4D emittance measurement with quads plus slits scan for PITZ (proposal)

Content:

- Motivation and methods
- General idea and principle
- Algorithm for coupling terms/4D emittance measurement with quads plus slits scan
- Experiment setup and simulation studies for PITZ
- Summary

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$$C = \begin{bmatrix} \langle xx \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\ \langle x'x \rangle & \langle x'x' \rangle & \langle x'y \rangle & \langle x'y' \rangle \\ \langle yx \rangle & \langle yx' \rangle & \langle yy \rangle & \langle yy' \rangle \\ \langle y'x \rangle & \langle y'x' \rangle & \langle y'y \rangle & \langle y'y' \rangle \end{bmatrix}$$

Background and Motivation

- **Beam asymmetry observed from experiment**

➔Due to field imperfection of RF coupler kick and solenoid.

➔Normal quads and skew quads can produce the beam wings structure, consistent with experiment results, induce the x and y plane beam coupling.

➔Gun quads are used for compensation the quads error field in the gun section, from experiment confirms work well.

But….
We still need to know….

- Try to find a reasonable and judgeable way to decide the optimized comenpensation quads strength and can optimize….standard procedure.
- Goal: minimize rms emittance.
- Start from: coupling beam dynamics and 4D beam emittance….

4D emittance measurement Methods

- Applied skewed quadrupoles in combination with a regular slit emittance measurement device.
- **A rotatable slit device in combination with regular quadrupoles.**
- **Multi-quads scan.**
- Peper pot.
- Single octupole plus two steerers.

*M. Maier†, X. Du, P. Gerhard, L. Groening, S. Mickat, H. Vormann, C. Xiao, COMPLETE TRANSVERSE 4D BEAM CHARACTERIZATION FOR ION BEAMS AT ENERGIES OF FEW MeV/U, TH2A03 Proceedings of LINAC2016, East Lansing, MI, USA.

*C. Xiao, L.Groening, P.Gerhard, M.Maier, S.Mickat, H.Vormann. Measurement of the transverse four-dimensional beam rms-emittance of an intense uranium beam at 11.4 MeV/u. Nuclear Instruments and Methods in Physics Research A 820(2016)14–22.

*C. Xiao, M. Maier, X. N. Du, P. Gerhard, L. Groening, S. Mickat, and H. Vormann. Rotating system for four-dimensional transverse rms-emittance measurements, PHYSICAL REVIEW ACCELERATORS AND BEAMS 19, 072802 (2016).

*Eduard Prat and Masamitsu Aiba. Four-dimensional transverse beam matrix measurement using the multiple-quadrupole scan technique. PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS 17, 052801 (2014).

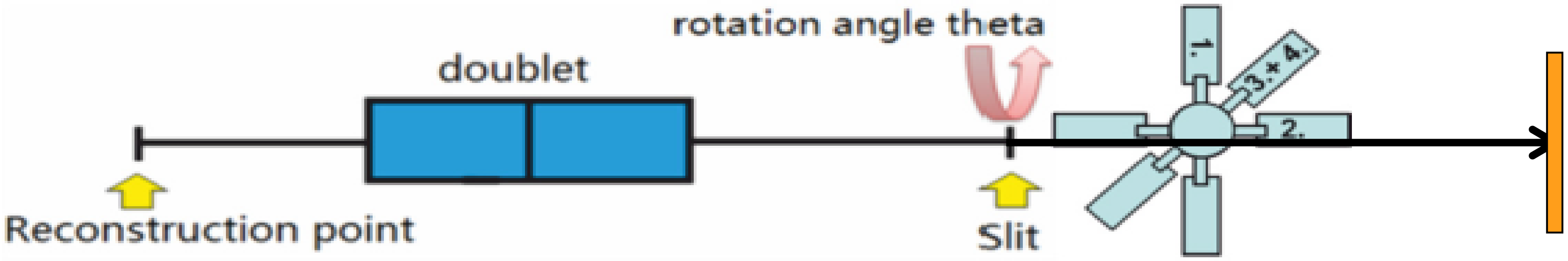
*R. P. Shanks, M. P. Anania, et al., Pepper-Pot Emittance Measurement of Laser-Plasma Wakefield Accelerated Electrons, https://pure.strath.ac.uk/portal/files/20403338/SPIE_paper.pdf.

*J. Ögren et al., Measuring the full transverse beam matrix using a single octupole, PHYSICAL REVIEW SPECIAL TOPICS—ACCELERATORS AND BEAMS 18, 072801 (2015).

Normal quads plus rotated slit → principle

Three slits 0 degree, 45 degree, 90 degree and two quads layout

Different doublet settings a and b



- ❑ We measured for quads settings both a and b:

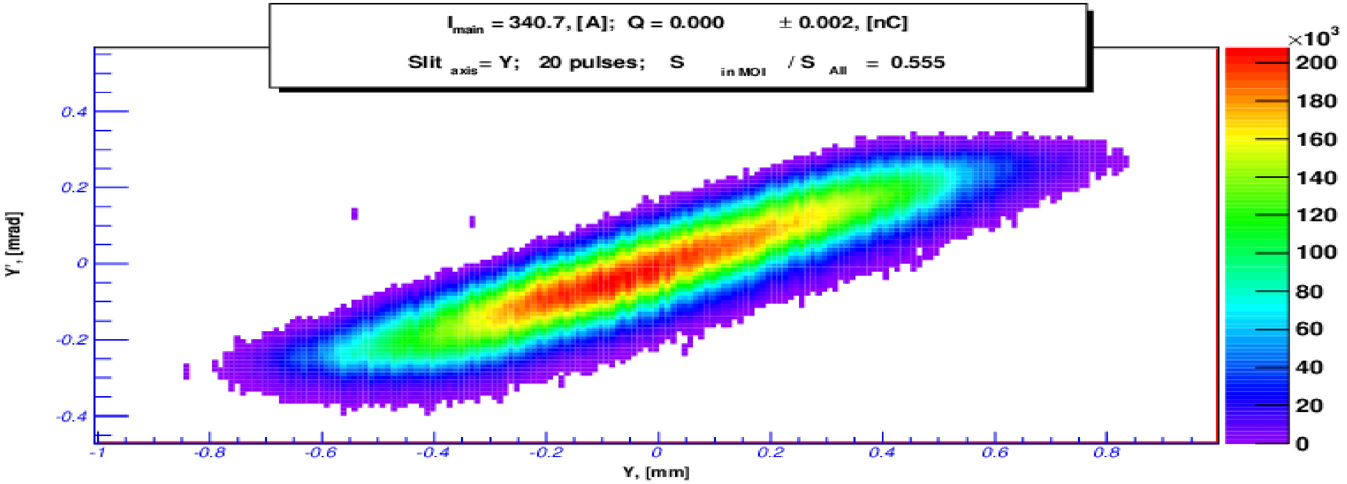
<xx>, <xx’>, <x’x’> horizontal slit scan

<yy>, <yy’>, <y’y’> vertical slit scan

<xx>, <xx’>, <x’x’> rotated slit in rotated coordinate

- ❑ With some algorithm to reconstruct the correlations terms at reconstruction point.

