# The Slice Emittance Measurement by Multi-Quadrupole Scanning Method

**based on TDS cavity**

# Outline

## Part I Simulation of Measurement

- Introduction
- Solenoid scan and zero-crossing phase of TDS
- The gradient of quadrupoles settings
- Reconstruction method for slice emittance
- Results

## Part II Measurement

- Measurement procedure
- Results

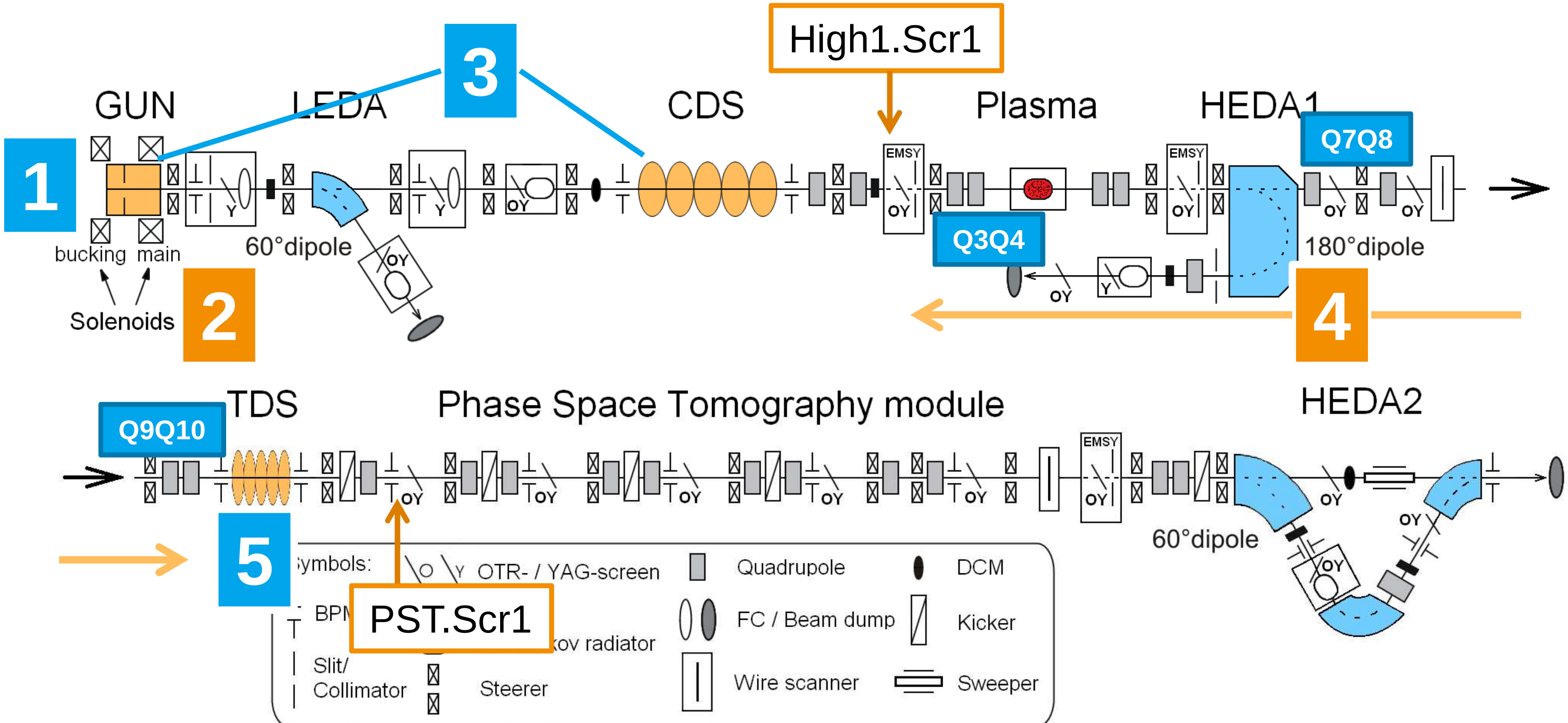
Conclusion

Outlook

# Part I Simulation of Measurement

## (everything is done in ASTRA)

# Overview




1

## Cathode distribution

| | | |
|---|---|---|
| Transverse profile | Uniform | - |
| Transverse rms size | 0.2 | mm |
| Temporal profile | Gaussian | - |
| Temporal length (FWHM) | 11 | ps |
| Charge | 100 | pC |

