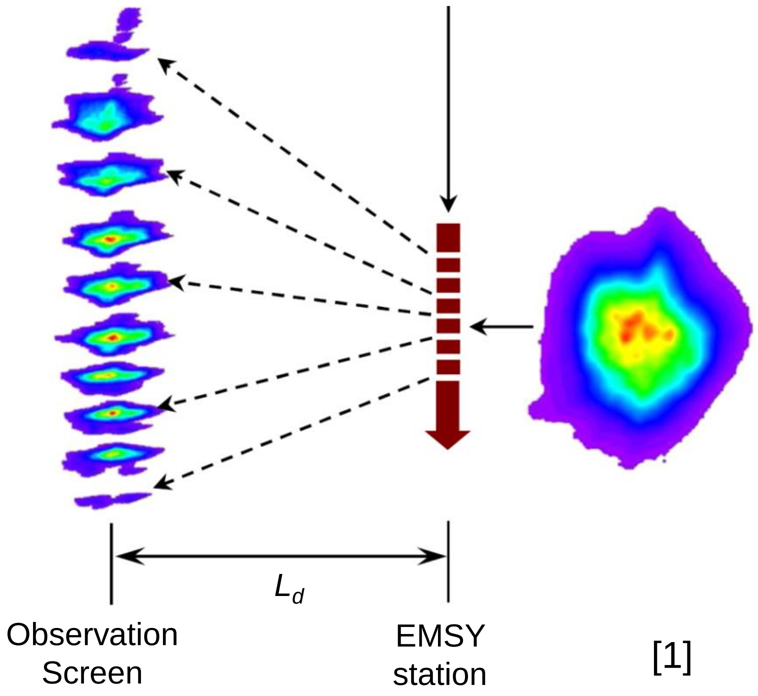
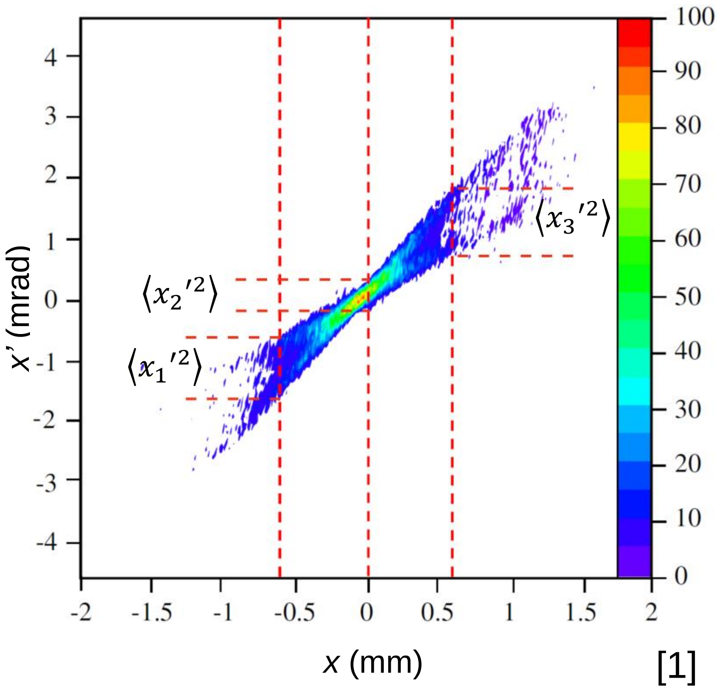
Simulation and numerical calculation: Beam size evolution after slit mask

Raffael Niemczyk, Zeuthen, July 26th 2018

Recap: Emittance measurement

Slit-based emittance measurement





> Cut out emittance-dominated beamlets from space charge-dominated beam with a slit

- Measure the **size**, **position** and **intensity** of each beamlet on screen

> **Reconstruct the phase space** at slit position

- Emittance via $\epsilon = \beta\gamma \frac{\sigma_x}{\sqrt{<x^2>}} \sqrt{\langle x_0{}^2 \rangle \langle x_0'^2 \rangle - \langle x_0 x_0' \rangle^2}$

[1] S. Rimjaem et al., Nucl. Instr. Meth. Phys. Res. A **671**, 62 – 75 (2012).

