Simulated measurement of the coupling term and 4D emittance with multi-quads scan for PITZ

Content:

- Motivation
- General idea, simulated experiment set up and procedure
- Reconstructed coupling terms/4D emittance from simulated experiment
- Summary and conclusions

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$$C = \begin{bmatrix} \langle xx \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\ \langle x'x \rangle & \langle x'x' \rangle & \langle x'y \rangle & \langle x'y' \rangle \\ \langle yx \rangle & \langle yx' \rangle & \langle yy \rangle & \langle yy' \rangle \\ \langle y'x \rangle & \langle y'x' \rangle & \langle y'y \rangle & \langle y'y' \rangle \end{bmatrix}$$

Background and Motivation

- **Beam asymmetry and imperfection observed from experiment**

➔Due to field imperfection of RF coupler kick and solenoid.

➔Normal quads and skew quads can produce the beam wings structure, consistent with experiment results, induce the x and y plane beam coupling.

➔Gun quads are used for compensation the quads error field in the gun section, from experiment confirms work well.

But….
We still need to know….

- Try to find a reasonable and judgeable way to decide the optimized comenpensation quads strength and can optimize….standard procedure.
- Goal: minimize rms emittance.
- Start from: coupling beam dynamics and 4D beam emittance….

4D emittance

4D beam matrix:

$$C = \begin{bmatrix} \langle xx \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\ \langle x'x \rangle & \langle x'x' \rangle & \langle x'y \rangle & \langle x'y' \rangle \\ \langle yx \rangle & \langle yx' \rangle & \langle yy \rangle & \langle yy' \rangle \\ \langle y'x \rangle & \langle y'x' \rangle & \langle y'y \rangle & \langle y'y' \rangle \end{bmatrix}$$

rms emittance(2D)

$$\epsilon_u = \sqrt{\det \sigma_{uu}},$$

$$\beta_u = \langle u^2 \rangle / \epsilon_u, \quad \gamma_u = \langle u'^2 \rangle / \epsilon_u, \quad \alpha_u = -\langle uu' \rangle / \epsilon_u,$$

u—x or y

4D emittance:

$$\varepsilon_{4D} = \varepsilon_1 \varepsilon_2 = \sqrt{\det(C)}$$

Coupling factor:

$$t = \frac{\varepsilon_x \varepsilon_y}{\varepsilon_1 \varepsilon_2} - 1 \geq 0$$

Beam brightness:

$$B = \frac{I}{\varepsilon_x \varepsilon_y} = \frac{I}{(1+t)\varepsilon_1 \varepsilon_2}$$

- Intrisic-emittances are invariant under symplectic transformations and the Intrisic-emittances are equal to the rms emittances if and only if inter plane correlations are zero.

For PITZ gun quads compensation:

- ✓ Try to minimize the coupling factor or correlation terms in the 4D beam maxtrix.
- ✓ So for each gun quads settings, we need to know how the correlation terms change➔ measure the Correlation terms in 4D matrix or 4D emittance.

Multi-quads scan for 4D emittance measurment

Multiple-optics/single-location

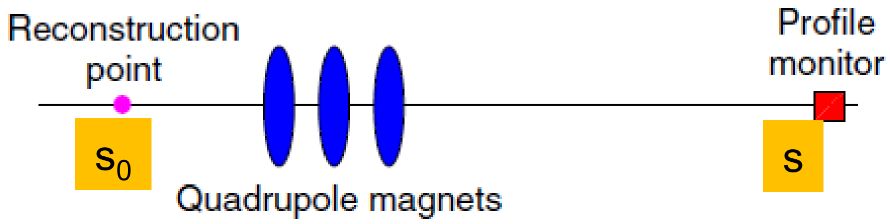


FIG. 1. Sketch of the measurement setup (not to scale).

$$= R\sigma_{s_0}^{4D} R^T$$

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ R_{21} & R_{22} & R_{23} & R_{24} \\ R_{31} & R_{32} & R_{33} & R_{34} \\ R_{41} & R_{42} & R_{43} & R_{44} \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{pmatrix}$$

$$\langle x^2\rangle_s = R_{11}^2\langle x^2\rangle_{s_0} + R_{12}^2\langle x'^2\rangle_{s_0} + 2R_{11}R_{12}\langle xx'\rangle_{s_0},$$

$$\langle y^2\rangle_s = R_{33}^2\langle y^2\rangle_{s_0} + R_{34}^2\langle y'^2\rangle_{s_0} + 2R_{33}R_{34}\langle yy'\rangle_{s_0},$$

$$\langle xy\rangle_s = R_{11}R_{33}\langle xy\rangle_{s_0} + R_{12}R_{33}\langle x'y\rangle_{s_0} + R_{11}R_{34}\langle xy'\rangle_{s_0} + R_{12}R_{34}\langle x'y'\rangle_{s_0}.$$

Measured <xx>, <yy>, <xy>

$$\langle x_{(i)}^2\rangle = R_{11}^{(i)\,2}\langle x_0^2\rangle + R_{12}^{(i)\,2}\langle x_0'^{\,2}\rangle + 2R_{11}^{(i)}R_{12}^{(i)}\langle x_0 x_0'\rangle.$$

$$\begin{pmatrix} \langle x_{(1)}^2\rangle \\ \langle x_{(2)}^2\rangle \\ \langle x_{(3)}^2\rangle \end{pmatrix} = \underbrace{\begin{pmatrix} R_{11}^{(1)\,2} & 2R_{11}^{(1)}R_{12}^{(1)} & R_{12}^{(1)\,2} \\ R_{11}^{(2)\,2} & 2R_{11}^{(2)}R_{12}^{(2)} & R_{12}^{(2)\,2} \\ R_{11}^{(3)\,2} & 2R_{11}^{(3)}R_{12}^{(3)} & R_{12}^{(3)\,2} \end{pmatrix}}_{A} \begin{pmatrix} \langle x_0^2\rangle \\ \langle x_0 x_0'\rangle \\ \langle x_0'^{\,2}\rangle \end{pmatrix} \qquad (3.2)$$

