Simulation of thermal momentum imaging

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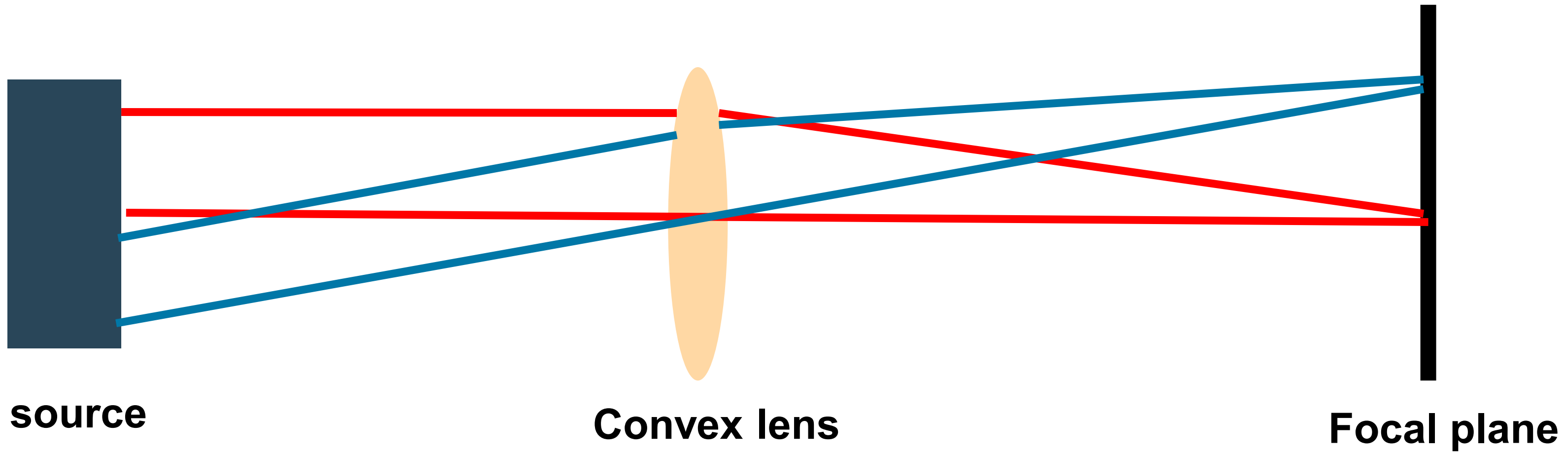
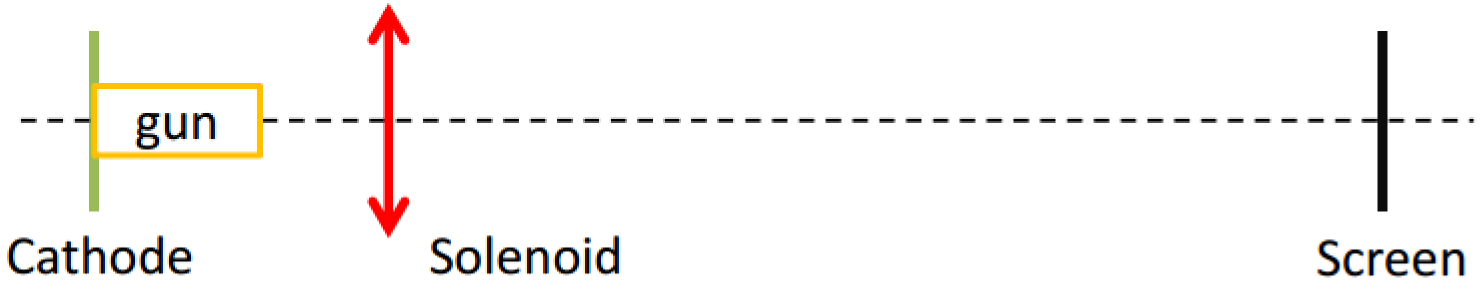
Imaging the momentum on the screen

Basic idea



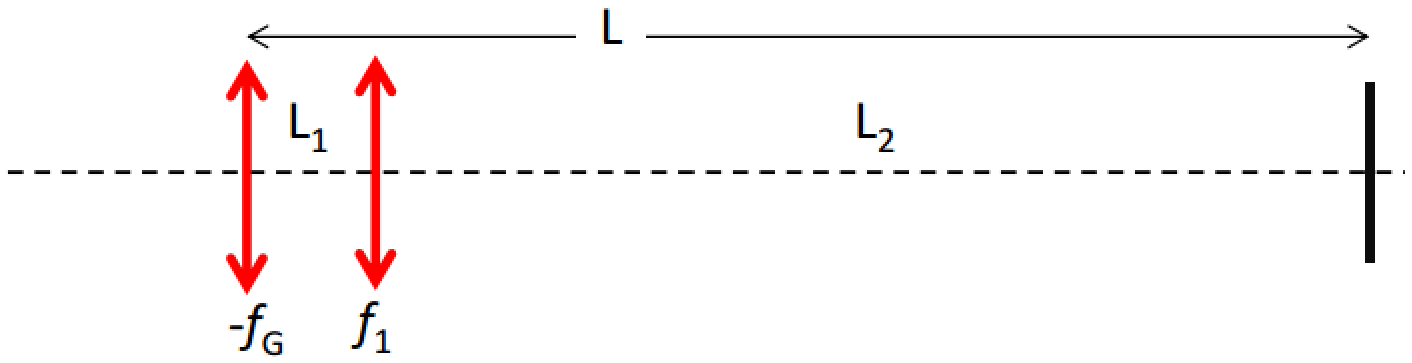
The parallel light will be focused on the same point at the focal plane through the convex lens. Similarly if a convex lens for beams can be found, we can image the beam on focal screen on the base of transverse momentum. This is the thermal momentum imaging.

$$\binom{x}{x'} = \begin{bmatrix} 0 & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \binom{x_0}{x_0'}$$



Theoretical analysis

Transport matrix



$$\begin{bmatrix}1 & L_2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\-k_1 & 1\end{bmatrix}\begin{bmatrix}1 & L_1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\k_G & 1\end{bmatrix}=\begin{bmatrix}1+k_G L-(1+k_G L_1)L_2 k_1 & L-L_1 L_2 k_1\\k_G-(1+k_G L_1)k_1 & 1-L_1 k_1\end{bmatrix}$$

$$M_{11}=1+k_G L-(1+k_G L_1)L_2 k_1=0$$

$$k_1=\frac{1+k_G L}{(1+k_G L_1)L_2}\approx\frac{1}{L_1}+\frac{1}{L_2}$$

$$M_{12}=L-L_1 L_2 k_1=\frac{L_2}{1+k_G L_1}$$

In order to maximized beam size on the screen for a given thermal momentum, L_2 should be maximized and L_1 should be minimized. We plan to use the high resolution screen at 5.28 m in PITZ beam line.

Theoretical analysis

Sensitivity on solenoid tuning

$$x = M_{11}x_0 + M_{12}x'_0$$

$$\sigma_x^2 = M_{11}^2\sigma_{x_0}^2 + M_{12}^2\sigma_{x'_0}^2$$

$$\frac{\partial\sigma_x^2}{\partial k_1}\Big|_{M_{11}=0} = \left(2M_{11}\sigma_{x_0}^2\frac{\partial M_{11}}{\partial k_1} + 2M_{12}\sigma_{x'_0}^2\frac{\partial M_{12}}{\partial k_1}\right)\Big|_{M_{11}=0} = 2M_{12}\sigma_{x'_0}^2\frac{\partial M_{12}}{\partial k_1}$$

$$\frac{\delta\sigma_x^2}{\sigma_x^2}\Big|_{M_{11}=0} = \frac{\Delta k_1\frac{\partial\sigma_x^2}{\partial k_1}\Big|_{M_{11}=0}}{(M_{12}\sigma_{x'_0})^2} = -2\Delta\frac{(1+k_G L)L_1}{L_2} = -2\Delta\left(k_G + \frac{1+k_G L_1}{L_2}\right)L_1$$

To minimize beam size sensitivity on solenoid tuning error, L2 should be maximized, and L1 should be minimized.