Method 1: Lazar Staykov

Numerical Solution of the beam envelope equation

- He starts from the beam envelope equation* (second order differential equation) [2]

$$\sigma_x'' = \frac{I_\mathrm{P}}{I_\mathrm{A}(\sigma_x + \sigma_y)\gamma^3} + \frac{\epsilon_{x,n}^2}{\sigma_x^3 \gamma^2}$$

- To calculate the evolution of the beamlet size after the slit mask (in x and y)
- Uses fourth-order Runge-Kutta algorithm [3]

$$\frac{dy}{dx} = f(x,y) \qquad y_{n+1} = y_n + \frac{1}{6}h[k_1 + 2k_2 + 2k_3 + k_4]$$

to calculate the evolution, but…

- … it can't be used for this problem. However, Euler method [3] can be used**:

$$\frac{dy}{dx} = f(x,y) \qquad y_{n+1} = y_n + hf(x_n, y_n), \text{ e.g.}$$

$$\sigma_{n+1}'' = \frac{I_\mathrm{P}}{I_\mathrm{A}(\sigma_x+\sigma_y)\gamma^3} + \frac{\epsilon_{x,n}^2}{\sigma_x^3\gamma^2} \quad \text{and} \quad \sigma_{n+1}' = \sigma_n' + h\sigma_n'' \quad \text{and} \quad \sigma_{n+1} = \sigma_n + h\sigma_n'$$

I_P: Peak current

I_A: Alfven current, 17 kA

$\sigma_{x,y}$: hor. and vert. beam size

$\epsilon_{x,y,n}$: hor. and vert. norm. emittance (constant along drift)

γ: Lorentz gamma

$$k_1 = f(x_n, y_n)$$
$$k_2 = f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_1)$$
$$k_3 = f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_2)$$
$$k_4 = f(x_n + h, y_n + hk_3)$$

[3]

*the one for asymmetric beams

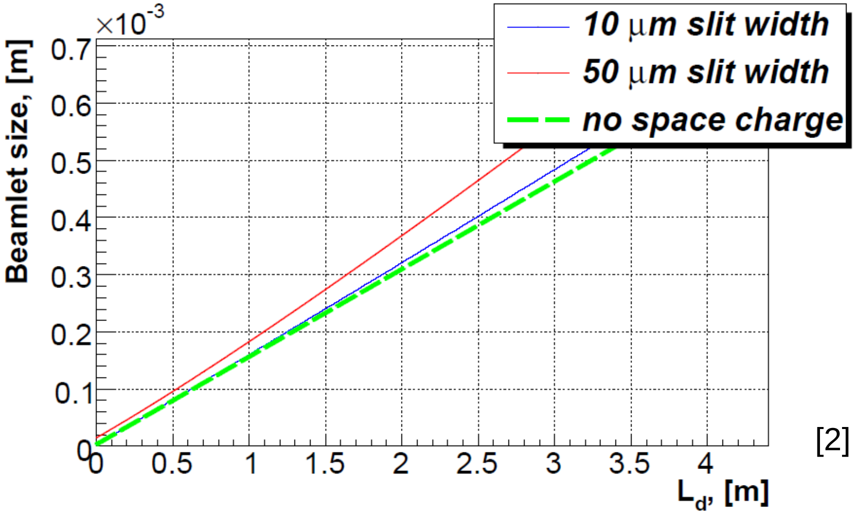
**if the first derivative is introduced

[2] L. Staykov, PhD thesis, Universität Hamburg, (2008)
[3] E. Süli and D. Mayers, An Introduction to Numerical Analysis, p. 310 – 328 (2003)