One slit scan emittance measurement results from PITZ:



Results

Plot system ver. Feb 21 2017 17:35:57

Laser:			
rms size	$\langle x^2\rangle_0$=0.11800,	$\langle y^2\rangle_0$=0.13100	[mm]
Electron beam:			
Momentum gun	6.63000	± 0.0059	[MeV/c]
Momentum booster	22.30000	± 0.0114	[MeV/c]
σ_{yag}	0.31860		[mm]
σ_{scan}	0.29426		[mm]
divergence	0.14251		[mrad]
covariance	0.03773		[mm mrad]
sheared div	0.01829		[mrad]
LDrift	3.64300		[m]
β	4.73331		[mm]
γ	1.11021		[mrad]
α	-2.06275		[mm mrad]
$\beta\gamma-\alpha^2$	1.00000		
$\varepsilon^{scaled}_{sheared}$	0.843		[mm mrad]
$\varepsilon^{noscaled}_{2D}$	0.798		[mm mrad]
$\varepsilon^{scaled}_{2D}$	0.864		[mm mrad]

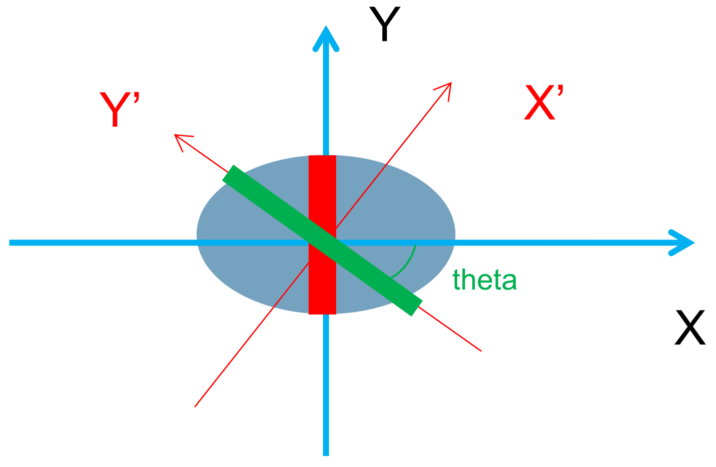
Comments: no comm

$$
\begin{pmatrix} \beta_{x0} \\ \alpha_{x0} \\ \gamma_{x0} \end{pmatrix} = \begin{pmatrix} \langle x_0^2\rangle / \epsilon_{x,\mathrm{rms}} \\ -\langle x_0 x_0'\rangle / \epsilon_{x,\mathrm{rms}} \\ \langle {x_0'}^2\rangle / \epsilon_{x,\mathrm{rms}} \end{pmatrix} .
$$

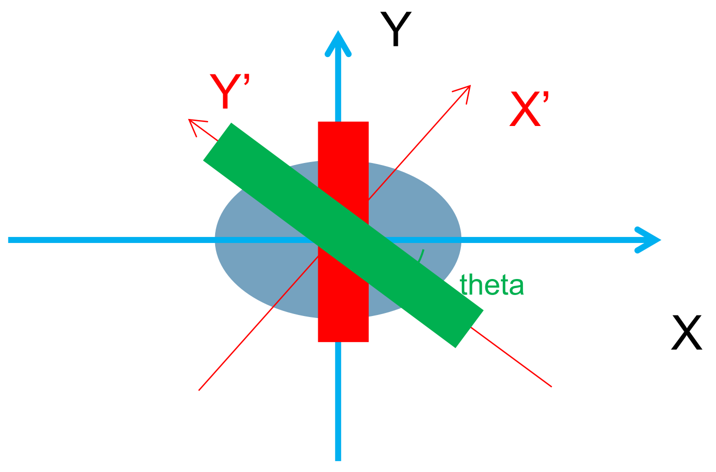
We can get
<xx>, <xx’>, <x’x’> horizontal slit scan
<yy>, <yy’>, <y’y’> vertical slit scan
<xx>, <xx’>, <x’x’> rotated slit in rotated coordinate

Rotated slit scan measurement

Beam at slit



Beam at screen



measured beamlets size at screen , Need to rotated theta transform to(X'-Y') coordinate.

Or just rotated the screen theta angle (maybe can do in software videoclient?).

✓ In the X'-Y' coordinate, we can consider it same as horizontal slit scan along X'. We can get <xx>, <xx'>, <x'x'> in X'-Y' coordinate at slit position.

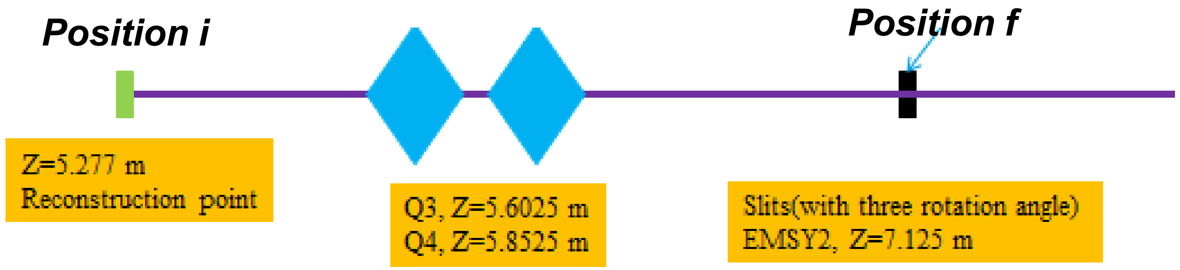
Algorithm

The transport of the beam matrix from location i to location f can be calculated as

$$C_f = MC_iM^T,$$

M is the transport matrix between location i and location f.