$$\begin{pmatrix} \langle x_0^2\rangle \\ \langle x_0 x_0'\rangle \\ \langle x_0'^{\,2}\rangle \end{pmatrix} = A^{-1}\begin{pmatrix} \langle x_{(1)}^2\rangle \\ \langle x_{(2)}^2\rangle \\ \langle x_{(3)}^2\rangle \end{pmatrix} \qquad (3.3)$$

$$\langle xy_{(i)}\rangle = R_{11}^{(i)}R_{33}^{(i)}\langle xy_{(0)}\rangle + R_{12}^{(i)}R_{33}^{(i)}\langle x'y_{(0)}\rangle + R_{34}^{(i)}R_{11}^{(i)}\langle xy'_{(0)}\rangle + R_{34}^{(i)}R_{12}^{(i)}\langle x'y'_{(0)}\rangle.$$

$$\underbrace{\begin{pmatrix} \langle xy_{(1)}\rangle \\ \langle xy_{(2)}\rangle \\ \langle xy_{(3)}\rangle \\ \langle xy_{(4)}\rangle \end{pmatrix}}_{l} = \underbrace{\begin{pmatrix} R_{11}^{(1)}R_{33}^{(1)} & R_{12}^{(1)}R_{33}^{(1)} & R_{34}^{(1)}R_{11}^{(1)} & R_{34}^{(1)}R_{12}^{(1)} \\ R_{11}^{(2)}R_{33}^{(2)} & R_{12}^{(2)}R_{33}^{(2)} & R_{34}^{(2)}R_{11}^{(2)} & R_{34}^{(2)}R_{12}^{(2)} \\ R_{11}^{(3)}R_{33}^{(3)} & R_{12}^{(3)}R_{33}^{(3)} & R_{34}^{(3)}R_{11}^{(3)} & R_{34}^{(3)}R_{12}^{(3)} \\ R_{11}^{(4)}R_{33}^{(4)} & R_{12}^{(4)}R_{33}^{(4)} & R_{34}^{(4)}R_{11}^{(4)} & R_{34}^{(4)}R_{12}^{(4)} \end{pmatrix}}_{K} \underbrace{\begin{pmatrix} \langle xy_{(0)}\rangle \\ \langle x'y_{(0)}\rangle \\ \langle xy'_{(0)}\rangle \\ \langle x'y'_{(0)}\rangle \end{pmatrix}}_{k}$$

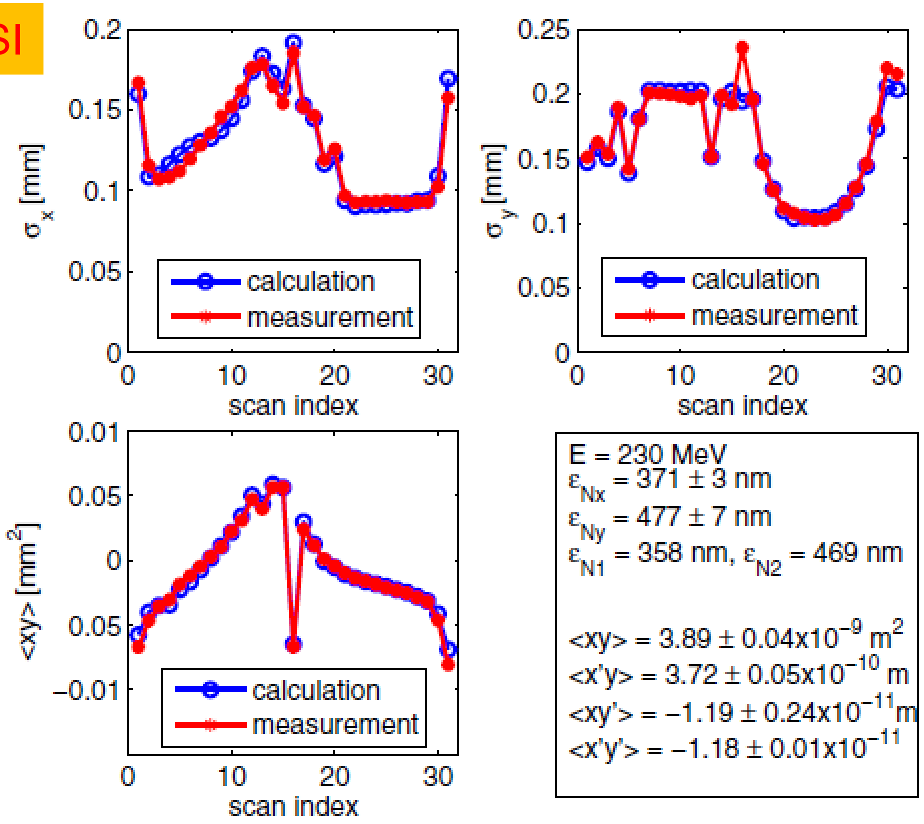
$$k = K^{-1} l. \qquad (3.26)$$

*Eduard Prat and Masamitsu Aiba. Four-dimensional transverse beam matrix measurement using the multiple-quadrupole scan technique. PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS 17, 052801 (2014).

*Florian L¨ohl, Measurements of the Transverse Emittance at the VUV-FEL, Diploma theis, 2005.