Application

- In principle, we can obtain the information about the initial momentum by analyze the spot on the screen placed at L **with one shot**. The rms divergence $\sigma_{x_0'}$ can be easily calculated through $\sigma_x(L)/M_{12}$. So the thermal emittance is

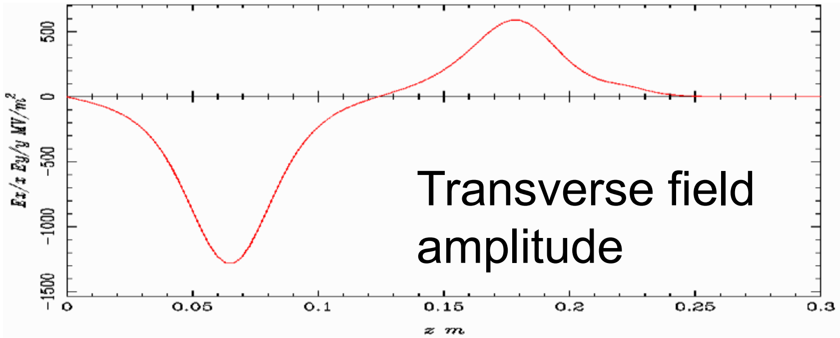
$$\varepsilon_{0,rms} = \sigma_{x_0}\sigma_{x_0'} = \sigma_{x_0}\frac{\sigma_x(L)}{M_{12}}$$

- Due to its convenience, we would like to use this method to do a **map investigation** about cathode on thermal emittance. Since the QE map is also able to obtain, it is interesting to discover a correlation between QE and thermal emittance. To do so, the RF emittance of the off-center beam can not be neglected.

RF induced emittance growth

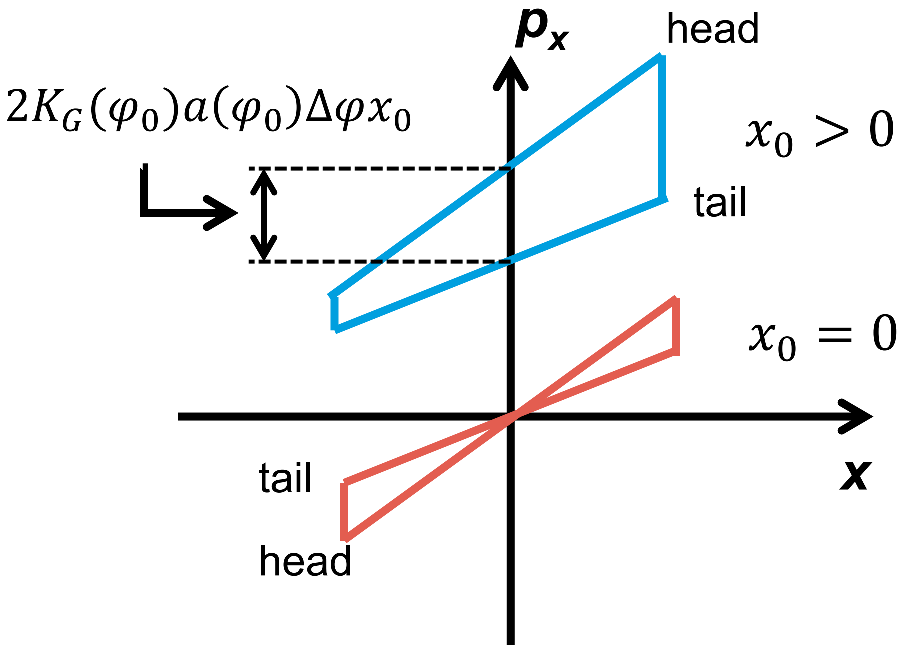
The transverse kick of RF field

The effect brought by transverse kick of RF field on emittance and further to the beam size at screen will be severe for the off-center emission. An example is presented here with offset x_0 and relative longitudinal position $\Delta\varphi$



$$\begin{aligned}\Delta p_x &= K_G(\varphi)x \\ &\approx K_G(\varphi_0)(1 + a(\varphi_0)\Delta\varphi)(x_0 + \Delta x) \\ &= K_G(\varphi_0)x_0(1 + a(\varphi_0)\Delta\varphi)(1 + \frac{\Delta x}{x_0})\end{aligned}$$

$$\Delta p_{xmax} - \Delta p_{xmin} = 2K_G(\varphi_0)a(\varphi_0)\Delta\varphi(x_0 + \Delta x)$$



- x_0 should not be too large – limit the area of measurement.
- $\Delta\varphi$ also should not be too large – limit the laser length, further exert influence on the space charge effect
- An optimized phase φ should be found to minimize parameter $a(\varphi)$ and further to the reduction of spatial and temporal error. It is the **key** to do the map investigation.

Looking for the optimized phase

The energy gain and transverse kick

The energy at the exit of gun (from Kim's paper)

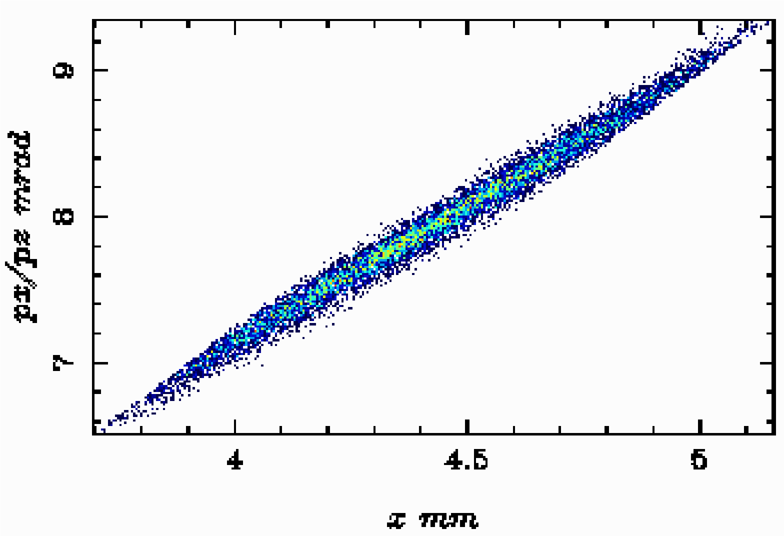
$$\gamma = 1 + \alpha\left(kz\sin(\phi_f) + \frac{1}{2}(\cos(\phi_f) - \cos(\phi_f + 2kz)\right)$$

$$\alpha = \frac{eE_0}{2m_0c^2k}, \qquad \phi_f = \omega t - kz + \phi_0$$

When $kz = 1.6\,\pi$, the largest γ is obtained at ϕ_f = **79.2 deg**

The transverse kick at the exit of gun

$$p_x = \left(\alpha k \sin(\phi_f)\right)x$$



To achieve the least dependence of p_x on ϕ_f, that is less sensitivity to the bunch length, ϕ_f should be chosen to be 90 deg, which means not at the maximum energy gain phase.

Looking for the optimized phase

The relationship between the phase and RF emittance

The corresponding RF induced emittance at the offset $\boldsymbol{x_0}$ and bunch length $\boldsymbol{\sigma_\phi}$

$$F(x,\phi) = f_1(x) f_2(\phi) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left(\frac{(x-x_0)^2}{2\sigma_x^2}\right) \frac{1}{\sqrt{2\pi\sigma_\phi^2}} \exp\left(\frac{(\phi-\phi_0)^2}{2\sigma_\phi^2}\right)$$

Assuming there is **no correlation** between the transverse and longitudinal phase space.

$$\begin{aligned} \varepsilon_{RF} &= \alpha k \sqrt{\langle x^2 - x_0^2\rangle\langle x^2 \sin^2\phi - x_0^2 \sin^2\phi_0\rangle - \langle x^2 \sin\phi - x_0^2 \sin\phi_0\rangle^2} \\ &= \alpha k \sigma_x \sqrt{[\frac{1}{2}(1-e^{-2\sigma_\phi^2}) - (2e^{-\sigma_\phi^2/2} - 1 - e^{-2\sigma_\phi^2})\sin^2\phi] x_0^2 + [\frac{1}{2}(1-e^{-\sigma_\phi^2}) + \frac{1}{2}(e^{-\sigma_\phi^2} - e^{-2\sigma_\phi^2})\cos 2\phi]\sigma_x^2} \end{aligned}$$

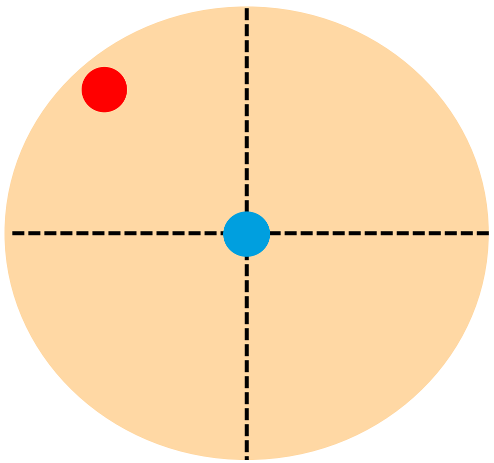
- When the bunch length is zero, the RF emittance will disappear
- Since σ_ϕ is always small (5 ps correspond to 0.04 for 1.3 GHz), the expression can simplify to be

$$\varepsilon_{RF} \approx \alpha k \sigma_x \sigma_\phi \cos\phi \sqrt{\sigma_x^2 + x_0^2}$$

To minimize the difference between various offset and the total RF emittance, ϕ should be 90 deg.