3

## Gun & booster at MMMG phase

| | | |
|---|---|---|
| Gun gradient | 53.75 | MV/m |
| Booster gradient | 17.3 | MV/m |

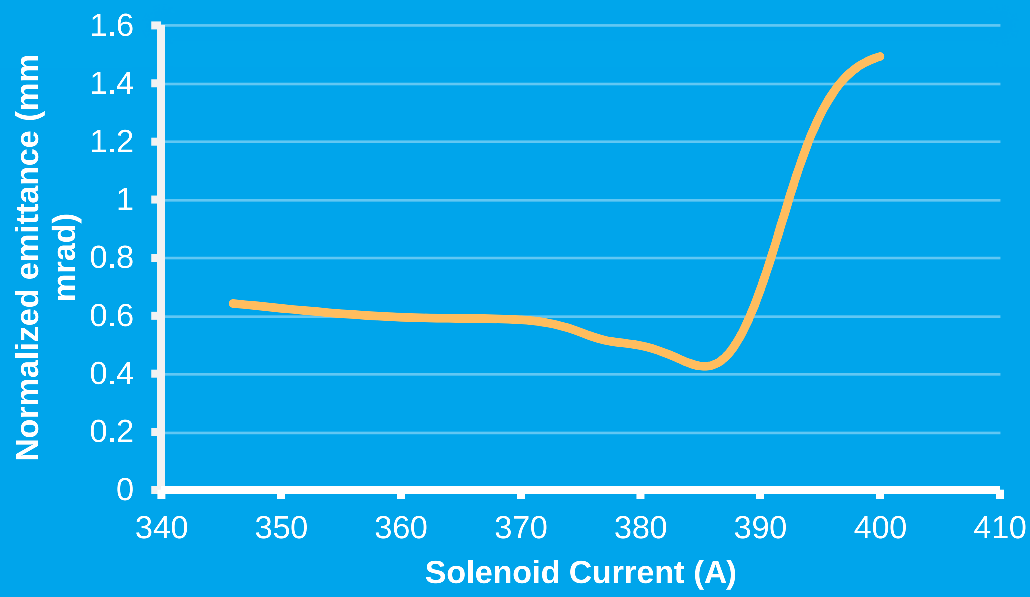
5

## TDS

| | | |
|---|---|---|
| TDS power | 0.680 | MW |
| TDS deflecting voltage | 0.962 | MV |

