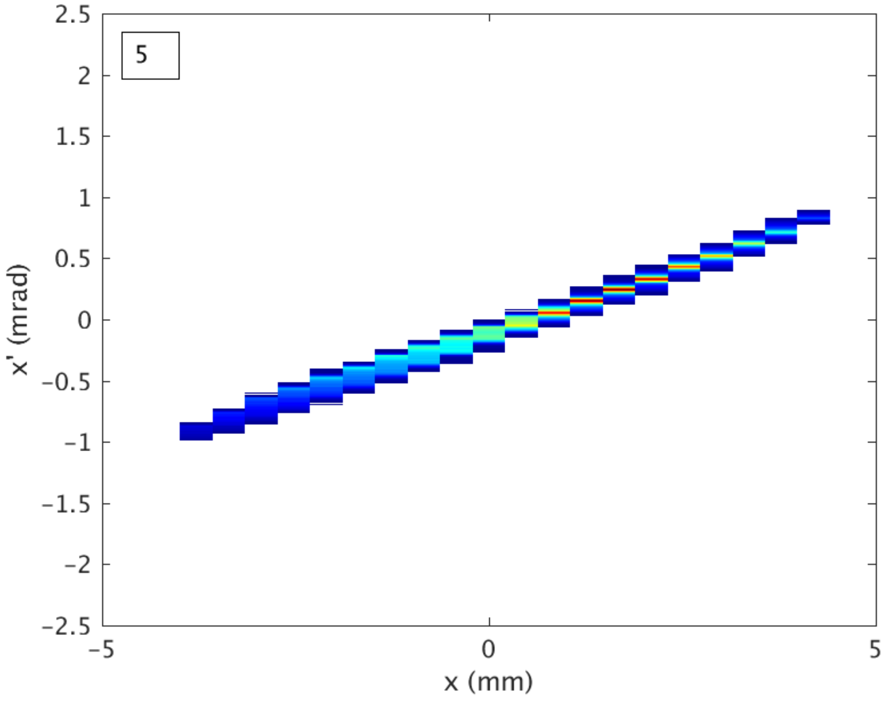

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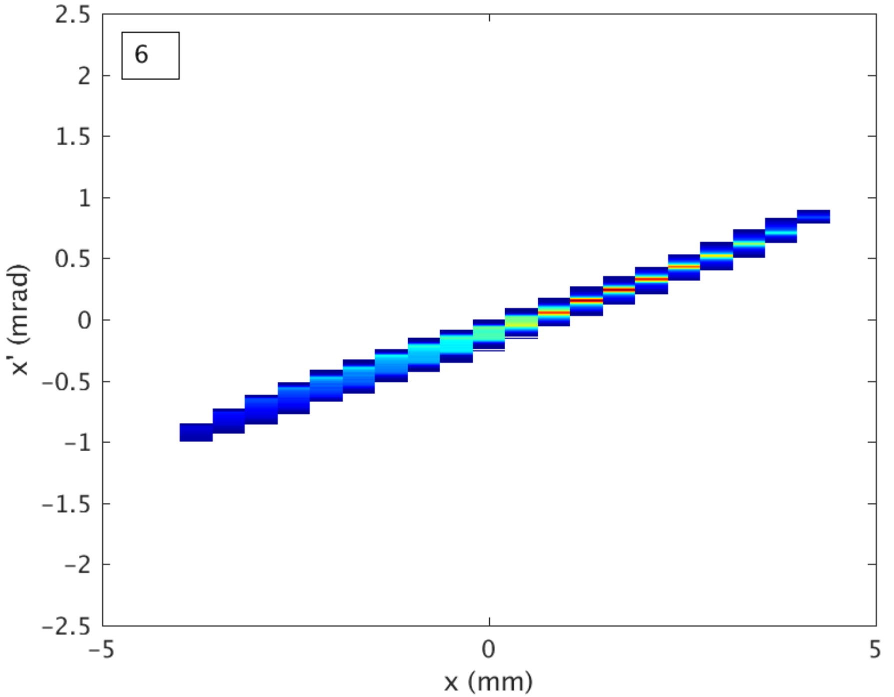