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} := \begin{bmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{bmatrix}.$$



The transports M^a or M^b of single particle coordinates from location i to location f using magnet settings a or b.

$$\begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_f^{a,b} = \begin{bmatrix} m_{11}^{a,b} & m_{12}^{a,b} & 0 & 0 \\ m_{21}^{a,b} & m_{22}^{a,b} & 0 & 0 \\ 0 & 0 & m_{33}^{a,b} & m_{34}^{a,b} \\ 0 & 0 & m_{43}^{a,b} & m_{44}^{a,b}. \end{bmatrix} \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_i, \quad (12)$$

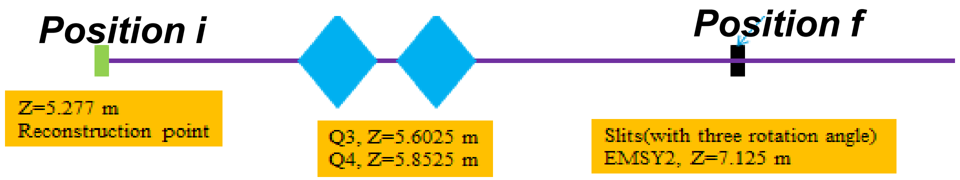
(1) We measured for quads settings both a and b:
<xx>, <xx'>, <x'x'> horizontal slit scan
<yy>, <yy'>, <y'y'> vertical slit scan
<xx>, <xx'>, <x'x'> rotated slit in rotated coordinate

(2) The uncoupled beam matrix at position i can be reconstructed from horitzontal slit scan for horizontal beam matrix at position f➔i.

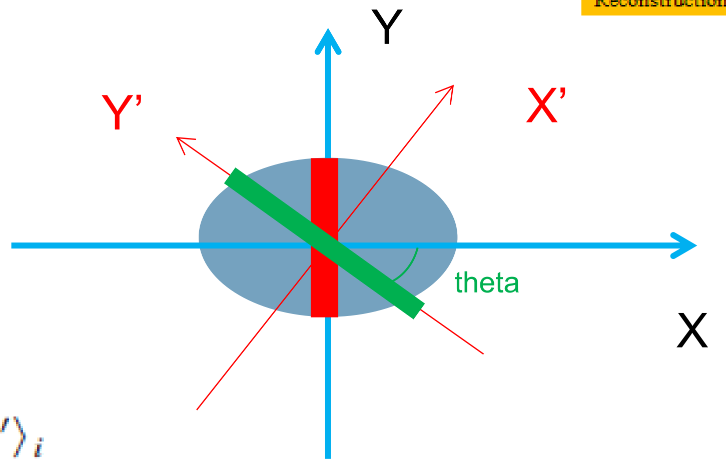
(3) The same procedure can be used for y slit scan for vertical beam matrix at position f➔i.

$$C = \begin{bmatrix} \langle xx \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\ \langle x'x \rangle & \langle x'x' \rangle & \langle x'y \rangle & \langle x'y' \rangle \\ \langle yx \rangle & \langle yx' \rangle & \langle yy \rangle & \langle yy' \rangle \\ \langle y'x \rangle & \langle y'x' \rangle & \langle y'y \rangle & \langle y'y' \rangle \end{bmatrix}$$

Coupling terms measurment



$$\begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_f^{a,b} = \begin{bmatrix} m_{11}^{a,b} & m_{12}^{a,b} & 0 & 0 \\ m_{21}^{a,b} & m_{22}^{a,b} & 0 & 0 \\ 0 & 0 & m_{33}^{a,b} & m_{34}^{a,b} \\ 0 & 0 & m_{43}^{a,b} & m_{44}^{a,b} \end{bmatrix} \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_i, \quad (12)$$



$$\begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_\theta^{a,b} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_f^{a,b}.$$

Beam correlation terms at f position

$$\langle xy\rangle_f^{a,b} = m_{11}^{a,b} m_{33}^{a,b}\langle xy\rangle_i + m_{11}^{a,b} m_{34}^{a,b}\langle xy'\rangle_i + m_{12}^{a,b} m_{33}^{a,b}\langle x'y\rangle_i + m_{12}^{a,b} m_{34}^{a,b}\langle x'y'\rangle_i,$$

$$\langle xy'\rangle_f^{a,b} + \langle x'y\rangle_f^{a,b} = (m_{11}^{a,b} m_{43}^{a,b} + m_{21}^{a,b} m_{33}^{a,b})\langle xy\rangle_i + (m_{11}^{a,b} m_{44}^{a,b} + m_{21}^{a,b} m_{34}^{a,b})\langle xy'\rangle_i + (m_{12}^{a,b} m_{43}^{a,b} + m_{22}^{a,b} m_{33}^{a,b})\langle x'y\rangle_i + (m_{12}^{a,b} m_{44}^{a,b} + m_{22}^{a,b} m_{34}^{a,b})\langle x'y'\rangle_i,$$

$$\langle x'y'\rangle_f^{a,b} = m_{21}^{a,b} m_{43}^{a,b}\langle xy\rangle_i + m_{21}^{a,b} m_{44}^{a,b}\langle xy'\rangle_i + m_{22}^{a,b} m_{43}^{a,b}\langle x'y\rangle_i + m_{22}^{a,b} m_{44}^{a,b}\langle x'y'\rangle_i.$$

$$\langle xx\rangle_\theta^{a,b} = \cos^2\theta\langle xx\rangle_f^{a,b} + 2\sin\theta\cos\theta\langle xy\rangle_f^{a,b} + \sin^2\theta\langle yy\rangle_f^{a,b},$$

$$\langle xx'\rangle_\theta^{a,b} = \cos^2\theta\langle xx'\rangle_f^{a,b} + \sin\theta\cos\theta\langle xy'\rangle_f^{a,b} + \sin\theta\cos\theta\langle x'y\rangle_f^{a,b} + \sin^2\theta\langle yy'\rangle_f^{a,b},$$

$$\langle x'x'\rangle_\theta^{a,b} = \cos^2\theta\langle x'x'\rangle_f^{a,b} + 2\sin\theta\cos\theta\langle x'y'\rangle_f^{a,b} + \sin^2\theta\langle y'y'\rangle_f^{a,b}.$$

$$\langle xx\rangle_\theta^{a,b} = \cos^2\theta\langle xx\rangle_f^{a,b} + 2\sin\theta\cos\theta\langle xy\rangle_f^{a,b} + \sin^2\theta\langle yy\rangle_f^{a,b},$$

$$\langle xx'\rangle_\theta^{a,b} = \cos^2\theta\langle xx'\rangle_f^{a,b} + \sin\theta\cos\theta\langle xy'\rangle_f^{a,b} + \sin\theta\cos\theta\langle x'y\rangle_f^{a,b} + \sin^2\theta\langle yy'\rangle_f^{a,b}$$

