K’n’K Seminar: Estimation of stat. error in emittance calculation

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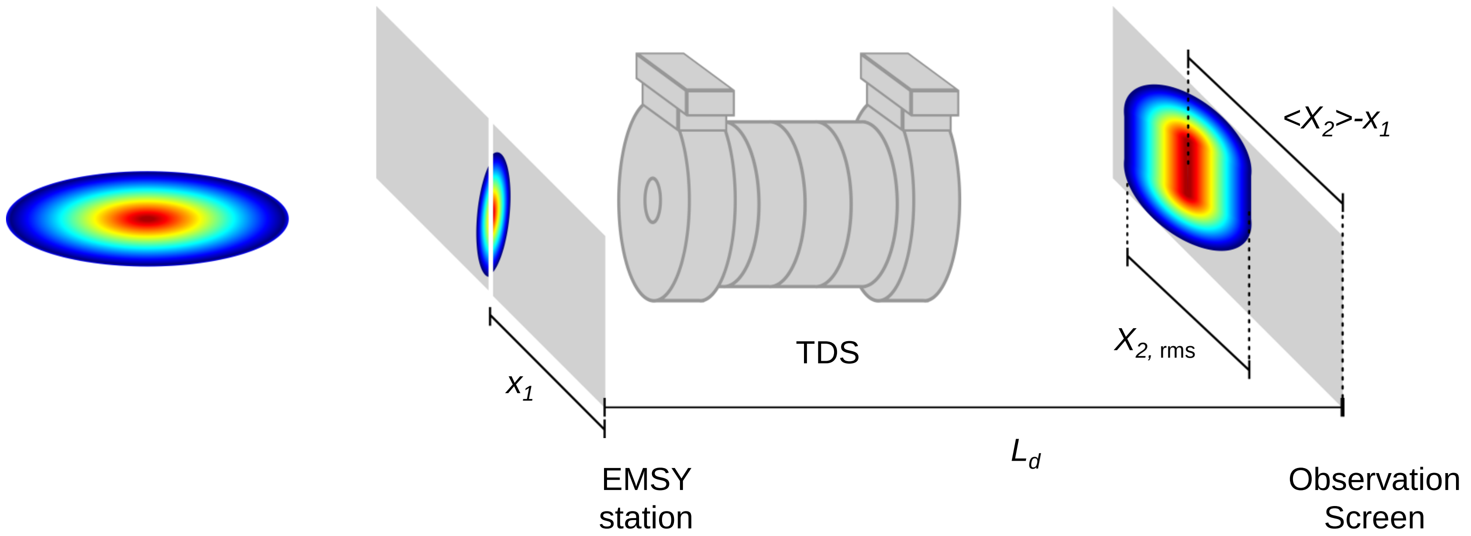
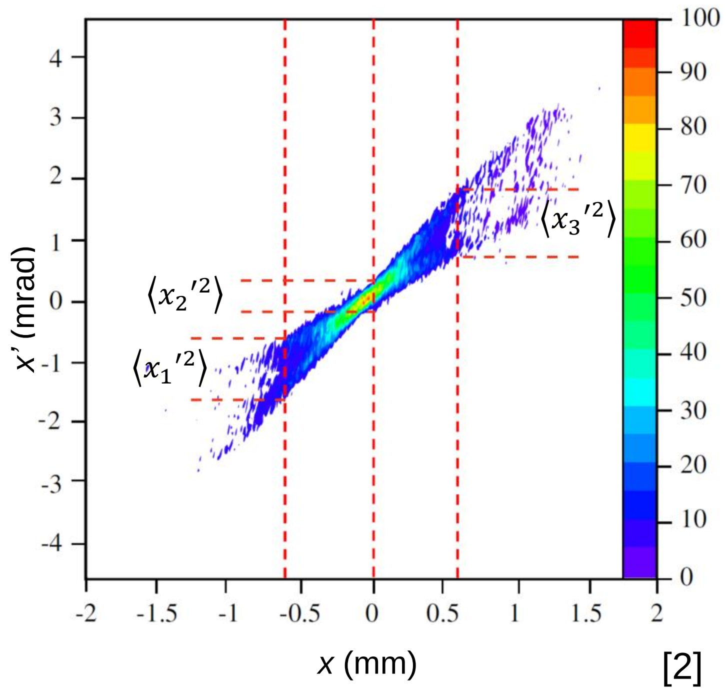
Recap: Slit Scan Method

Slit-Scan-based slice emittance measurements





[2]

> Cut out **emittance-dominated beamlets** from **space charge-dominated beam** with a slit
  - Measure the **size**, **position** and **intensity** of each beamlet on screen

> Reconstruct the phase space at slit position
  - Emittance via $\epsilon = \beta\gamma \frac{\sigma_x}{\sqrt{<x^2>}} \sqrt{\langle x_0{}^2 \rangle \langle x_0{}'^2 \rangle - \langle x_0 x_0{}' \rangle^2}$

[2] S. Rimjaem et al., Nucl. Instr. Meth. Phys. Res. A **671**, 62 – 75 (2012).