Multi-quads scan measruement at PSI and XFEL

PSI



➔The rms emittance increase due to transverse coupling is 5.4%

XFEL

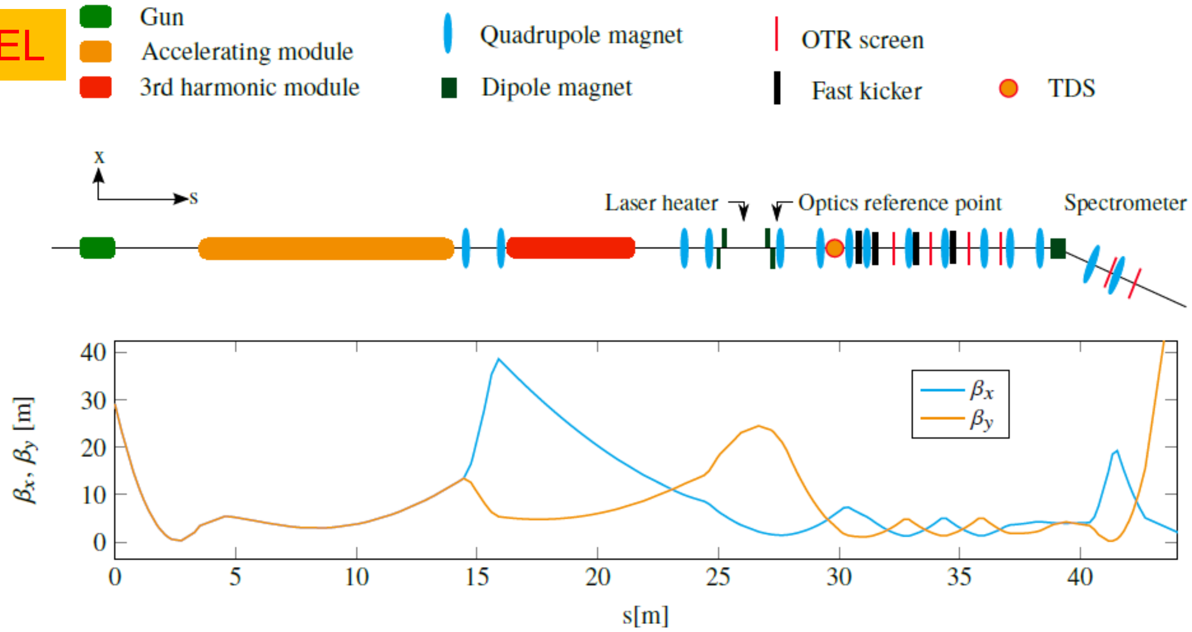


Figure 4: The default beta functions in the XFEL injector starting at the photo cathode and ending in the injector dump. Above the plot one can find a schematic layout of the XFEL injector. The last screen in the diagnostics section upstream the spectrometer dipole was the measurement position. All quadrupole magnets between the laser heater and the measurement screen were used for the scan of the phase advances.

Table 1: Reconstructed Parameters

ϵ_x	=	0.77 mm mrad
ϵ_y	=	0.71 mm mrad
$\langle xy\rangle_{s0}$	=	$51.14 \cdot 10^{-9}$ m^2
$\langle x'y\rangle_{s0}$	=	$-52.26 \cdot 10^{-9}$ m
$\langle xy'\rangle_{s0}$	=	$-11.57 \cdot 10^{-9}$ m
$\langle x'y'\rangle_{s0}$	=	$11.51 \cdot 10^{-9}$

*Eduard Prat and Masamitsu Aiba. Four-dimensional transverse beam matrix measurement using the multiple-quadrupole scan technique. PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS 17, 052801 (2014).

*M. Scholz, Proceedings of FEL2017, Santa Fe, NM, USA, TUP005

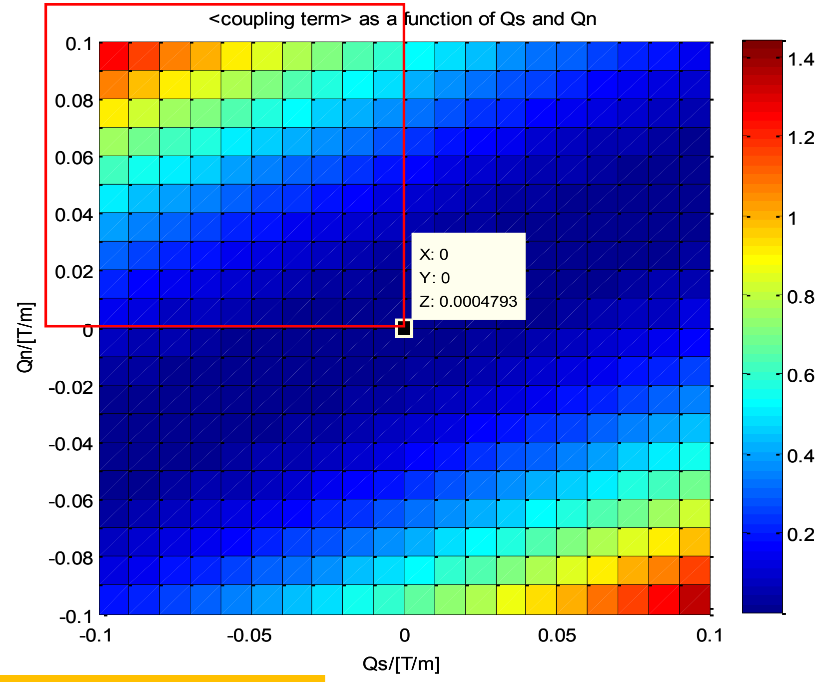
Beam coupling simulation studies due to quads error fields

Simulation settings:

- Gun 51.36MV/m, solenoid 350A, bunch charge 500 pC, laser rms size 0.3mm, gaussian beam. Beam momentum after gun:5.87 MeV/c
- Booster:17.2 MV/m,after booster:20.874 MeV/c
- Qs at z=0.18m, Qs is negative polarity;

  Qn at z=0.36m, Qn is related to solenoid, for normal current, Qn is positive polarity.
- Quads error scan in simulation (g[T/m],effective length 1cm) Qs = [-0.1:0.01:0.1] Qn = [-0.1:0.01:0.1];

Qs	Qn	rmsemitx	rmsemity	intriemit1	intriemit2	**Coupling term**
0	0	2.658e-05	2.555e-05	2.683e-05	2.529e-05	4.782e-04
-0.01	**0.01**	**3.091e-05**	**2.325e-05**	**3.145e-05**	**2.251e-05**	**0.0147**
-0.05	**0.05**	**6.387e-05**	**3.244e-05**	**6.308e-05**	**2.465e-05**	**0.322**
-0.1	0.1	8.269e-05	6.748e-05	6.726e-05	3.554e-05	1.3338



Qs and Qn is about 0.05T/m, the coupling contributes to the rms emittance is about ~10 ~15% .