Looking for the optimized phase

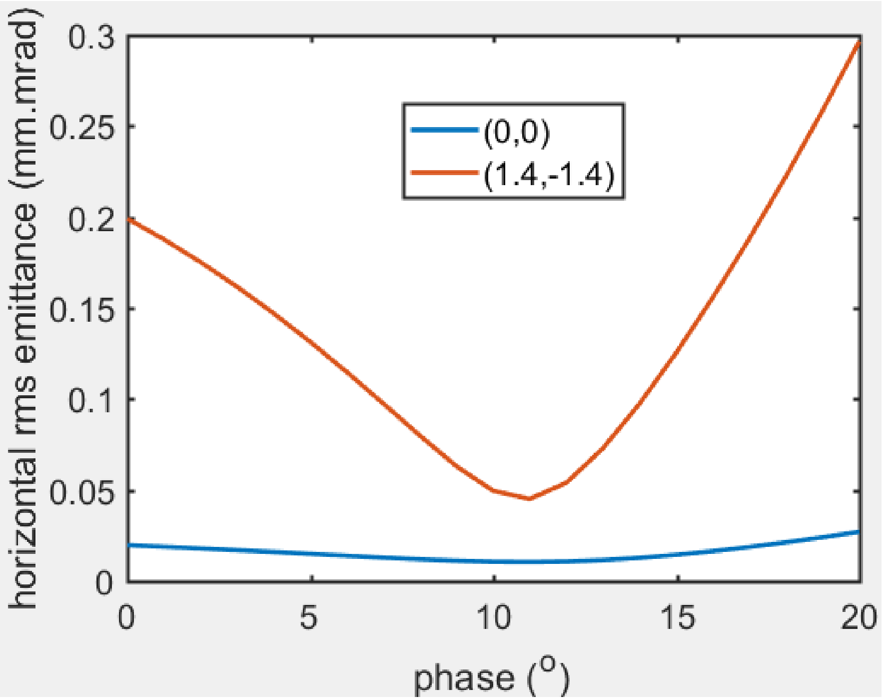
Demonstration of the relationship between RF emittance and phase, position



- The thermal emittance has been set to approach zero. The RF emittance varies with phase and position is shown. Here phase is regarded to the phase for maximum momentum gain.

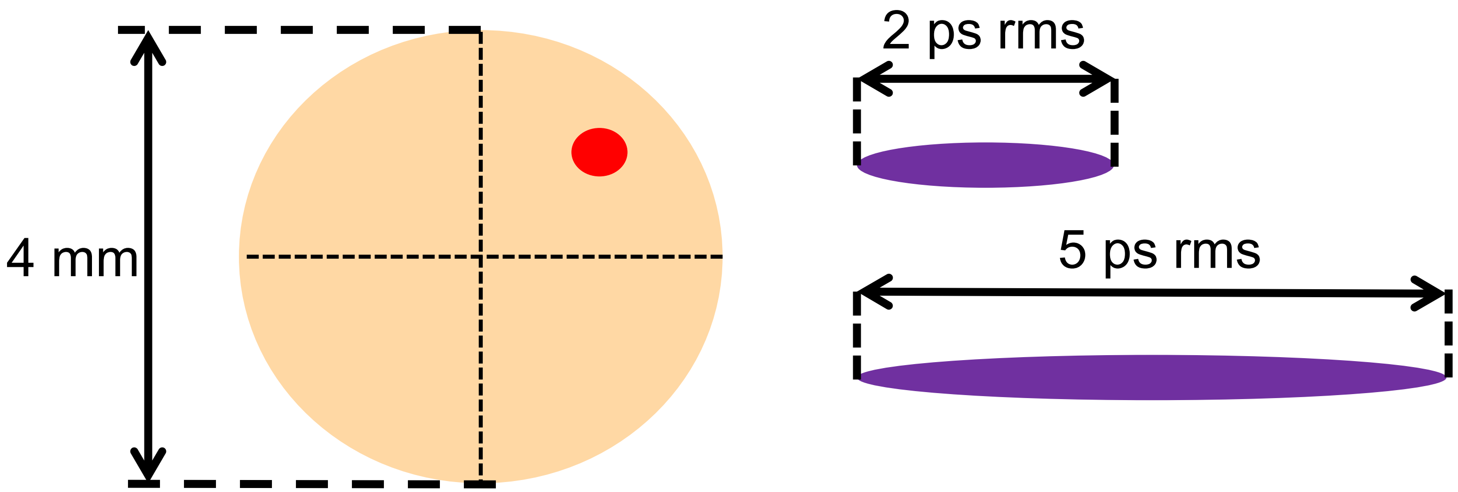
- A 'good' phase can diminish the RF emittance, especially for those off-center emission.

Parameter	Value
Peak field	60 MV/m
Laser length	5 ps rms
Laser radius	0.25 mm



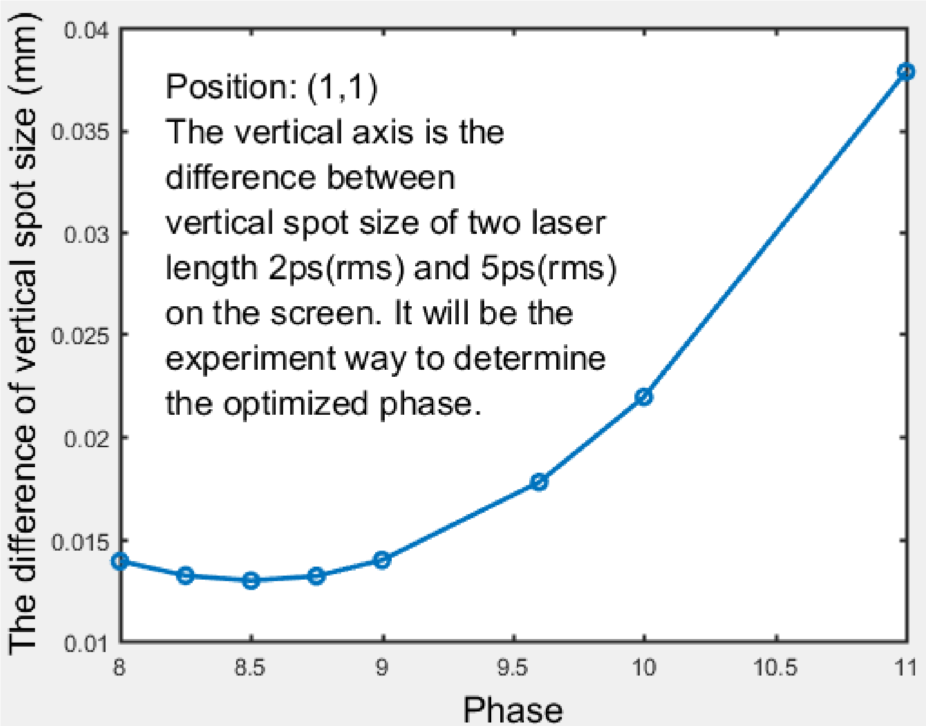
Looking for the optimized phase

Way to determine the optimized phase



Simulate and obtain the difference of beam size at the screen with two different laser pulse length (2 ps rms and 5 ps rms). The phase corresponding to minimum difference is the chosen one.