$$\langle x'x'\rangle_\theta^{a,b} = \cos^2\theta\langle x'x'\rangle_f^{a,b} + 2\sin\theta\cos\theta\langle x'y'\rangle_f^{a,b} + \sin^2\theta\langle y'y'\rangle_f^{a,b}.$$

$$\langle xy\rangle_f^{a,b} = \frac{\langle xx\rangle_\theta^{a,b} - \cos^2\theta\langle xx\rangle_f^{a,b} - \sin^2\theta\langle yy\rangle_f^{a,b}}{2\sin\theta\cos\theta}$$

$$\langle xy'\rangle_f^{a,b} + \langle x'y\rangle_f^{a,b} = \frac{\langle xx'\rangle_\theta^{a,b} - \cos^2\theta\langle xx'\rangle_f^{a,b} - \sin^2\theta\langle yy'\rangle_f^{a,b}}{\sin\theta\cos\theta}$$

$$\langle x'y'\rangle_f^{a,b} = \frac{\langle x'x'\rangle_\theta^{a,b} - \cos^2\theta\langle x'x'\rangle_f^{a,b} - \sin^2\theta\langle y'y'\rangle_f^{a,b}}{2\sin\theta\cos\theta}$$

$$\langle xy\rangle_f^{a,b} = m_{11}^{a,b}m_{33}^{a,b}\langle xy\rangle_i + m_{11}^{a,b}m_{34}^{a,b}\langle xy'\rangle_i + m_{12}^{a,b}m_{33}^{a,b}\langle x'y\rangle_i + m_{12}^{a,b}m_{34}^{a,b}\langle x'y'\rangle_i,$$

$$\langle xy'\rangle_f^{a,b} + \langle x'y\rangle_f^{a,b} = (m_{11}^{a,b}m_{43}^{a,b} + m_{21}^{a,b}m_{33}^{a,b})\langle xy\rangle_i + (m_{11}^{a,b}m_{44}^{a,b} + m_{21}^{a,b}m_{34}^{a,b})\langle xy'\rangle_i + (m_{12}^{a,b}m_{43}^{a,b} + m_{22}^{a,b}m_{33}^{a,b})\langle x'y\rangle_i + (m_{12}^{a,b}m_{44}^{a,b} + m_{22}^{a,b}m_{34}^{a,b})\langle x'y'\rangle_i,$$

$$\langle x'y'\rangle_f^{a,b} = m_{21}^{a,b}m_{43}^{a,b}\langle xy\rangle_i + m_{21}^{a,b}m_{44}^{a,b}\langle xy'\rangle_i + m_{22}^{a,b}m_{43}^{a,b}\langle x'y\rangle_i + m_{22}^{a,b}m_{44}^{a,b}\langle x'y'\rangle_i.$$

We measured for quads settings both a and b:
<xx>, <xx'>, <x'x'> horizontal slit scan
<yy>, <yy'>, <y'y'> vertical slit scan
<xx>, <xx'>, <x'x'> rotated slit

From these measurement results, we can get <xy>, <x'y'>,<xy'>+<x'y> at f position

All elements of the transport matrices $M_{xx}^{a,b}$ and $M_{yy}^{a,b}$ are known from magnet settings. The second moments $\langle xx\rangle_f^{a,b}$, $\langle xx'\rangle_f^{a,b}$, $\langle x'x'\rangle_f^{a,b}$, $\langle yy\rangle_f^{a,b}$, $\langle yy'\rangle_f^{a,b}$, and $\langle y'y'\rangle_f^{a,b}$ before rotation and $\langle xx\rangle_\theta^{a,b}$, $\langle xx'\rangle_\theta^{a,b}$, and $\langle x'x'\rangle_\theta^{a,b}$ after rotation can be measured. Combining Eq. (13) to Eq. (19), the solution

$$\begin{cases} \Gamma_{11}\langle xy\rangle_i + \Gamma_{12}\langle xy'\rangle_i + \Gamma_{13}\langle x'y\rangle_i + \Gamma_{14}\langle x'y'\rangle_i = \Lambda_1 \\ \Gamma_{21}\langle xy\rangle_i + \Gamma_{22}\langle xy'\rangle_i + \Gamma_{23}\langle x'y\rangle_i + \Gamma_{24}\langle x'y'\rangle_i = \Lambda_2 \\ \Gamma_{31}\langle xy\rangle_i + \Gamma_{32}\langle xy'\rangle_i + \Gamma_{33}\langle x'y\rangle_i + \Gamma_{34}\langle x'y'\rangle_i = \Lambda_3 \\ \Gamma_{41}\langle xy\rangle_i + \Gamma_{42}\langle xy'\rangle_i + \Gamma_{43}\langle x'y\rangle_i + \Gamma_{44}\langle x'y'\rangle_i = \Lambda_4 \\ \Gamma_{51}\langle xy\rangle_i + \Gamma_{52}\langle xy'\rangle_i + \Gamma_{53}\langle x'y\rangle_i + \Gamma_{54}\langle x'y'\rangle_i = \Lambda_5 \\ \Gamma_{61}\langle xy\rangle_i + \Gamma_{62}\langle xy'\rangle_i + \Gamma_{63}\langle x'y\rangle_i + \Gamma_{64}\langle x'y'\rangle_i = \Lambda_6 \end{cases}$$

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} \\ \Gamma_{21} & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & \Gamma_{34} \\ \Gamma_{41} & \Gamma_{42} & \Gamma_{43} & \Gamma_{44} \\ \Gamma_{51} & \Gamma_{52} & \Gamma_{53} & \Gamma_{54} \\ \Gamma_{61} & \Gamma_{62} & \Gamma_{63} & \Gamma_{64} \end{bmatrix} \begin{bmatrix} \langle xy\rangle_i \\ \langle xy'\rangle_i \\ \langle x'y\rangle_i \\ \langle x'y'\rangle_i \end{bmatrix} = \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \\ \Lambda_4 \\ \Lambda_5 \\ \Lambda_6 \end{bmatrix}.$$

$$\begin{bmatrix} \langle xy\rangle_i \\ \langle xy'\rangle_i \\ \langle x'y\rangle_i \\ \langle x'y'\rangle_i \end{bmatrix} = (\Gamma^T\Gamma)^{-1}\Gamma^T\Lambda \qquad \Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} \\ \Gamma_{21} & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & \Gamma_{34} \\ \Gamma_{41} & \Gamma_{42} & \Gamma_{43} & \Gamma_{44} \\ \Gamma_{51} & \Gamma_{52} & \Gamma_{53} & \Gamma_{54} \\ \Gamma_{61} & \Gamma_{62} & \Gamma_{63} & \Gamma_{64} \end{bmatrix}, \qquad \Lambda = \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \\ \Lambda_4 \\ \Lambda_5 \\ \Lambda_6 \end{bmatrix}$$