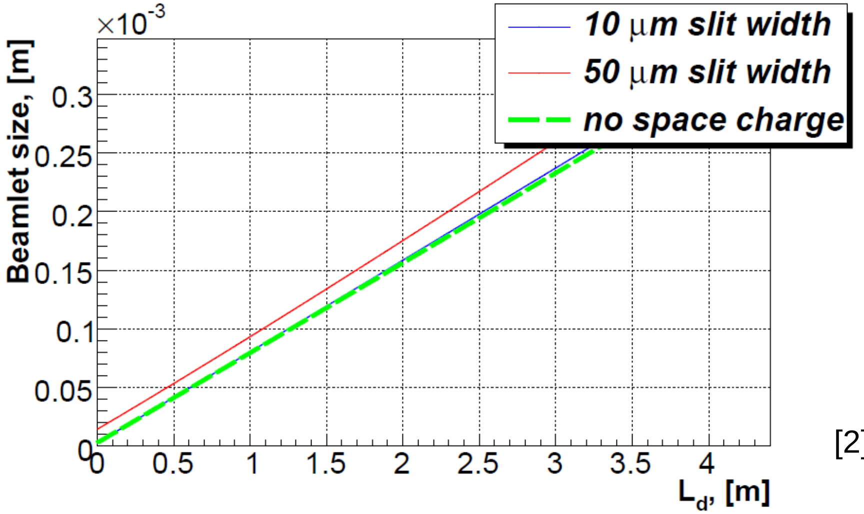
Method 1: Lazar Staykov

Numerical Solution of the beam envelope equation

$\epsilon = 0.9$ um
$I_P = 50$ A
$\sigma_x = 0.2$ mm
$\sigma'_x = 0$ (assumed)