Beam distribution at High1.scr1 for reconstruction

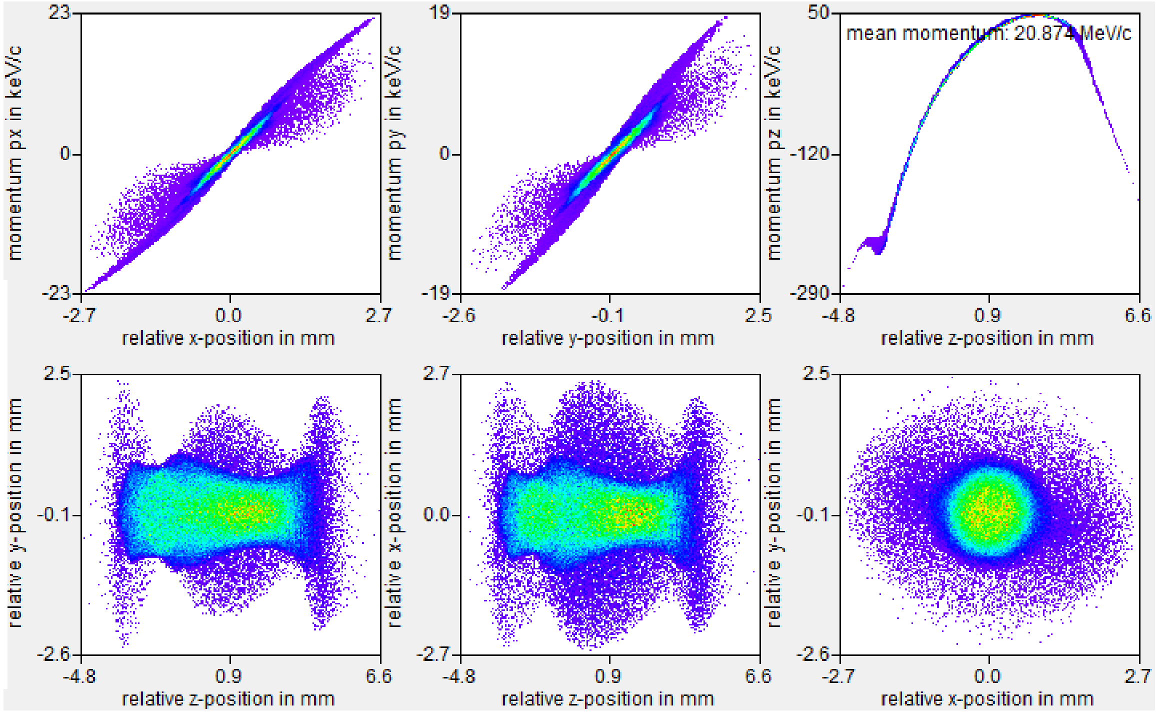
Qs=-0.01 T/m Qn=0.01 T/m

Qs	Qn	<xx>	<yy>	<x'x'>	<y'y'>	**<xy>**	<x'y>	<xy'>	<x'y'>	<xx'>	<yy'>	nemitx	nemity	**Coupling term**
-0.01	0.01	0.34953	0.222786	6.54E-08	3.64E-08	**-0.02761**	-1.3E-05	-1.4E-05	-6.6E-09	0.000148	8.7E-05	1.236357	0.94868	**0.014298**

$$C = \begin{bmatrix} \langle xx\rangle & \langle xx'\rangle & \langle xy\rangle & \langle xy'\rangle \\ \langle x'x\rangle & \langle x'x'\rangle & \langle x'y\rangle & \langle x'y'\rangle \\ \langle yx\rangle & \langle yx'\rangle & \langle yy\rangle & \langle yy'\rangle \\ \langle y'x\rangle & \langle y'x'\rangle & \langle y'y\rangle & \langle y'y'\rangle \end{bmatrix}$$

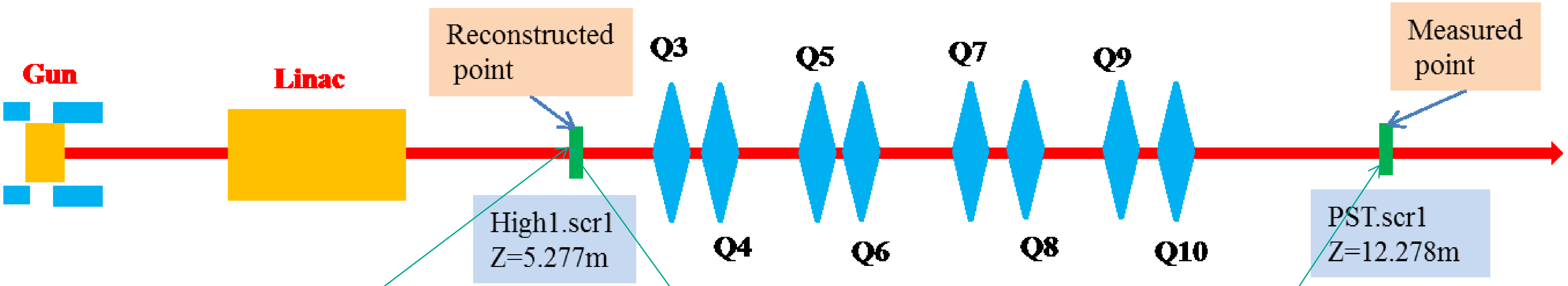
```
delt =

   1.0e-06 *

   0.349530194453562   0.148145594593491  -0.027605207459808  -0.014276831801143
   0.148145594593491   0.065411036071994  -0.013136063817492  -0.006648341028507
  -0.027605207459808  -0.013136063817492   0.222786264429634   0.086954889517126
  -0.014276831801143  -0.006648341028507   0.086954889517126   0.036359881993339
```