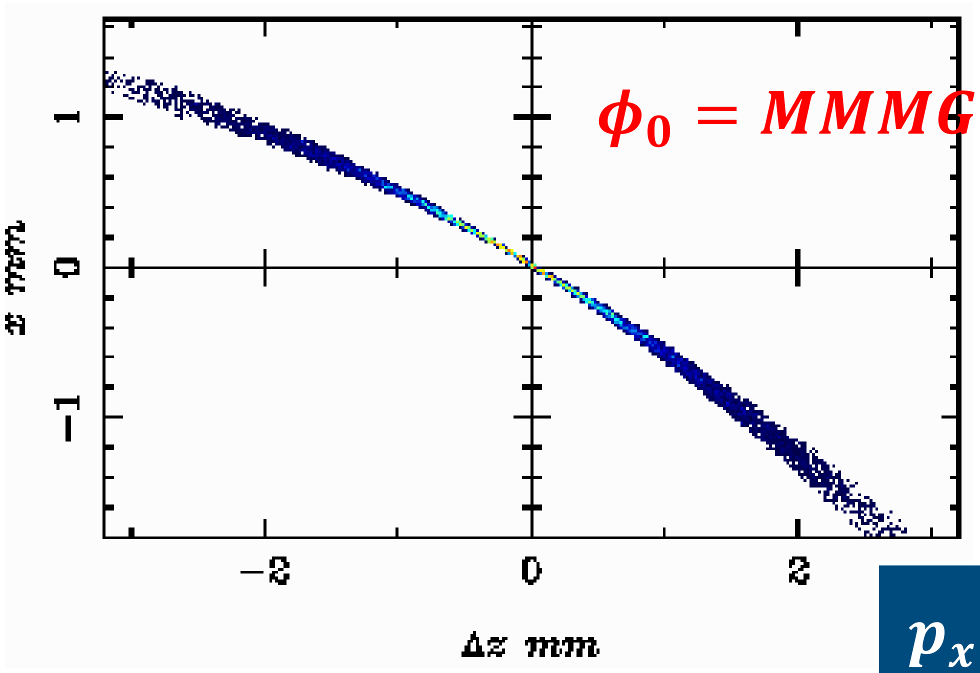
The center is not recommended for the test position due to its gentle change in beam size. We choose the test spot at (1,1) position. It is more sensitivity to the change and easier to determine the optimized phase. The right shows the simulation results. The optimized phase locates at 8.5 degree against the MMG phase.

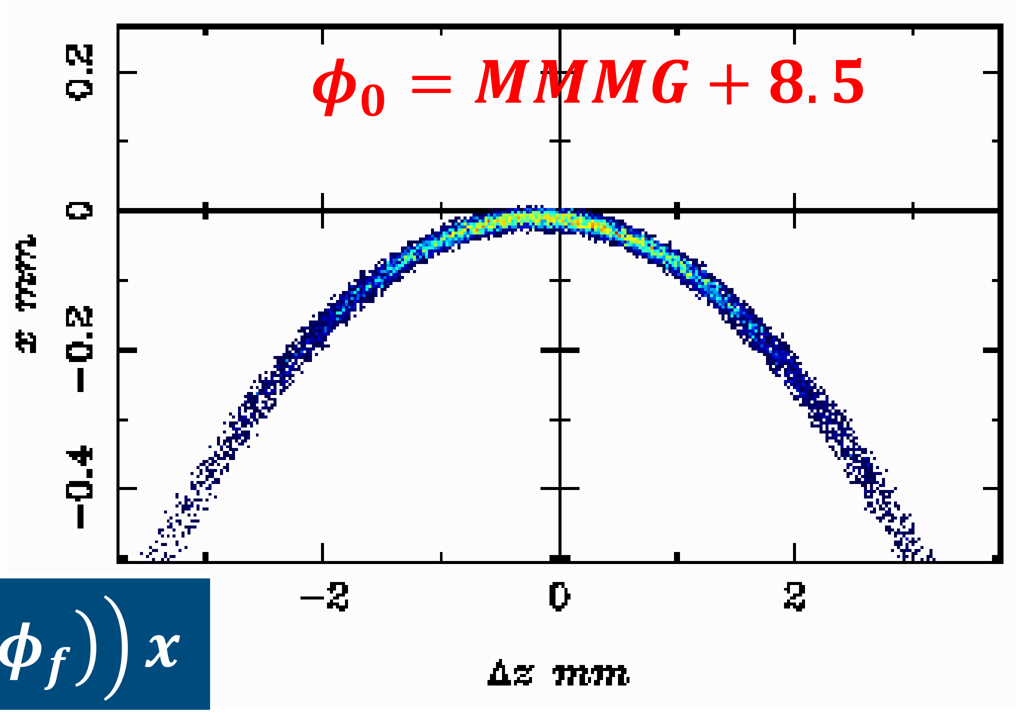


Looking for the optimized phase

Simulation proof

At $\phi_0 = MMMG$ and $\phi_0 = MMMG + 8.5$, with proper solenoid magnetic field to achieve M11=0, the beam shape in real space at 5.28 m is shown below. Here the thermal emittance is tuned to be nearly zero. Since $x = M_{12}x'$, the horizontal axis x represents for the transverse momentum.