Statistical error

Calculation and implementation in MATLAB

```
projected_PS = zeros( n_images, imageSizeX, n_steps);
average_corrected_steps = steps - mean(steps);
for ii = 1:n_steps
    for jj = 1:n_images
        projected_PS( jj, :, ii) = circshift( squeeze( sum
            round(average_corrected_steps(ii)./handles.MOI
    end
end
```

- We often take images ten times
  - Averaging and stat. error calculation possible
- I generate ten phase spaces from ten images (at every slit position)
- Then I calculate ten times each beam moment
  - $Vx = \langle {x_0}^2 \rangle, Vp = \langle {x_0}'^2 \rangle$ and $\mathrm{Cov} = \langle x_0 x_0' \rangle$
- I calculate the uncertainty of these values as

$$\Delta Vx = \mathrm{rms}(\, Vx \; - \mathrm{mean}(\, Vx))$$

```
% calculating mean, variance of the spacial distribution
EX(ii) = sum( dx.*x)./sum( dx);  % mean of x, in pixel
VX( ii) = xscale^2 * sum( dx.*( x - EX(ii)).^2)./sum( dx);  %

% calculating mean, variance of the angular distribution
EP(ii) = sum( dp.*p)./sum( dp); % mean of x', in pixel
VP( ii) = xPrimeScale^2 * sum( dp.*( p-EP(ii)).^2)./sum( dp);

N = sum( sum( PS_temp));

COV( ii) = xscale * xPrimeScale * sum( sum( PS_temp.*repmat( (
```

- Now calculate the geom. emittance via $\epsilon = \sqrt{\mathrm{Vx} * \mathrm{Vp} \; - \mathrm{Cov}^2}$
- .. and the stat. error with gaussian error propagation

$$\Delta\epsilon = \sqrt{\left(\frac{\mathrm{d}\epsilon}{\mathrm{d}Vx}\Delta Vx\right)^2 + \left(\frac{\mathrm{d}\epsilon}{\mathrm{d}Vp}\Delta Vp\right)^2 + \left(\frac{\mathrm{d}\epsilon}{\mathrm{d}Cov}\Delta Cov\right)^2} = \frac{1}{2\epsilon}\sqrt{(Vp * \Delta Vx)^2 + (Vx * \Delta Vp)^2 + (2 * Cov \; * \Delta Cov)^2}$$

```
% calculating geometric emittance
emitx = sqrt( mean(VX)*mean(VP) - mean(COV)^2);            % geom. emittance, in um
emitx_err = sqrt( (0.5 * VX_err * mean( VP)./emitx).^2 + (0.5 * VP_err * mean( VX)./emitx ).^2 + ( COV_err * mean( COV)./emitx).^2);
```

Statistical error calculation

An example of the calculation

$$\Delta\epsilon = \sqrt{\left(\frac{d\epsilon}{dVx}\Delta Vx\right)^2 + \left(\frac{d\epsilon}{dVp}\Delta Vp\right)^2 + \left(\frac{d\epsilon}{dCov}\Delta Cov\right)^2} = \frac{1}{2\epsilon}\sqrt{(Vp * \Delta Vx)^2 + (Vx * \Delta Vp)^2 + (2 * Cov * \Delta Cov)^2}$$

- The error of the emittance is extremely big!? Why??
- Alternative estimation of error:
  - RMS of the emittance values: Ten phase spaces give ten emittances, calculate all them and then the error via $\Delta\epsilon = \mathrm{rms}(\epsilon_i - \mathrm{mean}(\epsilon_i))$
- This gives $\Delta\epsilon = 0.0034\ \mu\mathrm{m}$ (rel. error $\sim 3.7\%$)
- Thanks to Hamed & Osip for discussions and cross-checks
- **What do we do?**

	Value	Error	Rel. error
Vx	$3.711\ \mathrm{mm}^2$	$0.014\ \mathrm{mm}^2$	0.37 %
Vp	$0.168\ \mathrm{mrad}^2$	$6.97\mathrm{E}-4\ \mathrm{mrad}^2$	0.42 %
Cov	0.783 mm mrad	0.003 mm mrad	0.37%
Emitx	0.092 μm	0.031 μm	33.97 %