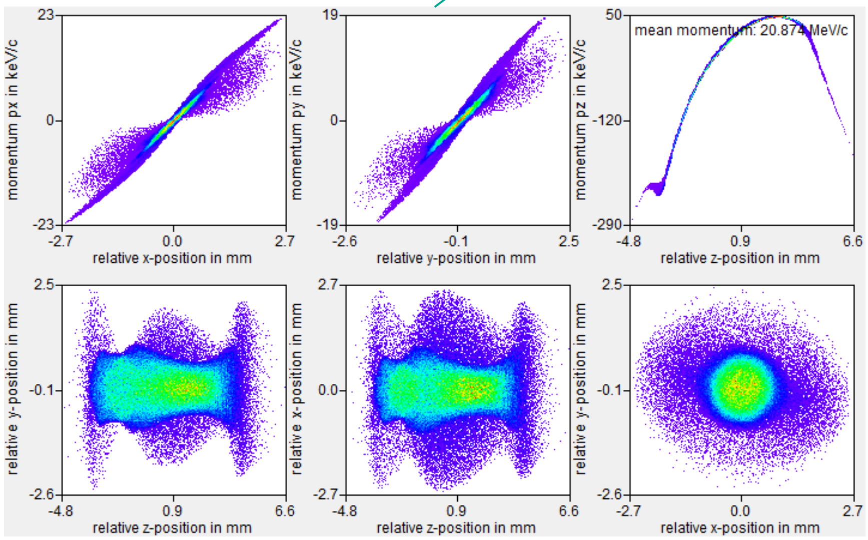
(Unit m rad)



Simulated experiment set up for PITZ

Multi-Quads, Q3-Q10 used for scan.





EMSY1 measurement:
Get twiss parameters for Matching set up.

With multi-quads settings simulated beam distribution.

(For experiment, the beam will be measured at here, <xx>, <yy>, <xy>).

Beam optics matching for 14 scans

➔The basic requirements on the quad scan for these measurements is to scan the phase advance between the optics reconstruction position and the measurement screen in one plane over 180 degree (if possible) and keep it constant in the second one.

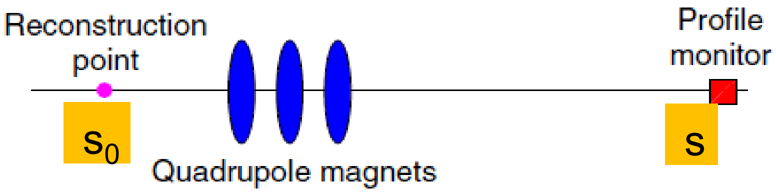
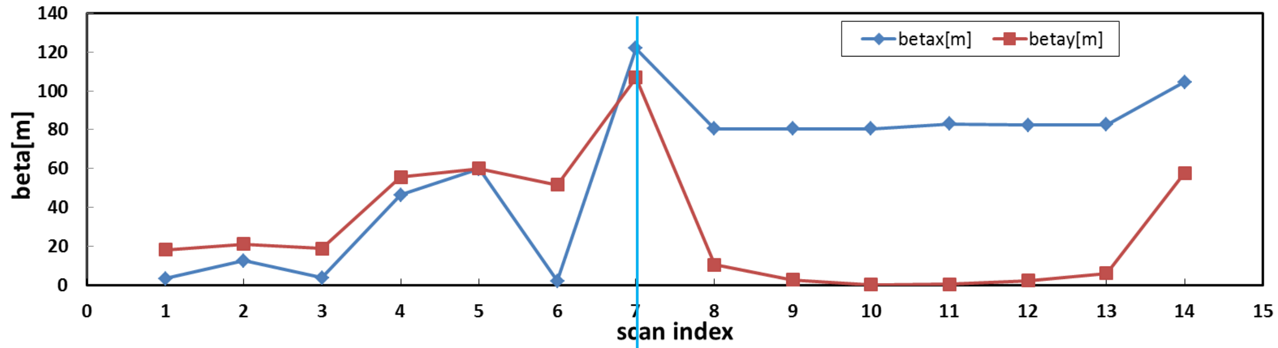
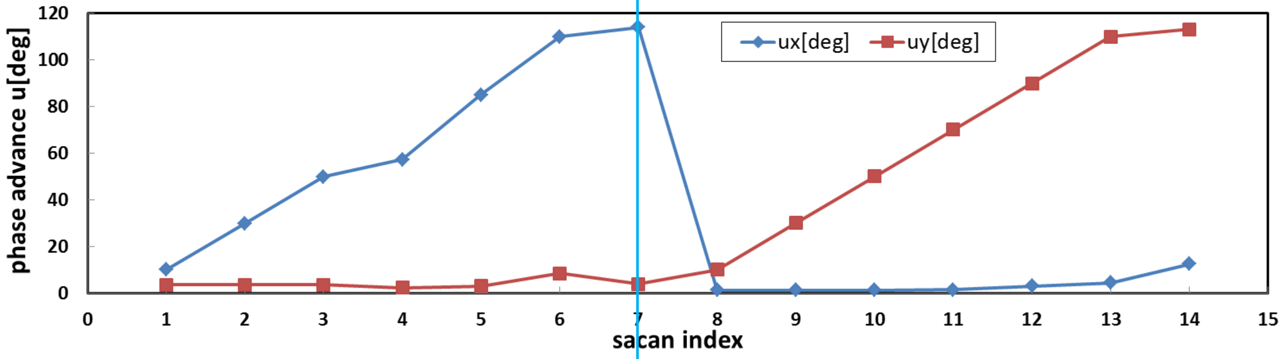
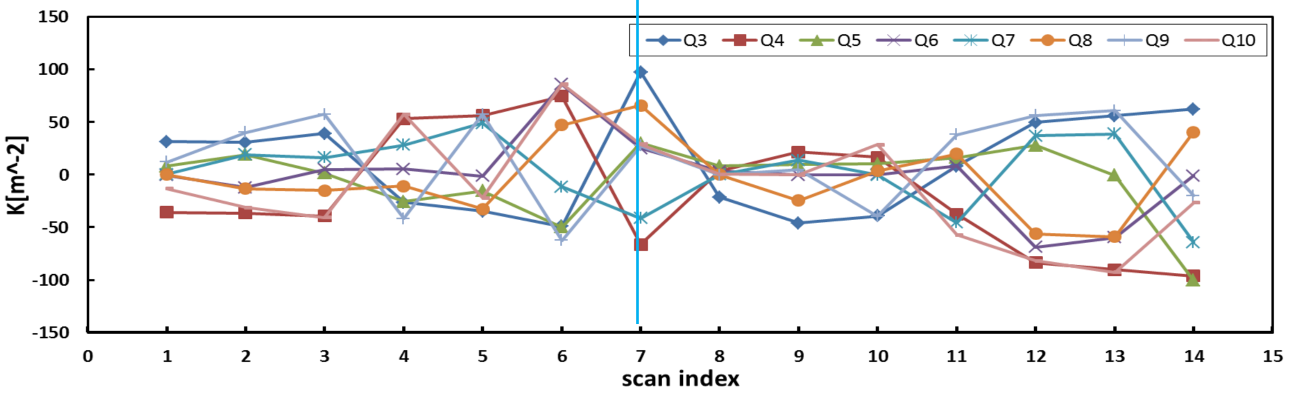


FIG. 1. Sketch of the measurement setup (not to scale).