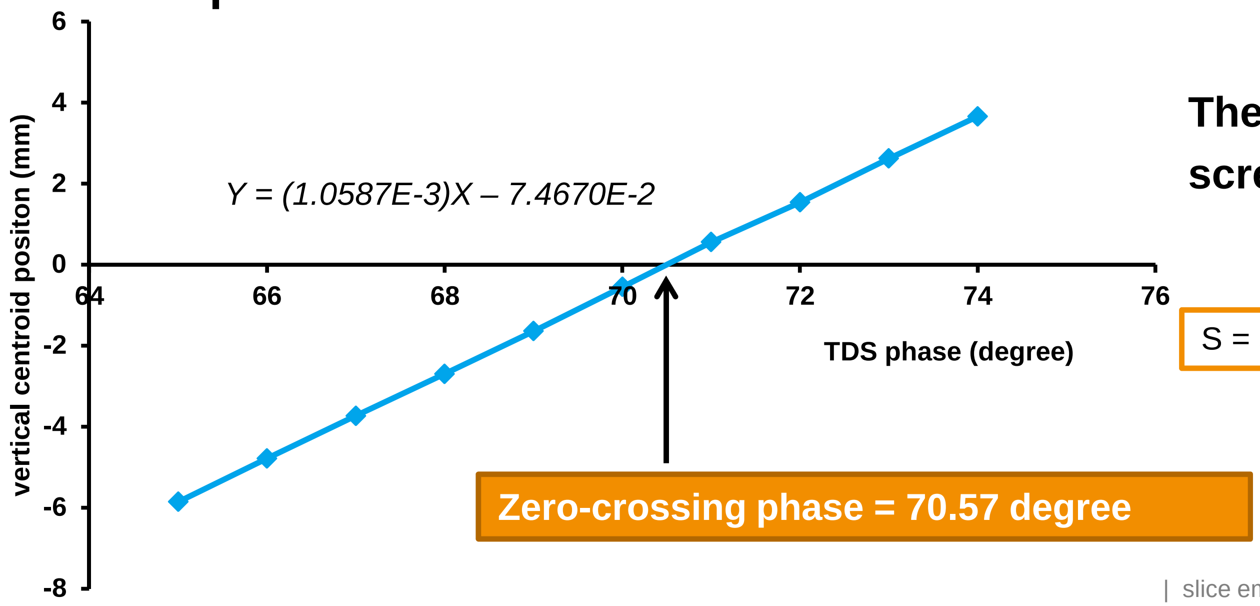
# Solenoid Scan 2

| | |
|---|---|
| **Optimized solenoid current** | **385A** |
| **Minimized emittance** | **0.426 mm mrad** |




# TDS Phase Scan

**TDS phase scan**




**The bunch centroid on the screen,** $\bar{y} = \frac{eV_0 L}{pc} \sin \emptyset_{RF}$**.**

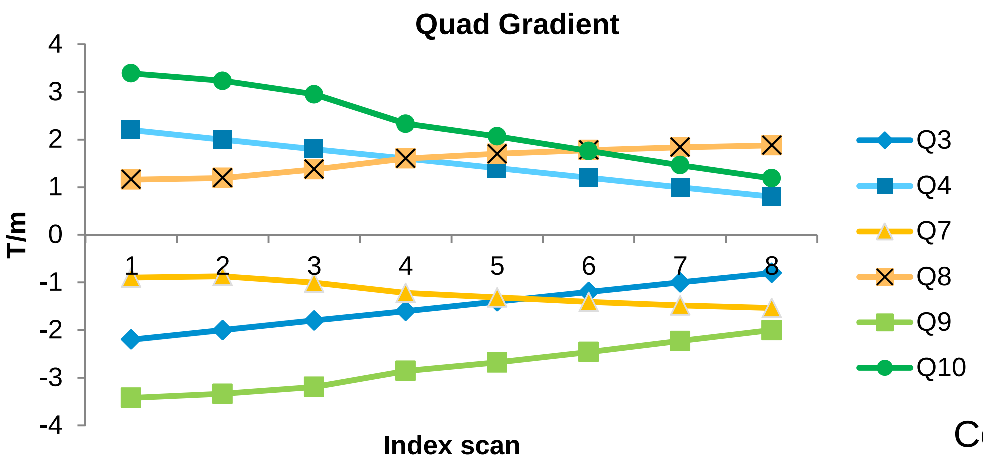
S = slope*360*f/$\beta c$ = 3.8125442

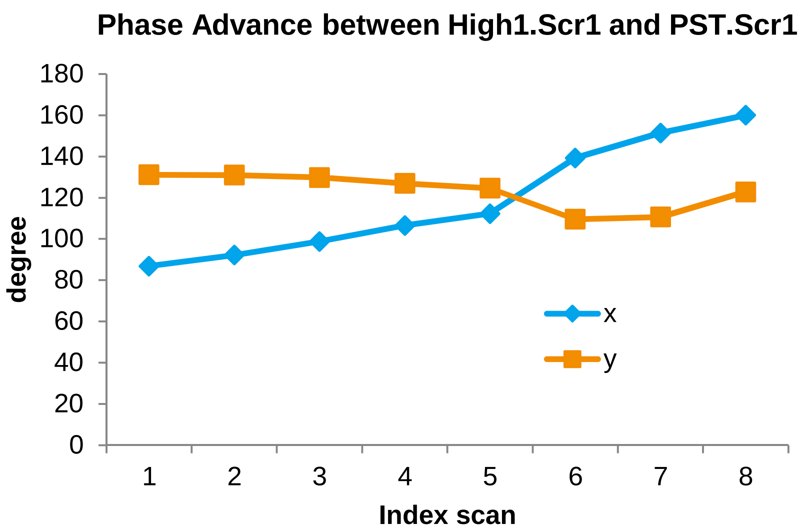
# Quadrupole Settings 4

Adjust Q3 and Q4, and compensate Q7-Q10 in order to get the same beam size at PST.Scr1.