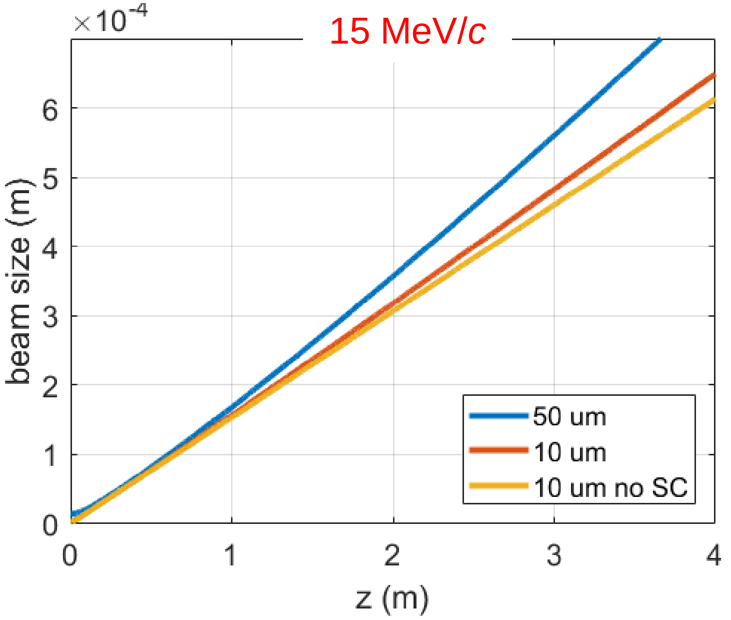


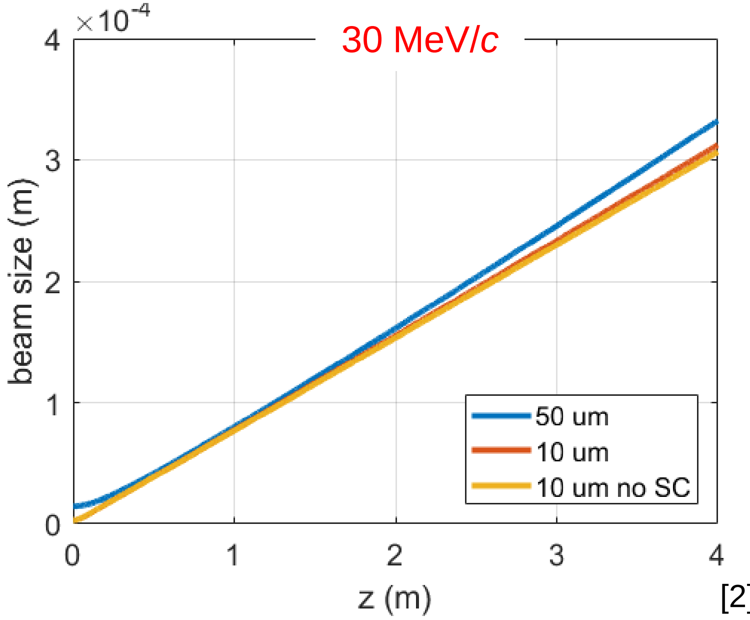
[2]



[2]

Lazar Staykov's result





My results, same parameters, Euler method

→ Almost same, but only almost