For current set up:
Phase advance scan from 10 to 120 degree





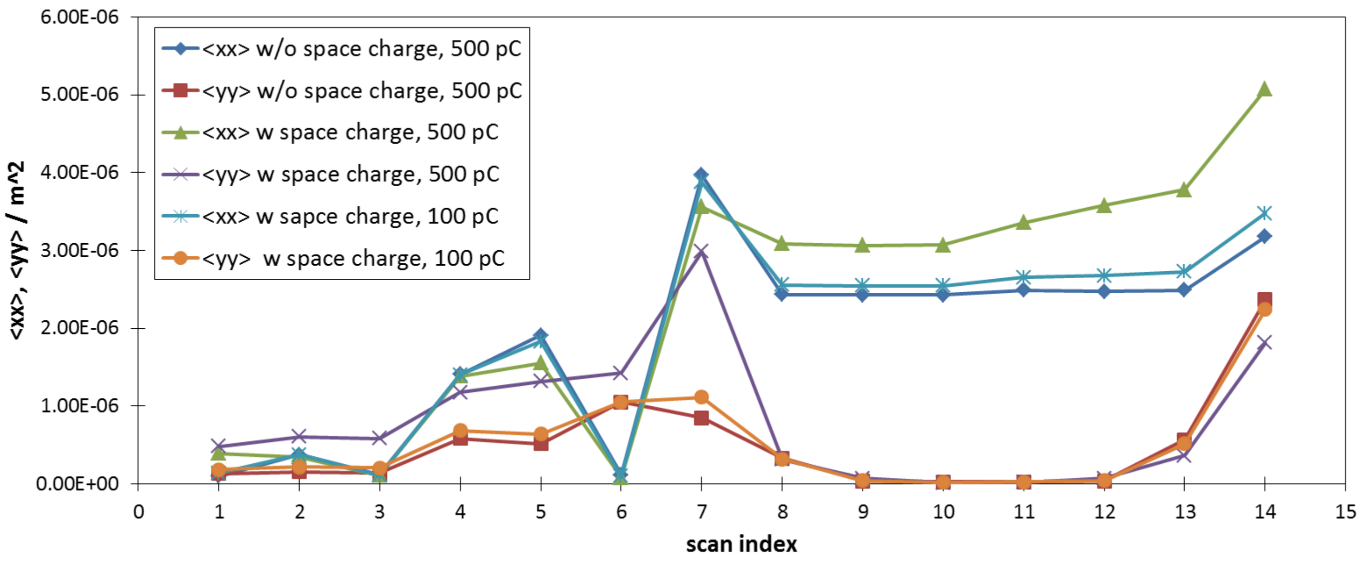


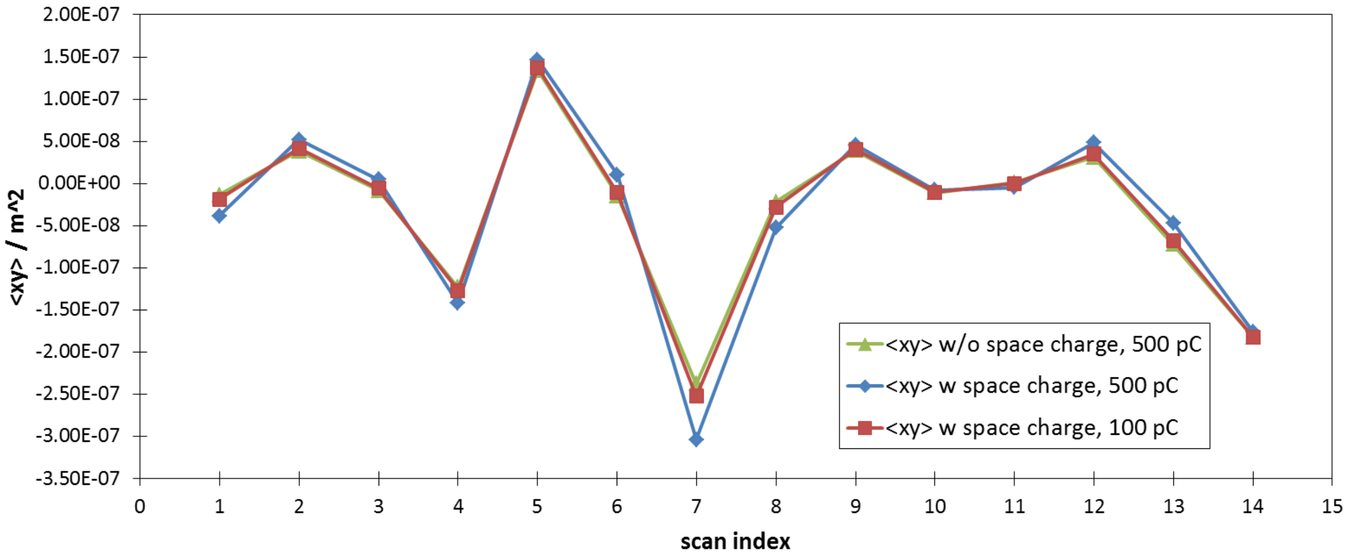
Simulated results of <xx>, <yy> and <xy> for 14 scans at PST.scr1

- ❑ Due to lower beam energy in PITZ, space charge effect need to be considered.