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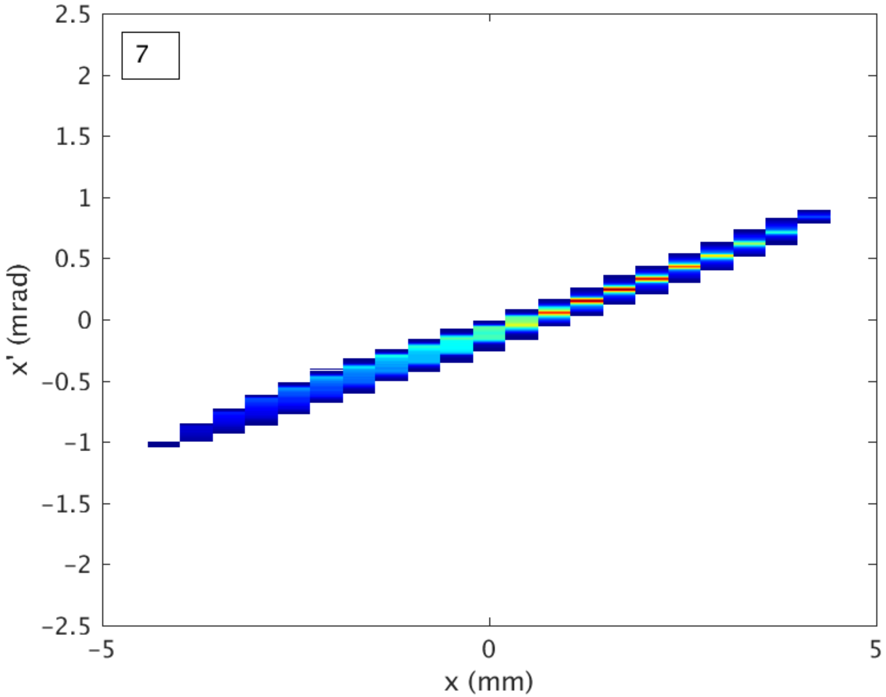

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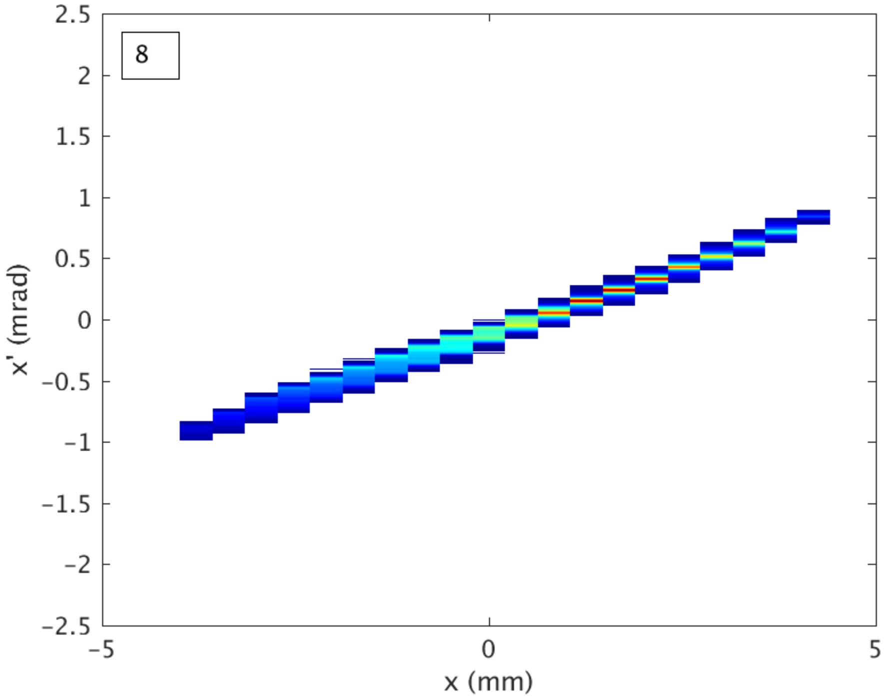