[2] L. Staykov, PhD thesis, Universität Hamburg, (2008)

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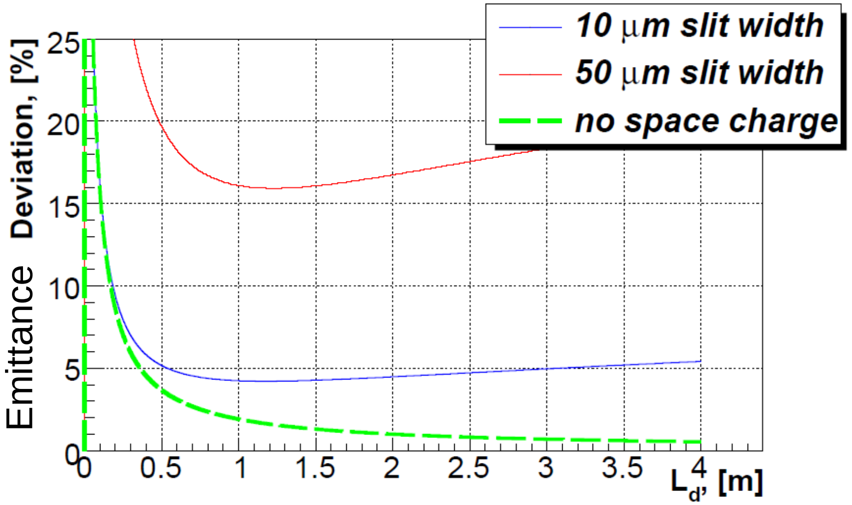
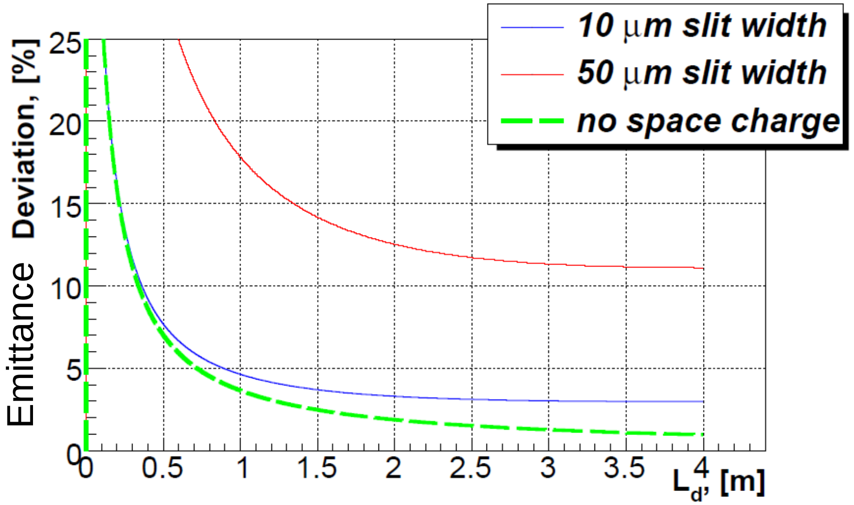