(1-3) for a

$$\Gamma_{11} = m^a_{11}m^a_{33}, \qquad \Gamma_{12} = m^a_{11}m^a_{34},$$
$$\Gamma_{13} = m^a_{12}m^a_{33}, \qquad \Gamma_{14} = m^a_{12}m^a_{34}, \qquad (21)$$

$$\Gamma_{21} = m^a_{11}m^a_{43} + m^a_{21}m^a_{33}, \qquad \Gamma_{22} = m^a_{11}m^a_{44} + m^a_{21}m^a_{34},$$
$$\Gamma_{23} = m^a_{12}m^a_{43} + m^a_{22}m^a_{33}, \qquad \Gamma_{24} = m^a_{12}m^a_{44} + m^a_{22}m^a_{34}, \qquad (22)$$

$$\Gamma_{31} = m^a_{21}m^a_{43}, \qquad \Gamma_{32} = m^a_{21}m^a_{44},$$
$$\Gamma_{33} = m^a_{22}m^a_{43}, \qquad \Gamma_{34} = m^a_{22}m^a_{44}, \qquad (23)$$

(4-6) for b

$$\Gamma_{41} = m^b_{11}m^b_{33}, \qquad \Gamma_{42} = m^b_{11}m^b_{34},$$
$$\Gamma_{43} = m^b_{12}m^b_{33}, \qquad \Gamma_{44} = m^b_{12}m^b_{34}, \qquad (24)$$

$$\Gamma_{51} = m^b_{11}m^b_{43} + m^b_{21}m^b_{33}, \qquad \Gamma_{52} = m^b_{11}m^b_{44} + m^b_{21}m^b_{34},$$
$$\Gamma_{53} = m^b_{12}m^b_{43} + m^b_{22}m^b_{33}, \qquad \Gamma_{54} = m^b_{12}m^b_{44} + m^b_{22}m^b_{34}, \qquad (25)$$

$$\Gamma_{61} = m^b_{21}m^b_{43}, \qquad \Gamma_{62} = m^b_{21}m^b_{44},$$
$$\Gamma_{63} = m^b_{22}m^b_{43}, \qquad \Gamma_{64} = m^b_{22}m^b_{44}, \qquad (26)$$

Calculated from measured beam matrix at position f with three slits scan.

$$\Lambda_1 = \langle xy\rangle^a_f, \quad \Lambda_2 = \langle xy'\rangle^a_f + \langle x'y\rangle^a_f, \quad \Lambda_3 = \langle x'y'\rangle^a_f, \qquad (27)$$

$$\Lambda_4 = \langle xy\rangle^b_f, \quad \Lambda_5 = \langle xy'\rangle^b_f + \langle x'y\rangle^b_f, \quad \Lambda_6 = \langle x'y'\rangle^b_f. \qquad (28)$$

Quads settings consideration

Condition number: $\kappa(\Gamma) := \|\Gamma\|_2 \|\Gamma^\dagger\|_2,$ $\Gamma^\dagger = (\Gamma^T\Gamma)^{-1}\Gamma^T$

Frobenius norm of gama matrix

$$\|\Gamma\|_2 := \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{k}(\Gamma_{i,j})^2}, \qquad \|\Gamma^\dagger\|_2 := \sqrt{\sum_{i=1}^{k}\sum_{j=1}^{n}(\Gamma^\dagger_{i,j})^2}$$

The numerical stability (degeneration of the system) is better if the condition number is small. Well-conditioned matrices have condition numbers which are close to 1.0.

In order to obtain reliable evaluation results a four-dimensional emittance measurement needs:

(i) one reference emittance measurement with 100% transmission efficiency between location i and location f to obtain projected beam parameters at location i (on-diagonal section of beam matrix of Ci).

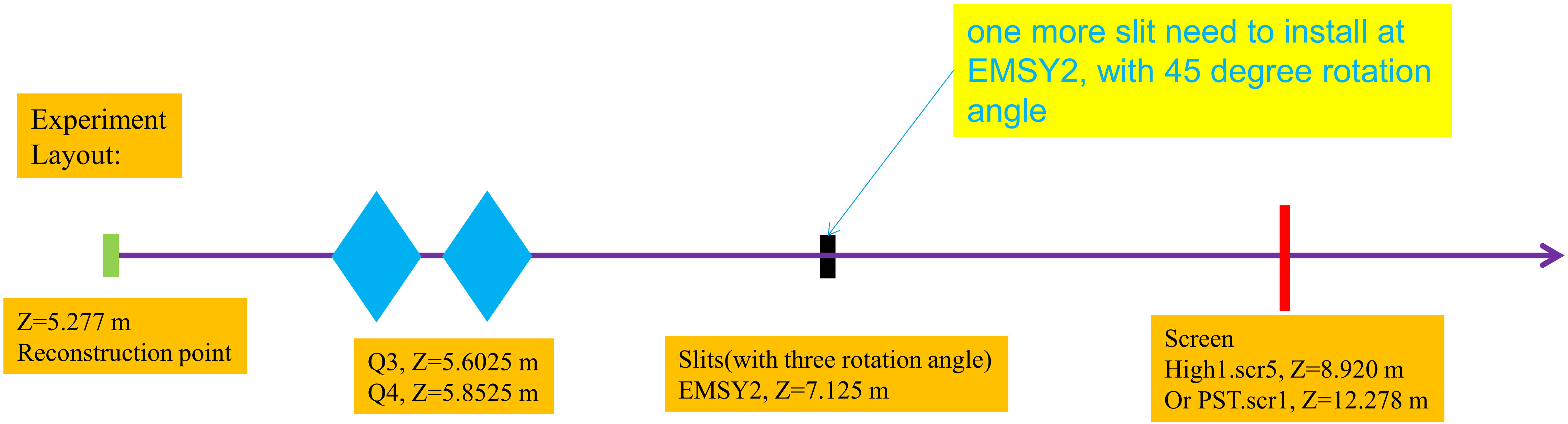
(ii) all quadrupoles varied numerically in a brute-force method in order to check each setting for full transmission efficiency from location i to location f, and for reasonable beam sizes on slit and screen

(iii) All settings from safety islands are combined to determine combinations of two settings a and b corresponding to a low condition number.