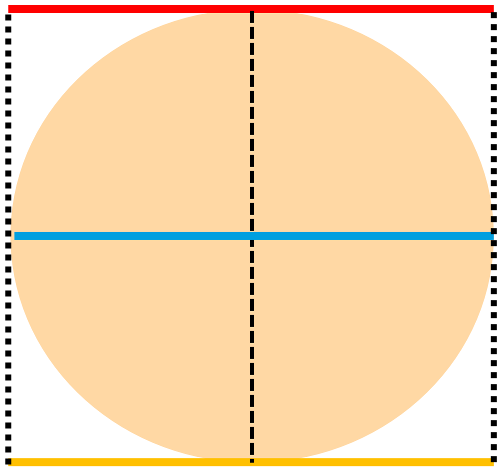


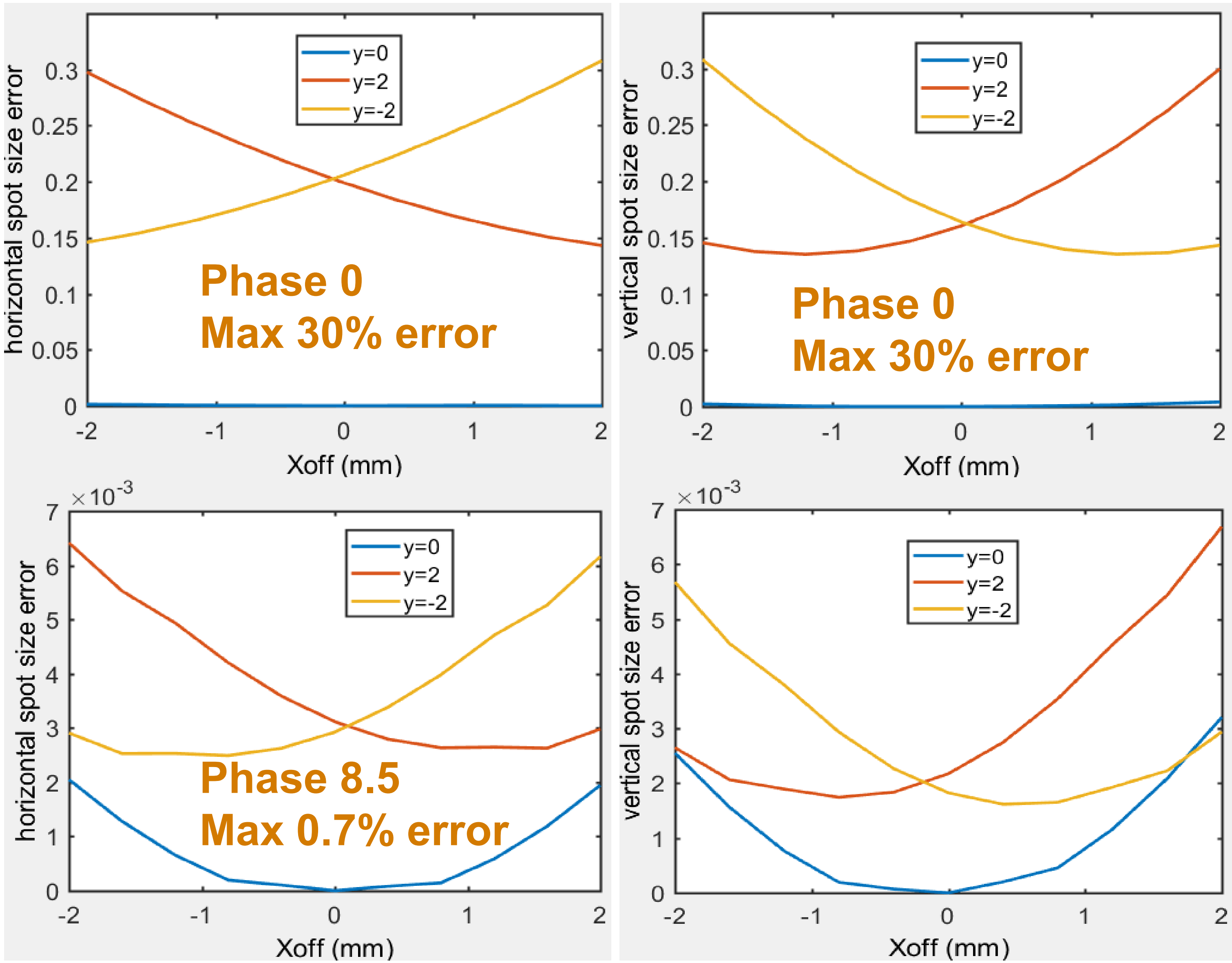


$$p_x = \left(\alpha k \sin(\phi_f)\right) x$$

Advantage of optimized phase

Spatial error



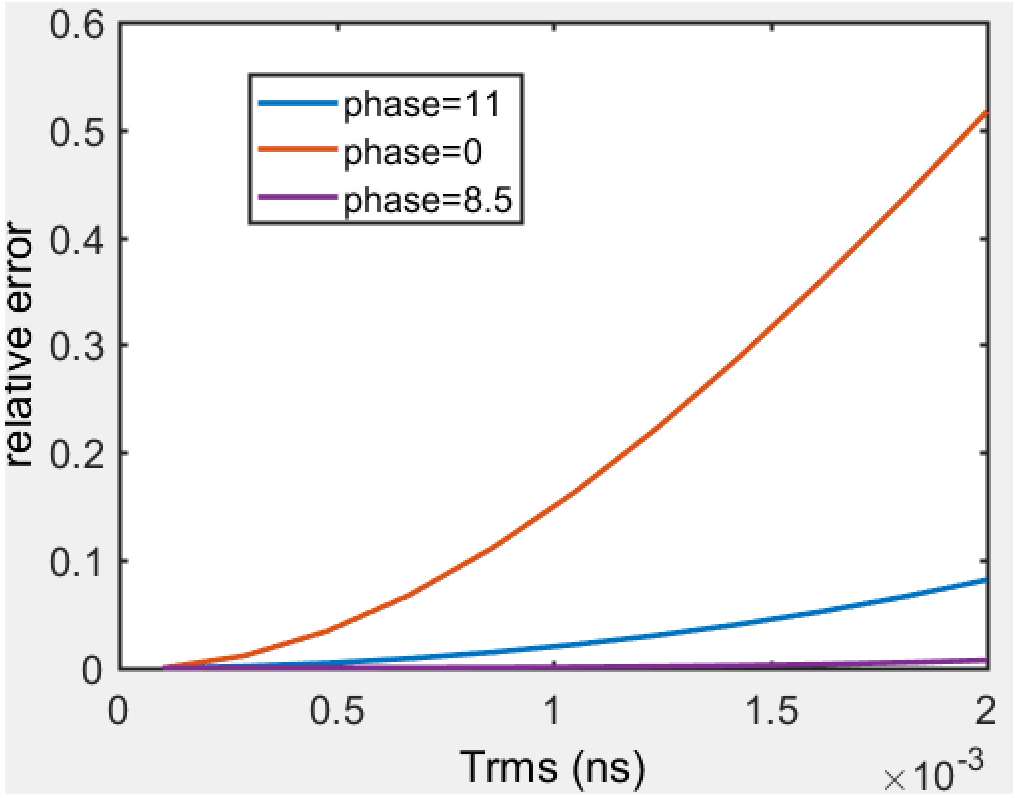


The spatial error is induced by RF emittance growth. Working at the optimized phase will significantly reduce the spatial error.

Advantage of optimized phase

Temporal error

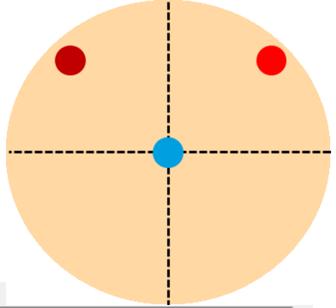
The off-center emission is delicate to the beam length. The right picture shows the development of beam size error with the beam length at the very off-center point (2,-2). A non optimized phase, like phase 0, will bring about tremendous error with 2 ps rms length. However, a moderate beam length is required to alleviate the space charge effect. Therefore, working at optimized phase is necessary, since it can reduce the error to only **0.7%** at 2 ps rms. It provides a wider range choice to the laser length.

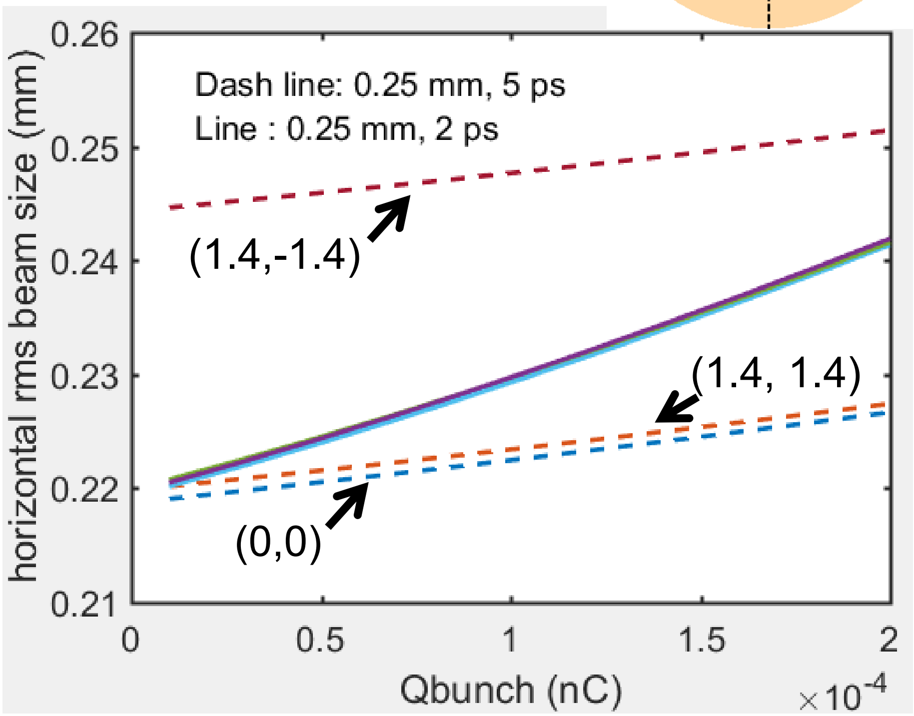


Space charge effect

A choice of radius and length

Laser profile:
Transverse – truncated gaussian (1 σ)
Longitudinal - gaussian





The new high resolution screen manage to detect electron bunch down to **10 fc**.