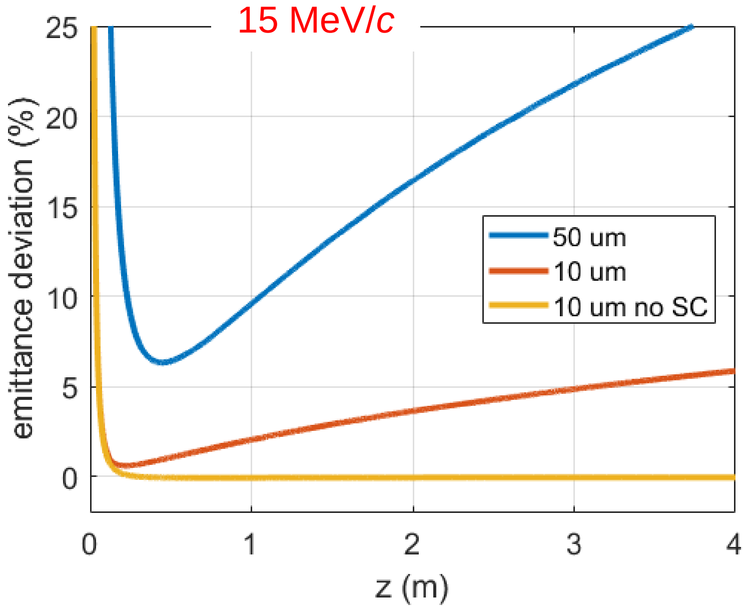
Lazar Staykov's result

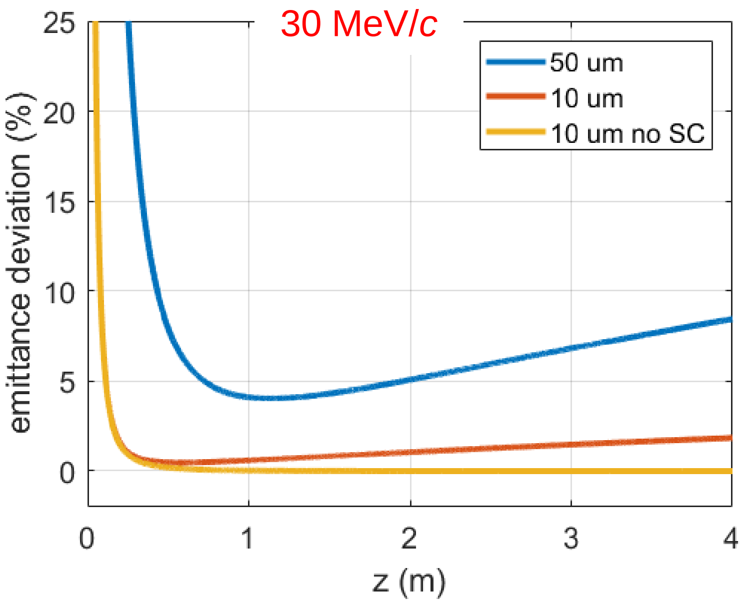
Calculation according to

$$\epsilon_{n,rms} = \beta\gamma\sqrt{< x^2 >< x'^2 >}$$

$$< x'^2 > = < x^2 >_b / L_d^2$$

Beamlet size





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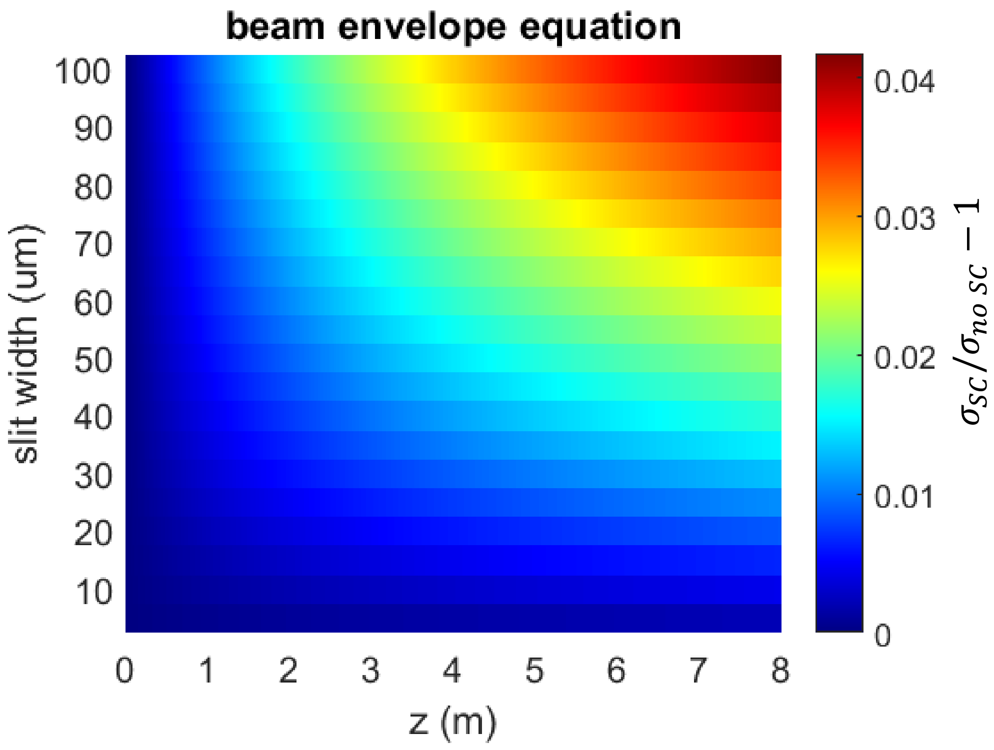
Now with our beam parameters!