Measurement is quick and data analysis is easy➔
We can try several settings of quads and to find out the reliable experiment result.

For PITZ two quads plus slits scan set up

- Three slits are required: rotation 0 degree(horizontal), rotation 90 degree(vertical), arbitrary angle(not 0 and 90, such as 30 degree, 45 degree).
- For current setup, one more slit need to instal at EMSY2, with 45 degree rotation angle➔easy to set up the experiment and also can use fastscan software.

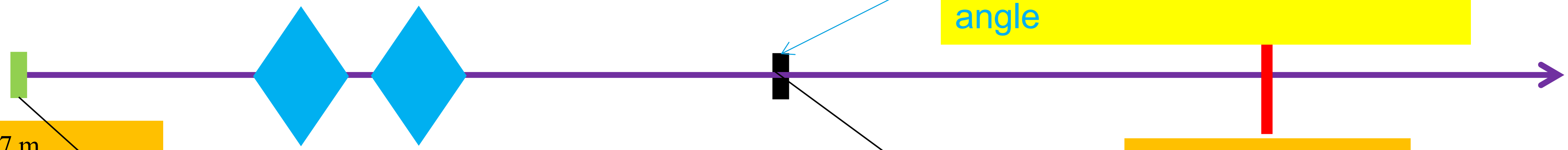


Simulation studies

Initial beam 4d beam matrix, unit mm rad

Qs	Qn	<xx>	<yy>	<x'x'>	<y'y'>	**<xy>**	<x'y>	<xy'>	<x'y'>	<xx'>	<yy'>	nemitx	nemity	**Coupling term**
-0.01	0.01	0.34953	0.222786	6.54E-08	3.64E-08	**-0.02761**	-1.3E-05	-1.4E-05	-6.6E-09	0.000148	8.7E-05	1.236357	0.94868	**0.014298**

Experiment Layout:



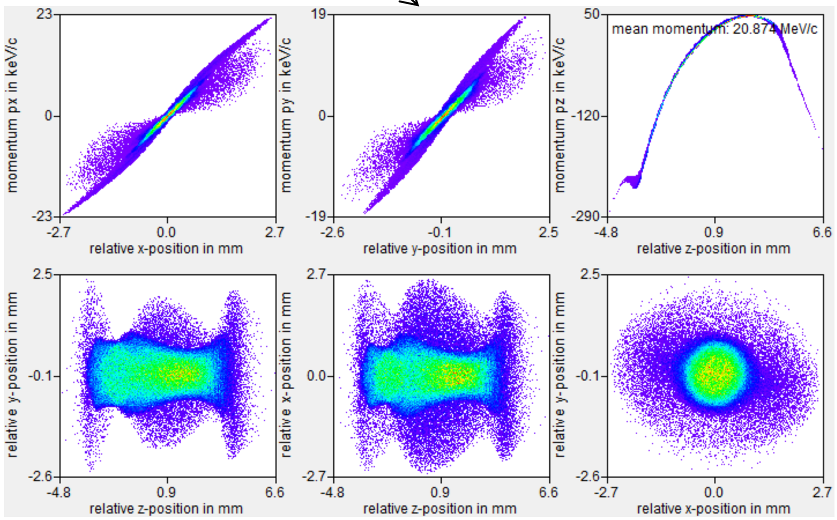
one more slit need to install at EMSY2, with 45 degree rotation angle

Z=5.277 m
Reconstruction point

Q3, Z=5.6025 m
Q4, Z=5.8525 m

Slits(with three rotation angle)
EMSY2, Z=7.125 m

Screen
High1.scr5, Z=8.920 m



Transport beam from EMSY1 to EMSY2 with a&b quads settings , get the uncorrelated terms of beam matrix.
<xx>, <xx'>, <x'x'> ➔horizontal slit scan
<yy>, <yy'>, <y'y'> ➔vertical slit scan
<xx>, <xx'>, <x'x'> ➔rotated slit scan in rotated coordinate

(Unit m rad)

Simulaltion set up

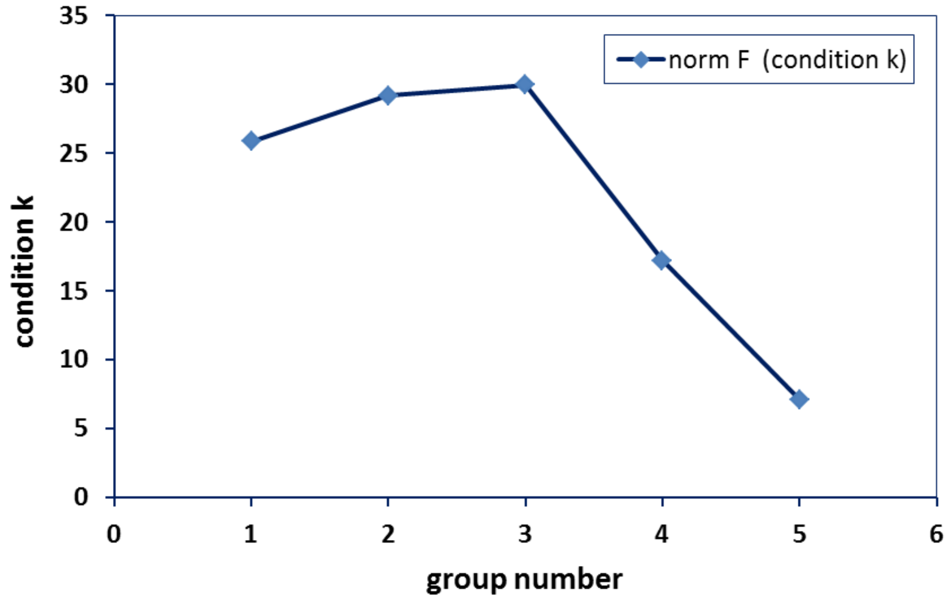
5 groups quads settings

	Q3, k / m^2	Q4, k /m^2
group1		
a	-21.5	19
b	-15	16
group2		
	Q3, k / m^2	Q4, k /m^2
c	-10	8
d	10	-12
group3		
	Q3, k / m^2	Q4, k /m^2
e	-10	8
f	-21.5	19
group4		
	Q3, k / m^2	Q4, k /m^2
e	-25	33
f	-22	38
group5		
	Q3, k / m^2	Q4, k /m^2
e	-45	43
f	-24	40