- ❑ Simulation are done for three different cases:

- ❖ **W/o space charge, 500 pC**
- ❖ **W space charge, 500 pC**
- ❖ **W space charge, 100 pC (same optics as 500 pC, same particles distribution)**





Reconsctructed results

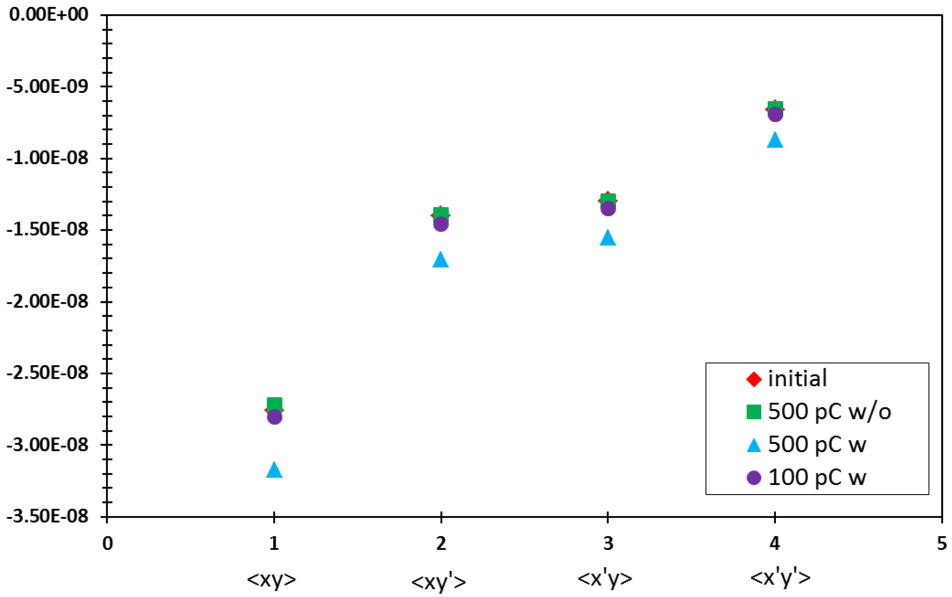
2D emittance and coupling terms

	Emitx [m*rad]	Emity [m*rad]	Coupling term
Initial beam (At z=5.277m)	3.094e-08	2.487e-08	0.014039
W/o spcae charge (500 pC)	3.024e-08	2.320e-08	0.014298
W space charge (500 pC)	5.087e-08	3.266e-08	0.007501
W space charge (100 pC)	3.429e-08	2.761e-08	0.0112931

Correlation terms

Unit m rad

	<xy>	<xy'>	<x'y>	<x'y'>
Initial (goal)	-2.76E-08	-1.40E-08	-1.30E-08	-6.60E-09
500 pC w/o	-2.72E-08	-1.40E-08	-1.30E-08	-6.52E-09
500 pC w	-3.17E-08	-1.71E-08	-1.55E-08	-8.69E-09
100 pC w	-2.81E-08	-1.46E-08	-1.35E-08	-6.94E-09



→In quads scan, the rms emittance will be over estimated by space charge effect. Also the correlation terms (absolute value) are over estimated.

Space charge effect investigation

- The notion of space-charge dominated flow is quantified by comparing the space charge and emittance terms in the rms beam envelope equation for an ultrarelativistic beam in a drift space ($\gamma \gg 1, \beta = v/c \approx 1$).

$$\sigma_x'' = \frac{{\epsilon_{nx}}^2}{\gamma^2 {\sigma_x}^3} + \frac{I}{\gamma^3 I_0(\sigma_x + \sigma_y)}$$

I is the peak beam current, I_0 =ec/r_e is the characteristic current, ϵ_{nx} is the normalized rms emittance.

taking the ratio of the second to the first terms on the right-hand side of the envelope equation, we have a measure of the degree of space-charge dominance over emittance in driving the evolution of the beam envelope

$$R_{0x} = \frac{I{\sigma_x}^3}{{\epsilon_{nx}}^2 I_0(\sigma_x + \sigma_y)} \approx 2{k_p}^2{\beta_x}^2$$

$$R_{0y} = \frac{I{\sigma_y}^3}{{\epsilon_{ny}}^2 I_0(\sigma_x + \sigma_y)} \approx 2{k_p}^2{\beta_y}^2$$

plasma wave number associated with the beam density n_b

$$k_p = \frac{\omega_p}{c} = \sqrt{\frac{4\pi r_e n_b}{\gamma^3}}$$

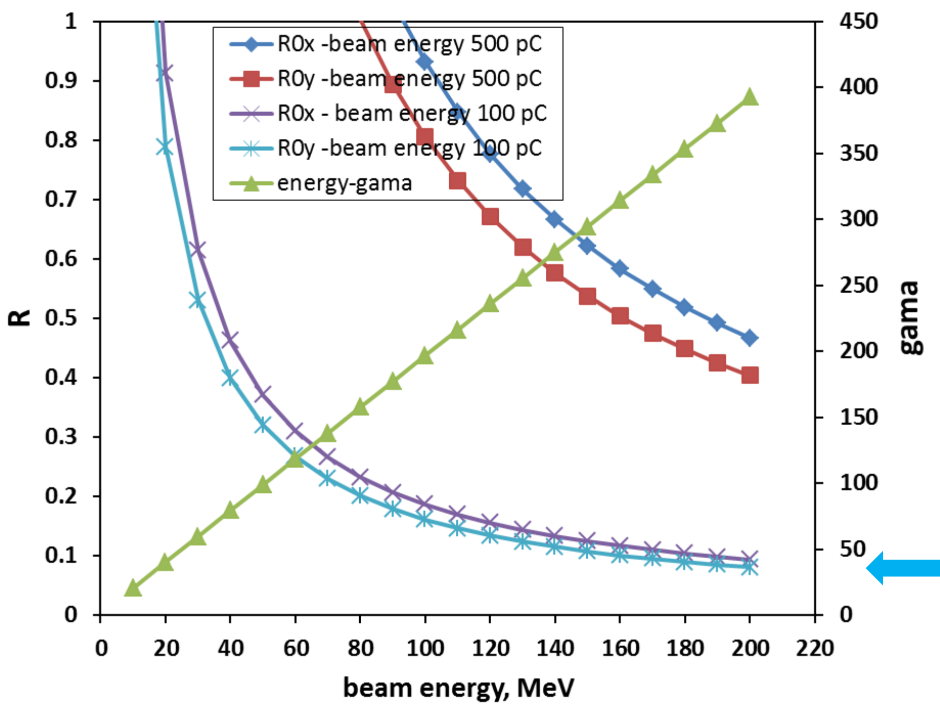
*S. G. Anderson and J. B. Rosenzweig et al., Space-charge effects in high brightness electron beam emittance measurements, PRSAB, VOLUME 5, 014201 (2002)