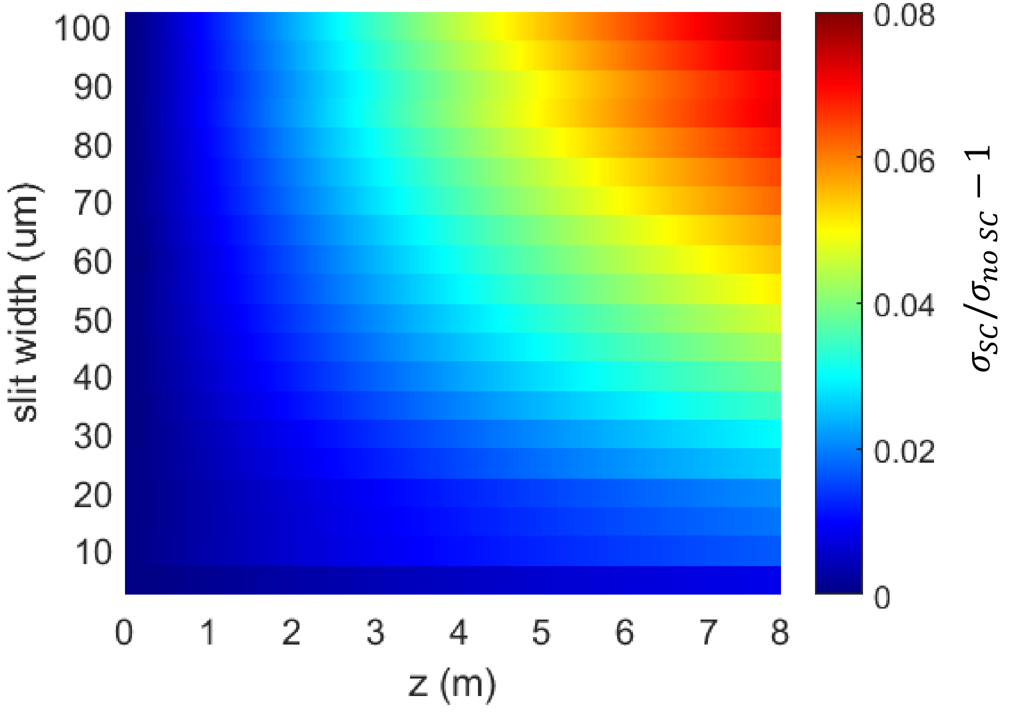
Compare to Method 2: ASTRA Simulations

$\epsilon = 1.55$ um $\quad \sigma_x = 0.36$ mm
$q = 500$ pC $\quad \sigma'_x = 1.6$ mrad
$I_P = 34$ A $\quad \sigma_t(\text{cathode}) = 4.68$ ps
$\sigma_t(\text{EMSY2}) = 5.13$ ps

Numerical solution of beam envelope equation:



Results from ASTRA simulation*



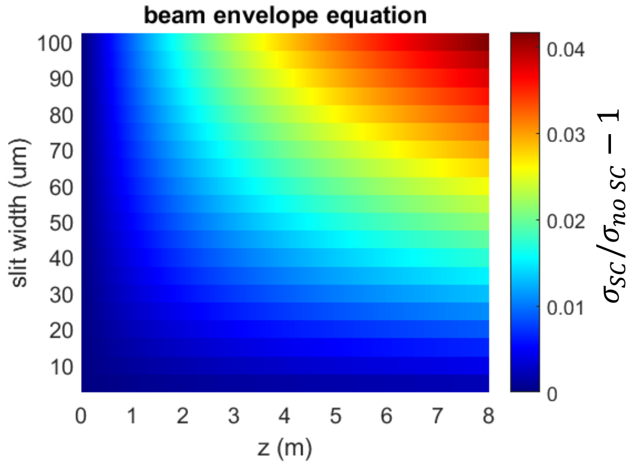
→ Almost same, but only almost

*slit mask is 1 mm thick

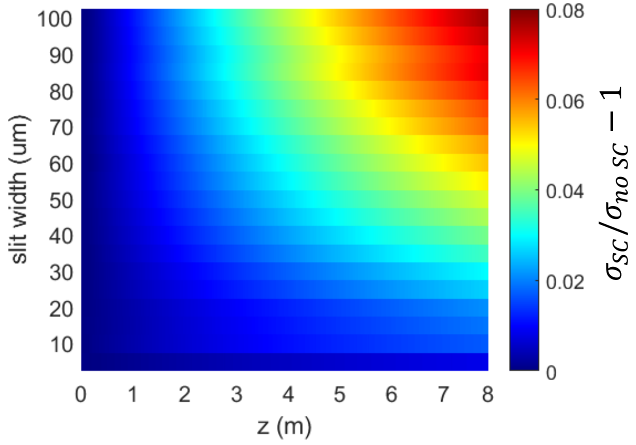
Are these values reasonable?