Condition number for each group

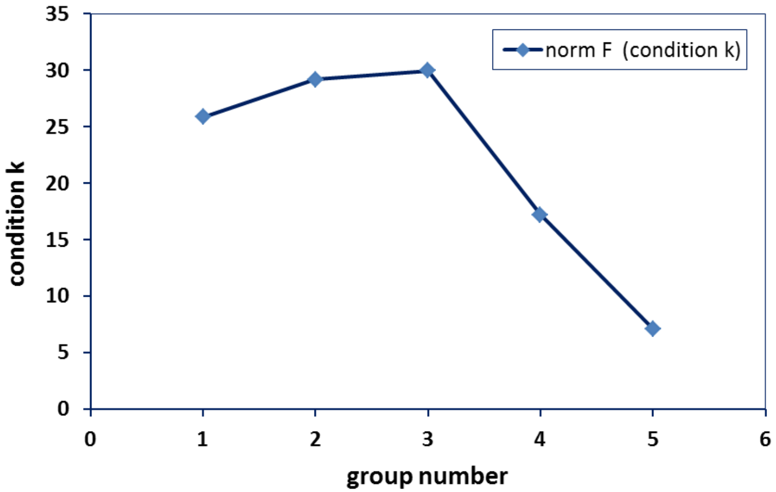
$$\kappa(\Gamma) := \|\Gamma\|_2 \|\Gamma^\dagger\|_2, \qquad \Gamma^\dagger = (\Gamma^T\Gamma)^{-1}\Gamma^T$$

$$\|\Gamma\|_2 := \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{k}(\Gamma_{i,j})^2}, \qquad \|\Gamma^\dagger\|_2 := \sqrt{\sum_{i=1}^{k}\sum_{j=1}^{n}(\Gamma^\dagger_{i,j})^2}$$



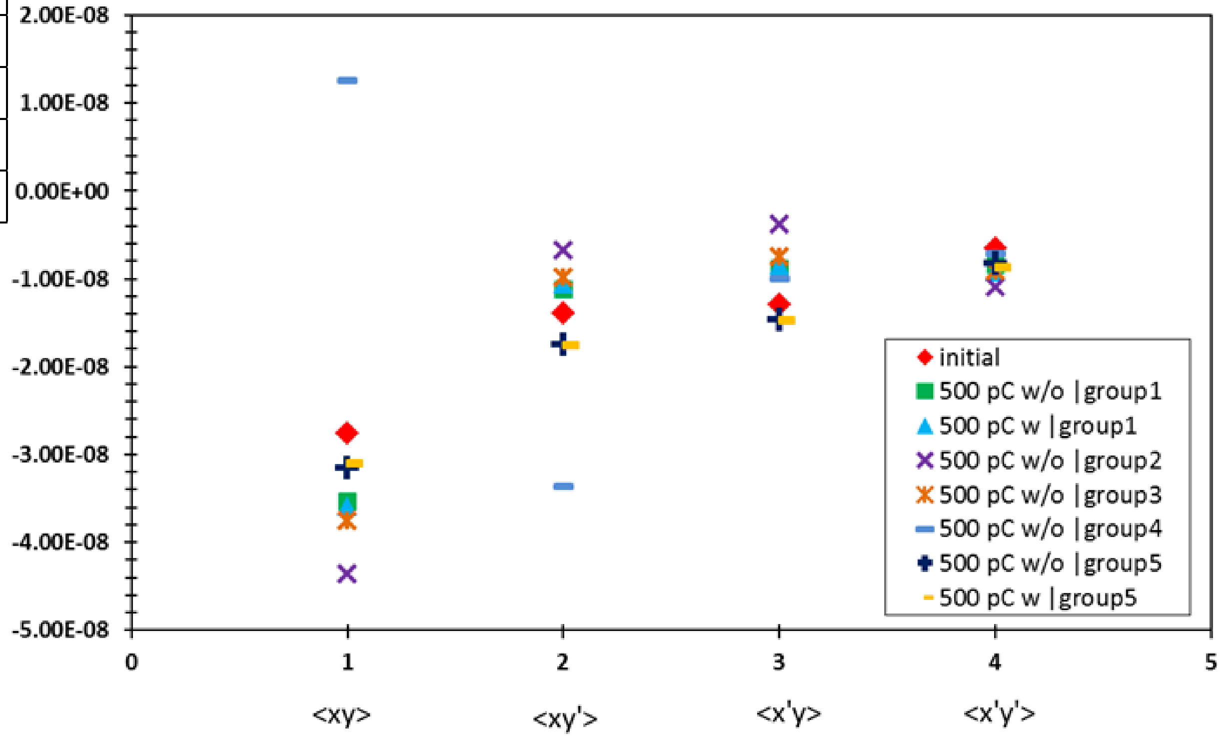
Note: In simulation, for rotated slit is equavilent to rotate the beam in same angle. Beam rotated and then get <xx>, <xx'> and <x'x'> in rotated coordinate.