Initial beam R calculation

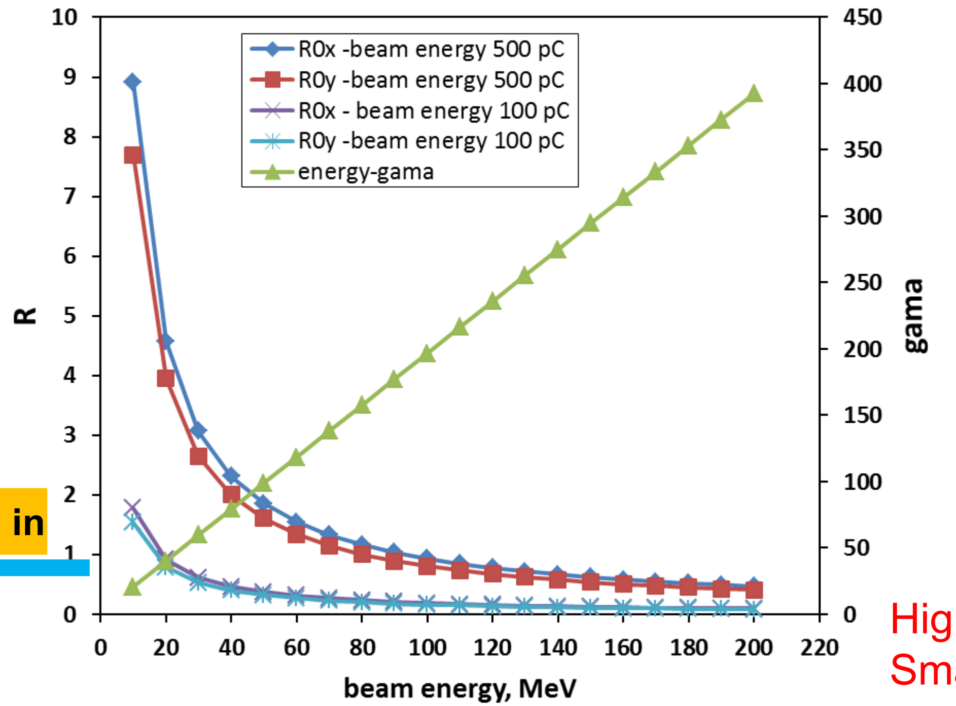
- I=24.5 A, x rms size 0.591 mm, y rms size 0.472 mm, P = 20.874 MeV/c
- norm x rms emittance 1.236 mm mrad, norm y rms emittance 0.949 mm mrad

R_{0x}= 4.37
R_{0y}= 3.78

Assuming current beam with different enegy and charge, other parameters are same.the space charge effect ratio R depends on the beam energy for different bunch charge are shown in the plot.
➔ For 100 pC, the R is smaller than 1, space charge has less effect.



Zoom in



High energy
Small bunch charge

Summary and conclusions

- Multi-quads scan method for 4D emittance measurment is studied by simulation for PITZ set up and considering for optimizing the gun quads compensation.

- Quads settings for beam matching for each scan are critical and need beam match for all scans.
  - Phase advance
  - Beam size at the measured screen

- The reconstructed 4D beam matrix from simulation are consistent with initial beam without space charge effect or less bunch charge➔principle and experiment set up confirmed.

- This method is precise for low bunch charge 4D beam matrix measurement for PITZ (~100 pC).

Restructed 4D matrix for different cases

Unit m rad

$$C = \begin{bmatrix} \langle xx \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\ \langle x'x \rangle & \langle x'x' \rangle & \langle x'y \rangle & \langle x'y' \rangle \\ \langle yx \rangle & \langle yx' \rangle & \langle yy \rangle & \langle yy' \rangle \\ \langle y'x \rangle & \langle y'x' \rangle & \langle y'y \rangle & \langle y'y' \rangle \end{bmatrix}$$

Initial beam (At 5.277m)	delt = 1.0e-06 * 0.349530194453562 0.148145594593491 -0.027605207459808 -0.014276831801143 0.148145594593491 0.065411036071994 -0.013136063817492 -0.006648341028507 -0.027605207459808 -0.013136063817492 0.222786264429634 0.086954889517126 -0.014276831801143 -0.006648341028507 0.086954889517126 0.036359881993339
W/o spcae charge (500 pC)	delt_recon1 = 1.0e-06 * 0.364271280270197 0.149338490153527 -0.027216831878540 -0.013956515825966 0.149338490153527 0.063852925296102 -0.012972525636867 -0.006516220034252 -0.027216831878540 -0.012972525636867 0.262817263848690 0.101730120523666 -0.013956515825966 -0.006516220034252 0.101730120523666 0.041731926885671
W space charge (500 pC)	delt_recon1 = 1.0e-06 * 0.359563329189557 0.172780508258457 -0.031736523493811 -0.017079149156129 0.172780508258457 0.090223422486976 -0.015537631659967 -0.008686619551019 -0.031736523493811 -0.015537631659967 0.271745906936861 0.138403183469094 -0.017079149156129 -0.008686619551019 0.138403183469094 0.074416987879002
W space charge (100 pC)	delt_recon1 = 1.0e-06 * 0.363709434071274 0.154342435894709 -0.028063601974417 -0.014579967965038 0.154342435894709 0.068729543401361 -0.013463007272344 -0.006940997076881 -0.028063601974417 -0.013463007272344 0.260849610616667 0.106025697857509 -0.014579967965038 -0.006940997076881 0.106025697857509 0.046019761660485