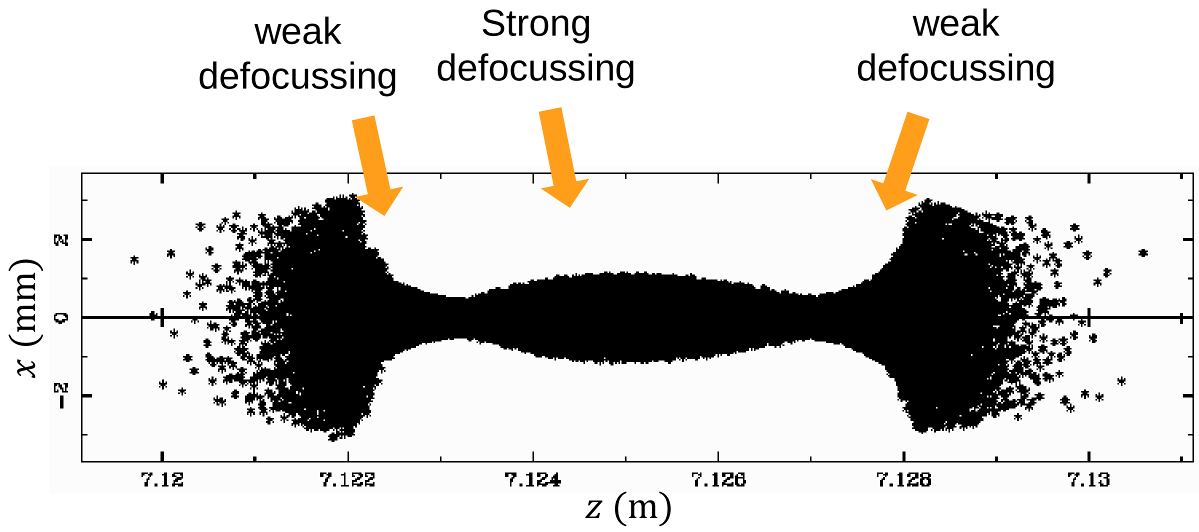
ASTRA suggests even smaller peak current!

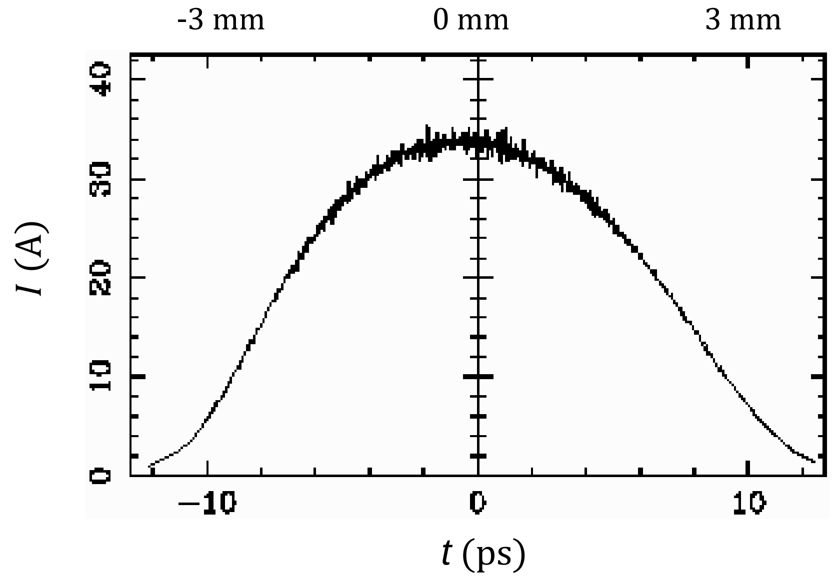
Numerical calculation



ASTRA







- Peak current (as suggested by ASTRA) is 34 A. This value has been used for the calculation
- The head and tail of the bunch should be less defocused by space charge → beam envelope equation should systematically yield too high values

Outlook

Beamlet evolution – stand alone simulation and comparison to numerical calculation

- Space charge lead to an additional growth of beamlet size after slit mask
- Space-charge-caused beam size growth generates systematic error in emittance reconstruction, i.e. the reconstructed emittance is **too high** (as assumed)
- ASTRA simulation suggest, that systematic errors stay below 5 % (at least for this charge, 500 pC, 50 um slit)

To do (for me): Do emittance reconstruction and check behaviour with rf deflector in between

Bottom Line: ASTRA simulation shows bigger growth than beam envelope equation (not expected)