Coupling terms reconstruction results



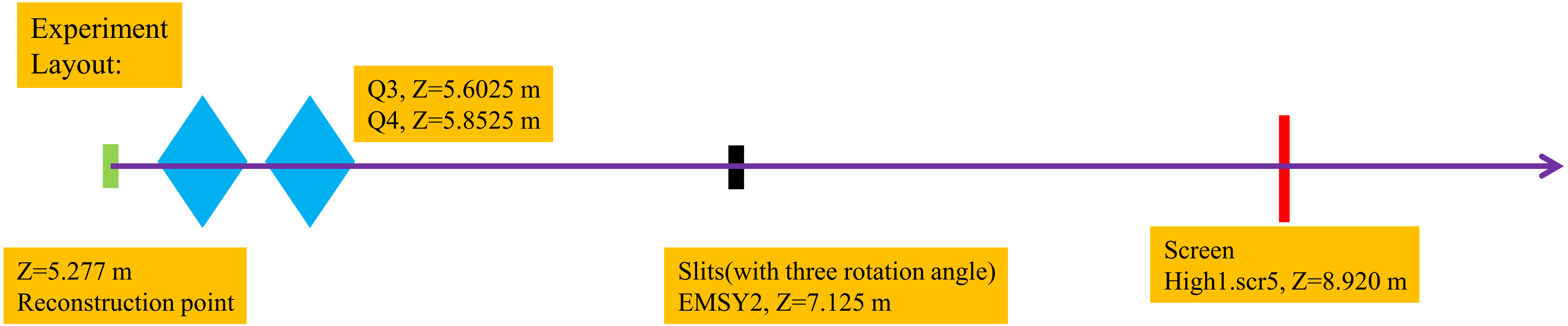
Reconstruted correlation terms

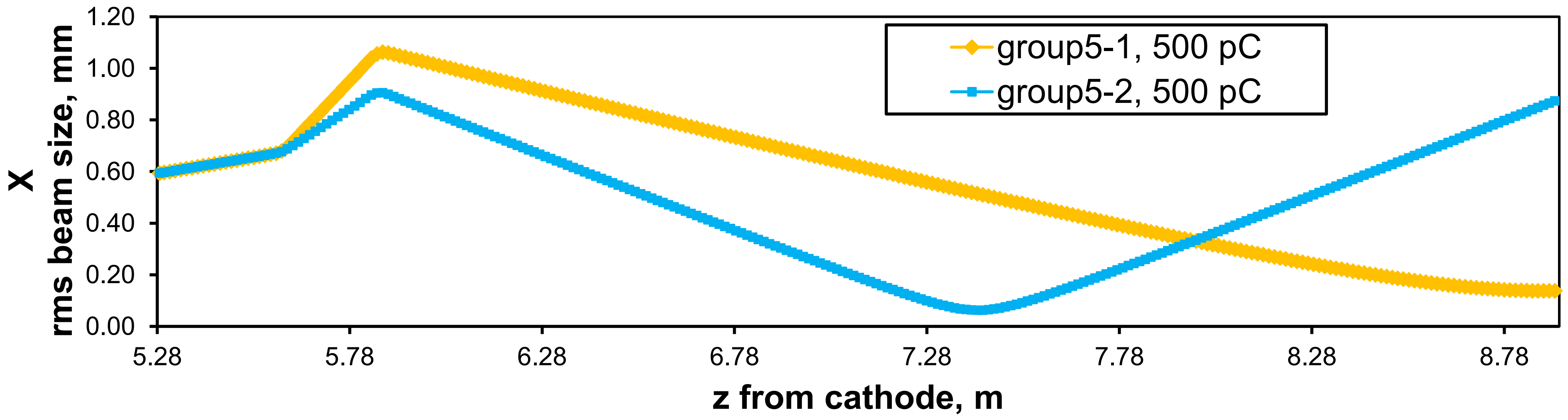
	<xy>	<xy'>	<x'y>	<x'y'>
Initial (goal)	-2.76E-08	-1.40E-08	-1.30E-08	-6.60E-09
500 pC w/o/ group1	-3.53192E-08	-1.1172E-08	-8.87733E-09	-8.37941E-09
500 pC w/ group1	-3.59878E-08	-1.0633E-08	-8.48221E-09	-9.23953E-09
500 pC w/o group2	-4.36E-08	-6.85E-09	-3.87E-09	-1.09E-08
500 pC w/o group3	-3.76E-08	-9.84E-09	-7.55E-09	-9.18E-09
500 pC w/o group4	1.26E-08	-3.37E-08	-9.94E-09	-7.12E-09
500 pC w/o group5	-3.15E-08	-1.75E-08	-1.47E-08	-8.31E-09
500 pC w group5	-3.10E-08	-1.77E-08	-1.48E-08	-8.76E-09



➔For group 5 quads settings, with k =7.13, we can get closer result to the initial value.
➔space charge has less effect.

Group 5 quads settings beam transport





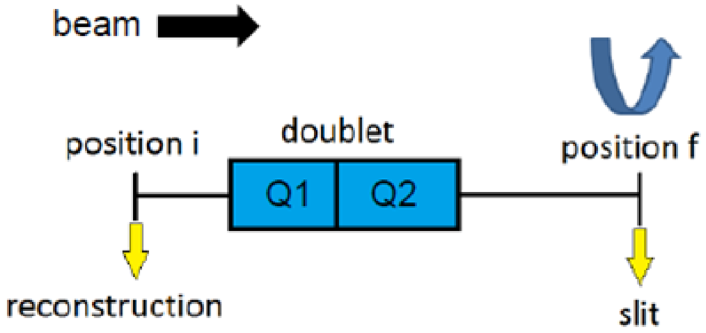
Summary and out look

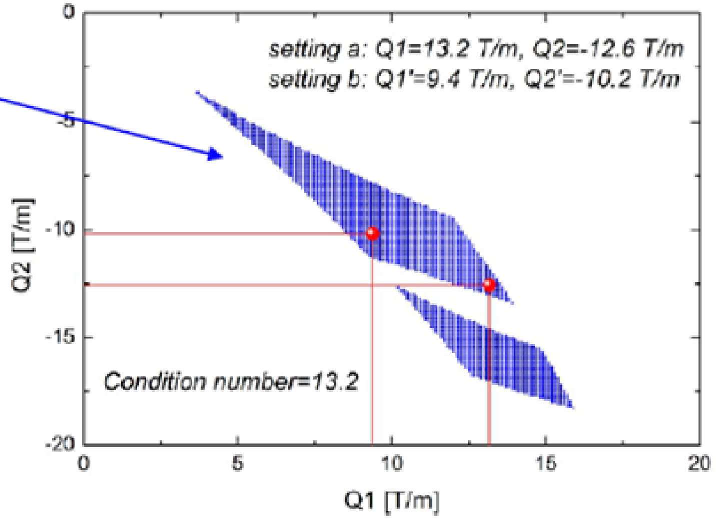
- Quads plus roated slit scan (slit+grid) now used for heavy ions Linac 4D emittance measruement and confirmed it works.
- Quads plus roated slit scan is easy to implement into PITZ and also the fastscan can be used.➔more convenient and data processing consistent with slit scan measurement (standard emittance measurement).
- Simulation studies are done➔show possible to get approxiate results with good quads settings (when condion numbet k close to 1).
- One more slit needed to install at EMSY2 with 45 degree rotation.
- Try with experiment to investigate coupling terms measurement and RMS emittance minimization with gun quads by this mehod.

Quads settings ?

- do initial emittance measurements at 0° and 90°, i.e. in ver. and hor. plane
- backtransform to quad doublet entrance
- vary in brute force way two quad strengths Q1, Q2 and check for (analytically!) :
  - 100% transmission
  - reasonable beam size at slit
  - reasonable beam size at grid
- store settings Q1, Q2 that passed this test
- build all possible pairs of settings, check their *K*
- pick pair with lowest *K* for measurements:

  1. slit at 0° with setting Q1[a], Q2[a]
  2. slit at θ with setting Q1[a], Q2[a]
  3. slit at 90° with setting Q1[a], Q2[a]
  4. slit at θ with setting Q1[b], Q2[b]





*L. Groening / GSI Darmstadt, 4d Phase Space Measurements 9/29/2017