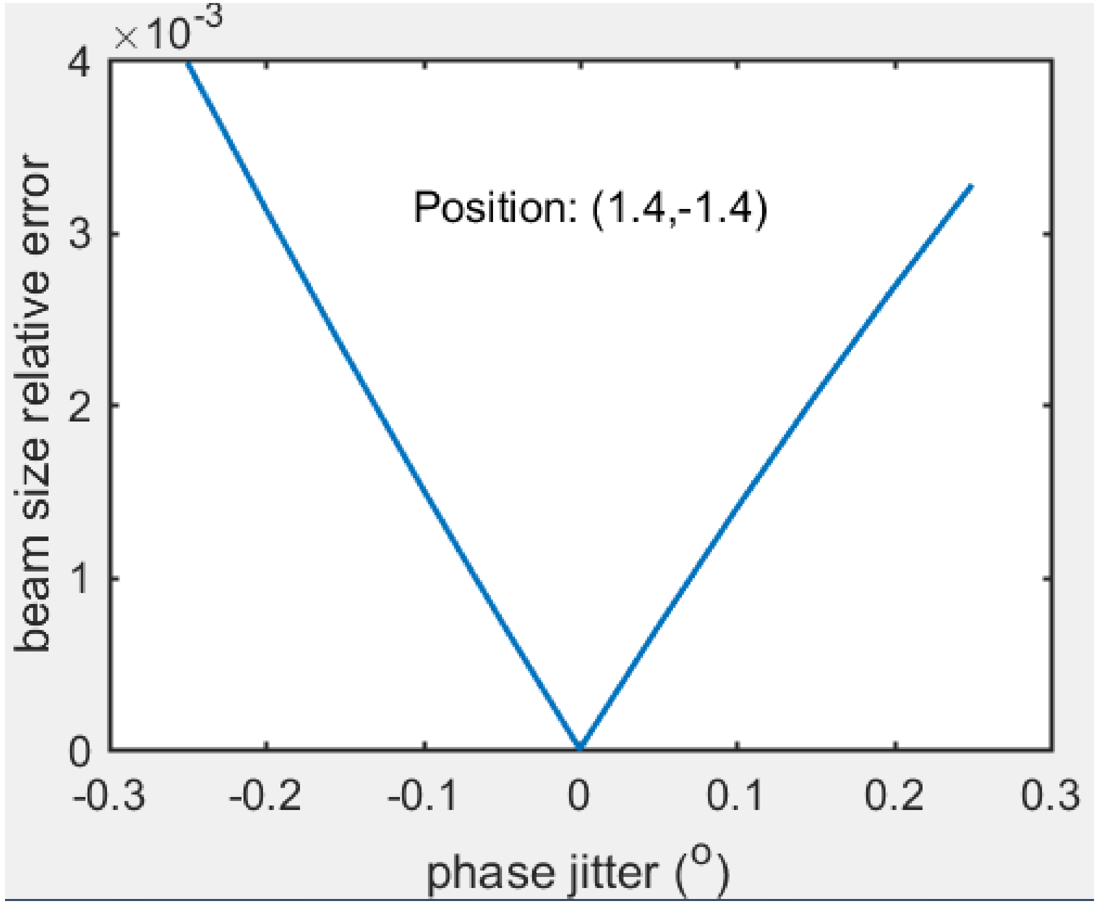
Initial radius (mm)	Laser length (ps rms)	Spot size change*(R(q)-R(0))*(mm)/ Charge *q* (nc)
0.25	5	47.28
0.25	**2**	**107.5**
0.5	1	121.6

On the one hand, we wish a small initial spot size for a better resolution to do a map investigation about cathode, so we need a longer beam to diminish space charge effect. On the other hand, the large laser length will result in severe spatial error in the interested area, shown in the above picture. Therefore, a compromise is achieved to choose **0.25 mm** as initial radius with **2 ps** rms length.

Gun phase jitter

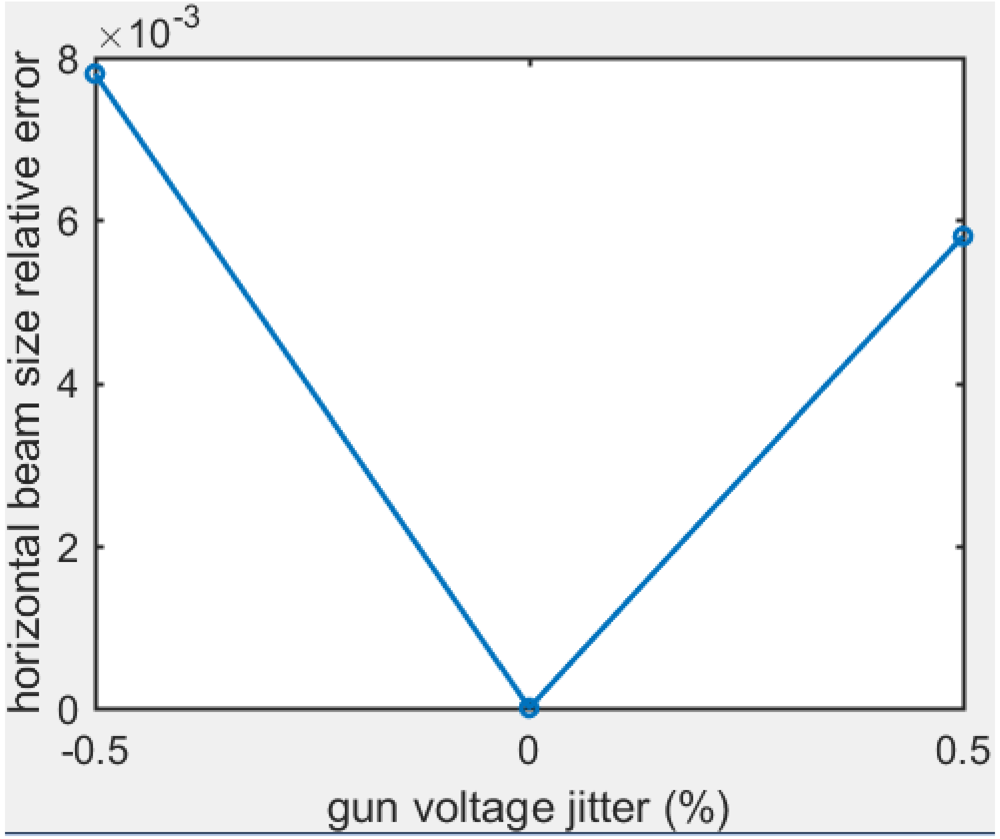
Phase jitter : 0.04 degree rms

The error at sensitive point is less than **0.4%**. So the error brought by the phase jitter is negligible.



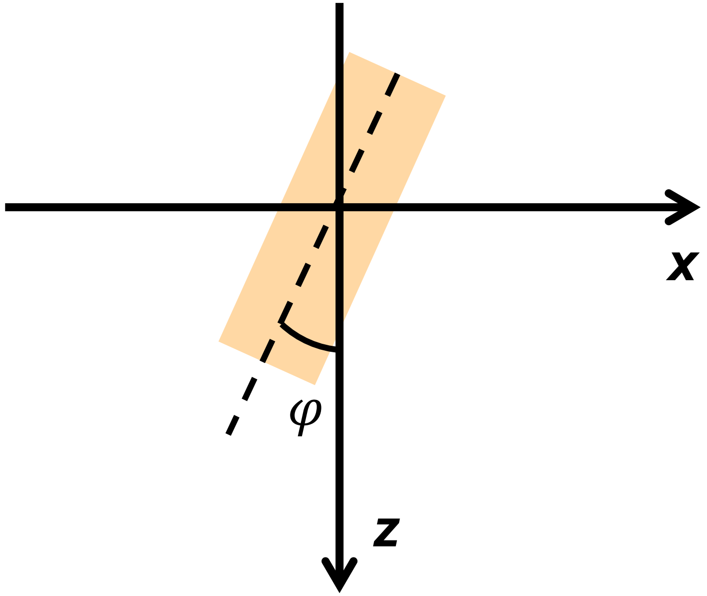
Gun voltage jitter error

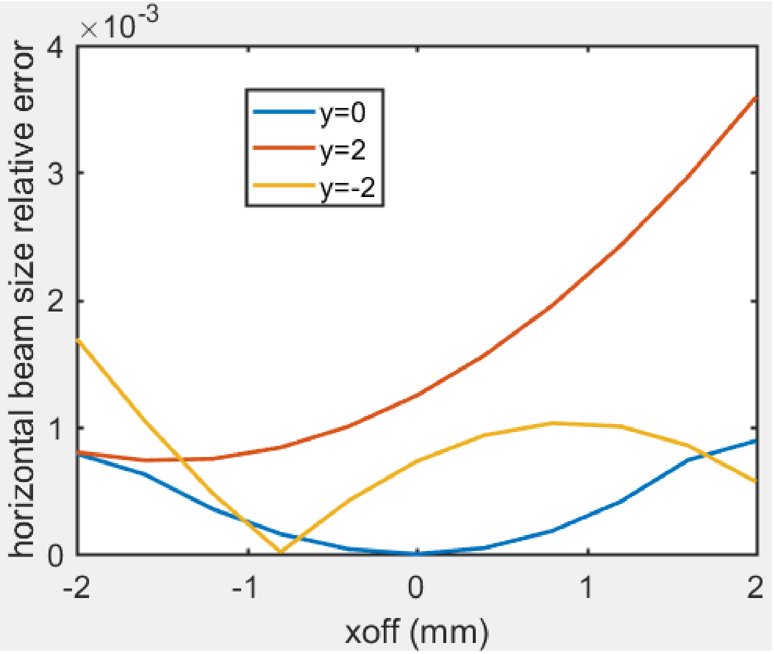
With 0.5% voltage jitter, the final beam size has only **0.8%**, also very small.