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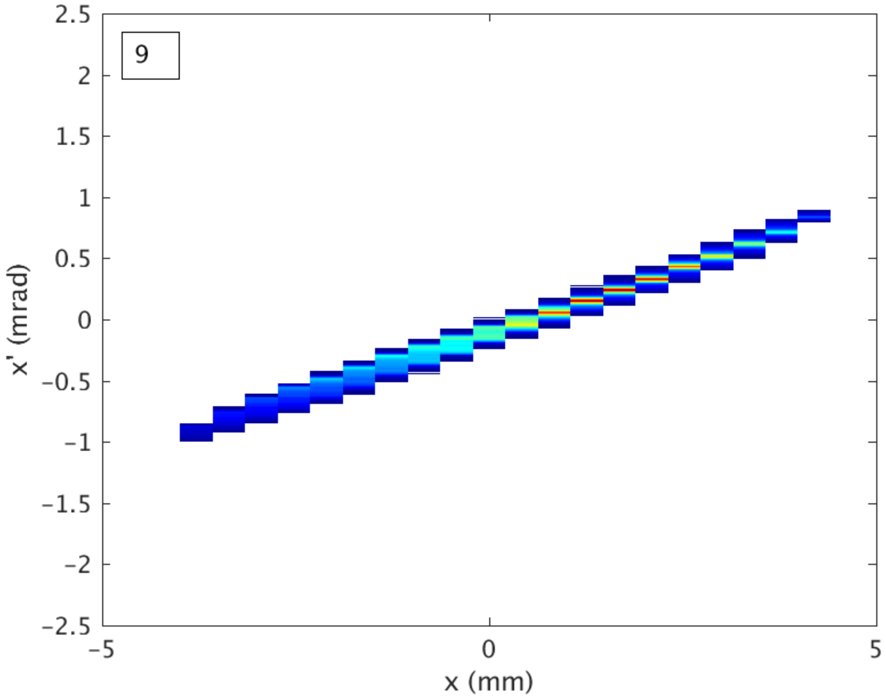

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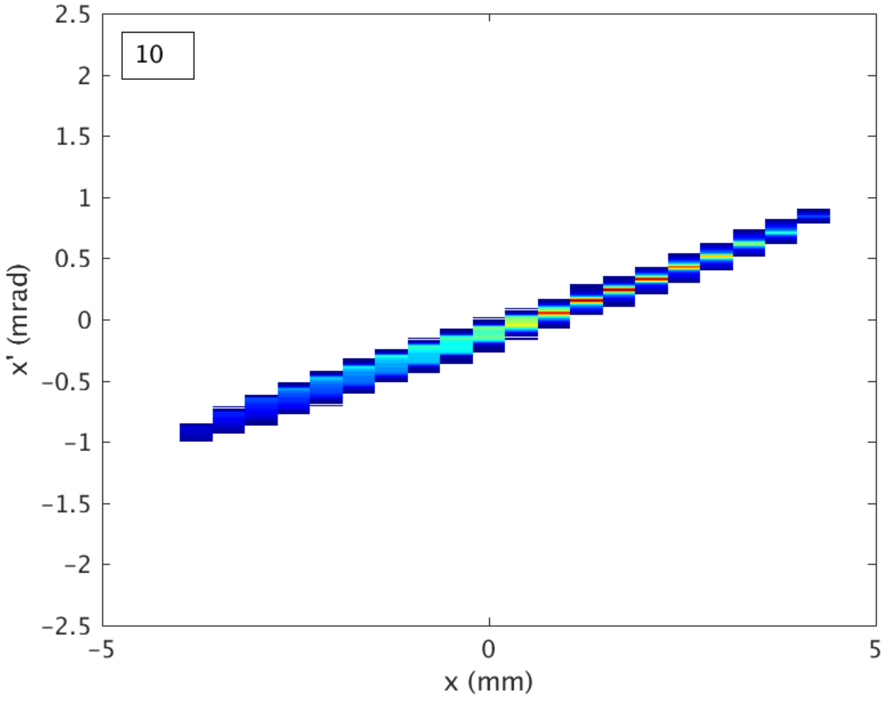

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