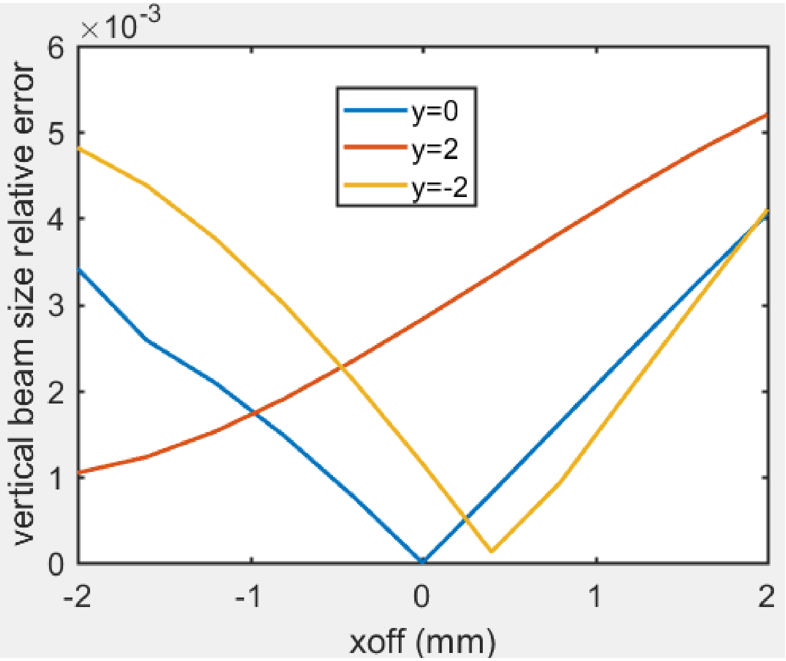


X-Z/Y-Z solenoid rotation

Spatial error





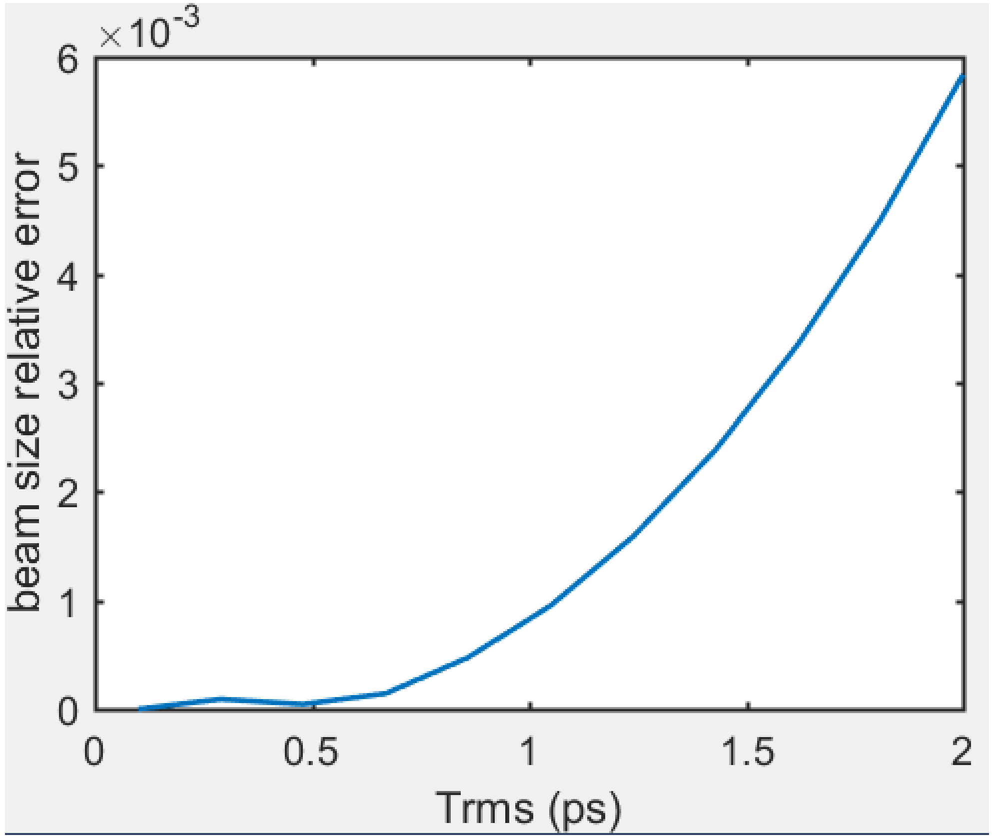


The solenoid rotation is one of the results of alignment error. Mainly it will induce the image's position deviation from the center on the screen. But also it will bring about the change of beam size. The above picture shows the results with 0.5 mrad x-z rotation. The maximum error is less than **0.6%**. As a comparison, the real rotation angle is believed to be better than 0.1 mrad.

X-Z/Y-Z solenoid rotation

Temporal error

At sensitive point (2,-2) the 0.5 mrad rotation induce **0.6%** error at 2 ps rms. So the method is robust enough to the solenoid rotation.



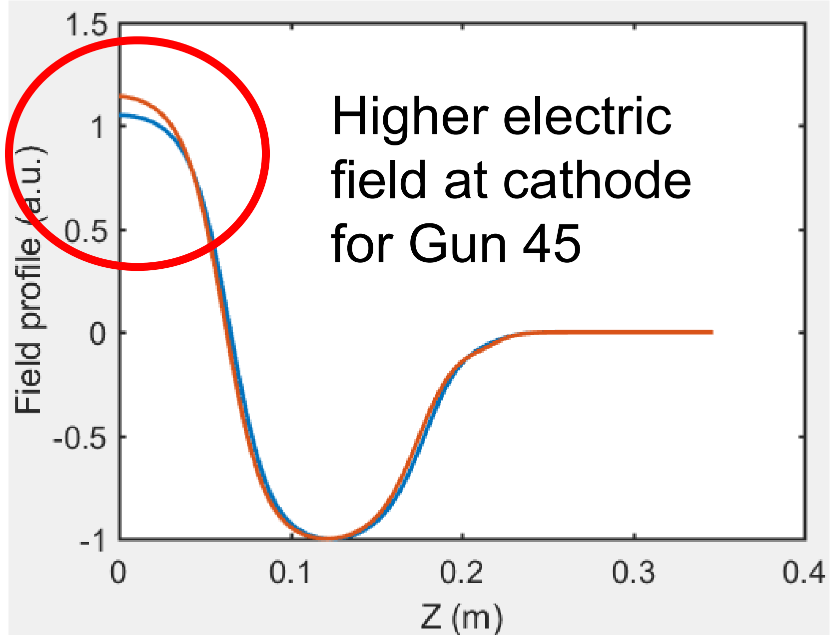
Sensitivity to the Gun field profile

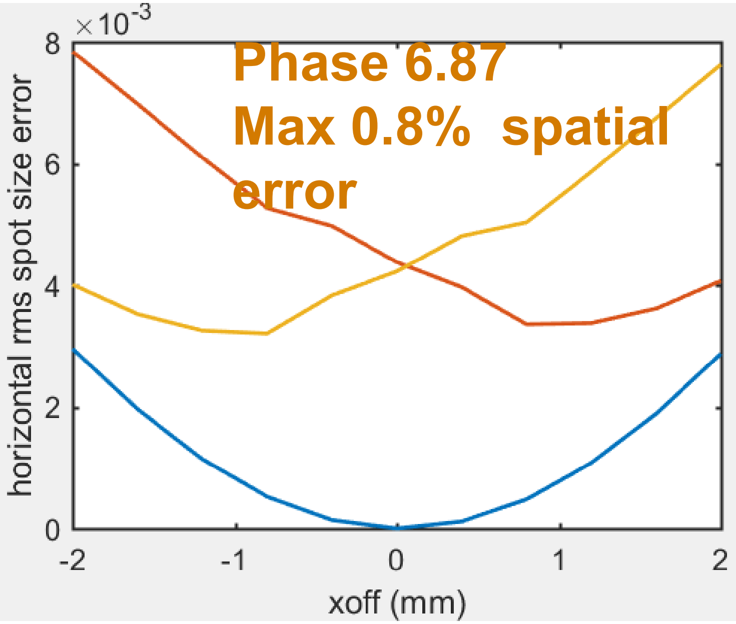
Gun 42 and Gun 45

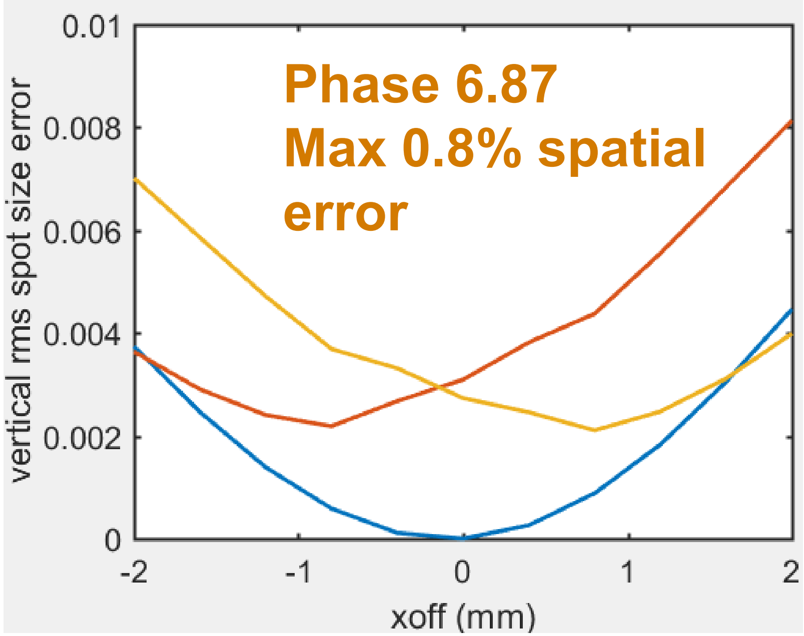
Explore the method’s sensitivity to the gun field profile:

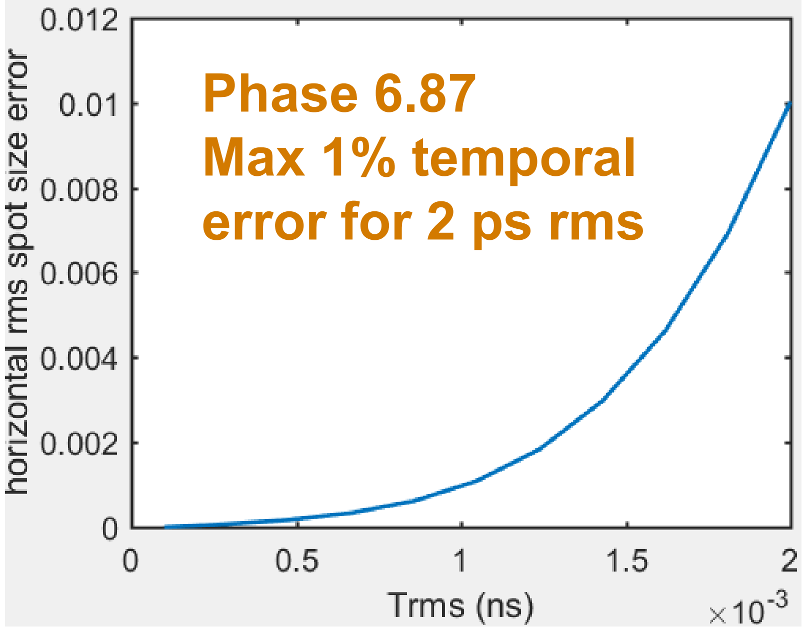
- The optimized phase
- The spatial and temporal error
- The value of M12
- Space charge effect

Gun 45 is chosen since its field profile is relatively most different from Gun 42 among Gun 41-46. The maximum electric field is tuned to achieve the same beam final energy at MMG phase









Sensitivity to the Gun field profile

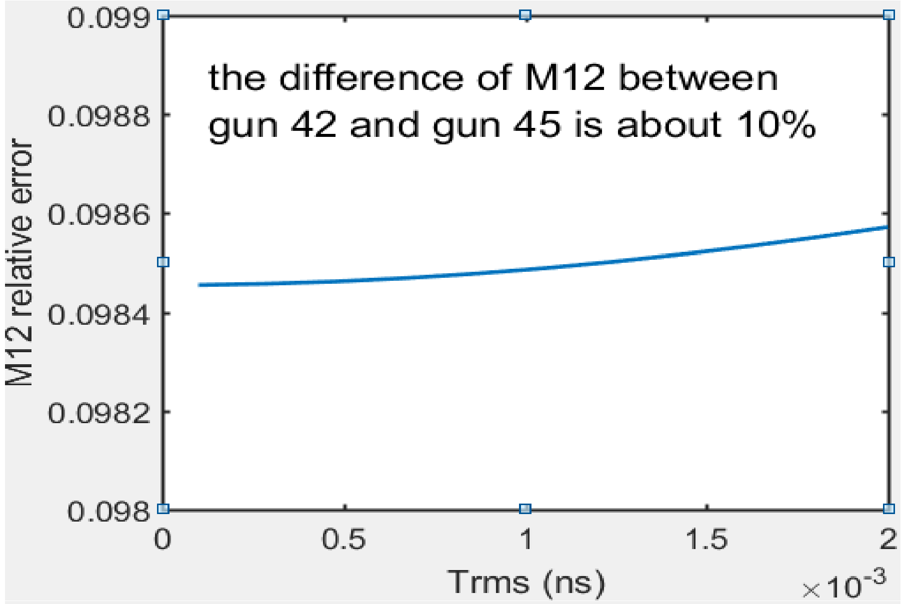
The change of M12 and space charge effect

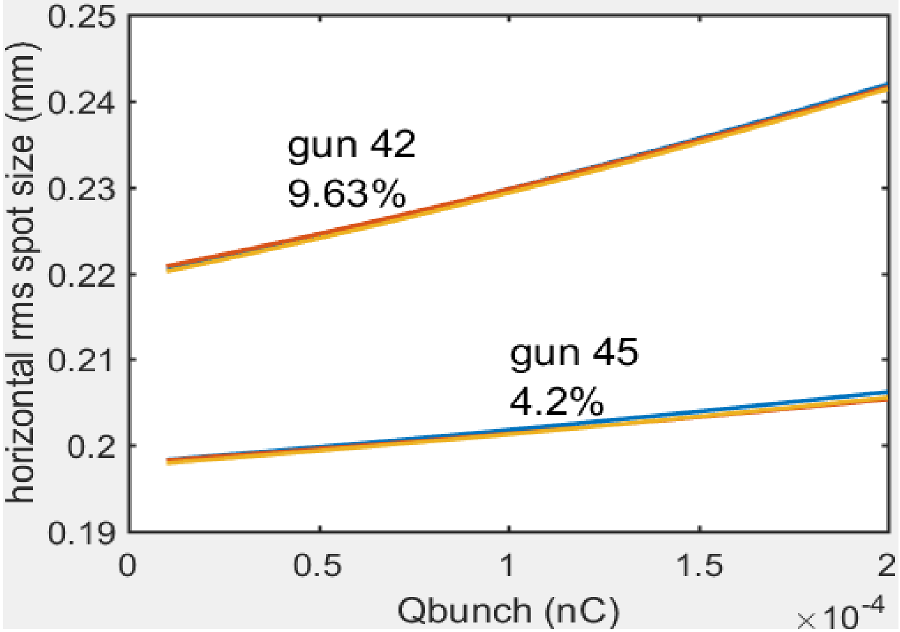
- The value of M12 is dependent on gun field profile. The M12 of gun 45 is 10% smaller than gun 42.

$$M_{12} = \frac{L_2}{1 + k_G L_1}$$

- The space charge effect is less significant in gun 45. The higher electric field at cathode can effectively reduce the space charge.

Initial radius (mm)	Laser length (ps rms)	Spot size change*(R(q)-R(0))*(mm)/ Charge *q* (nc)
0.25	5	47.28
0.25	2	107.5
0.5	1	121.6
0.25	2	41.551





Summary

- The principle of thermal momentum imaging is introduced. It provides a new way to measure thermal emittance with single shot.
- The effect of RF emittance on off-center thermal emittance measurement is discussed. Working at the optimized phase can effectively reduce the RF emittance spatially and temporally. An experiment way to determine the optimized phase is proposed.
- Space charge will enlarge the beam size and overestimate the thermal emittance.
- The gun phase jitter, voltage jitter and solenoid rotation has small effect on the accuracy.
- The exact value of M12 is related to the real gun field. Higher electric field at cathode can effectively reduce the space charge effect and further reduce the main error of this method.

Thanks for your attention