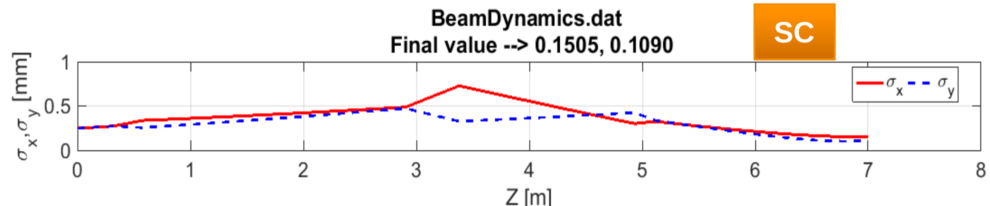
| SC (unit:mm) | | |
|---|---|---|
| index | X-rms | Y-rms |
| 1 | 0.127 | 0.121 |
| 2 | 0.119 | 0.116 |
| 3 | 0.117 | 0.110 |
| 4 | 0.134 | 0.103 |
| 5 | 0.137 | 0.103 |
| 6 | 0.144 | 0.106 |
| 7 | 0.151 | 0.109 |
| 8 | 0.157 | 0.112 |

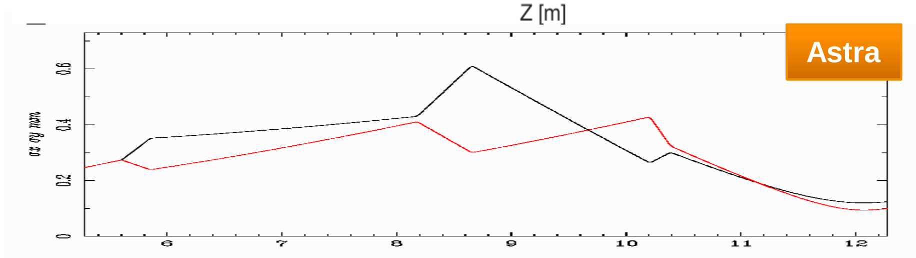
| ASTRA (unit:mm) | | |
|---|---|---|
| index | X-rms | Y-rms |
| 1 | 0.099 | 0.103 |
| 2 | 0.095 | 0.102 |
| 3 | 0.098 | 0.098 |
| 4 | 0.112 | 0.085 |
| 5 | 0.117 | 0.087 |
| 6 | 0.126 | 0.088 |
| 7 | 0.131 | 0.088 |
| 8 | 0.137 | 0.095 |




Comparison rms beam size along z between SC Code and Astra



# Calculation of Sliced Emittance by Reconstruction Method

The normalized emittance is given by

$$\epsilon_n = \beta\gamma\sqrt{\langle x_0^2\rangle \langle x_0'^2\rangle - \langle x_0^2 x_0'\rangle}$$

3 unknown variables

For a single slice

$$\sigma_i^2 = \left(R_{11}^i\right)^2 \left\langle x_0^2\right\rangle + \left(R_{12}^i\right)^2 \left\langle x_0'^2\right\rangle + 2R_{11}^i R_{12}^i \left\langle x_0 x_0'\right\rangle$$

where $\sigma$ is rms beam size on screen

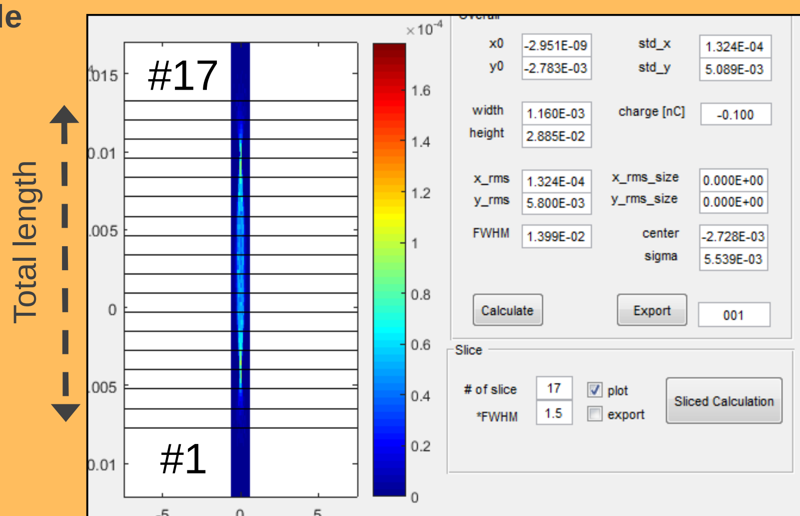
For number of measurement, i = 1,2 ... 8 ( number of quad settings )

R11 and R12

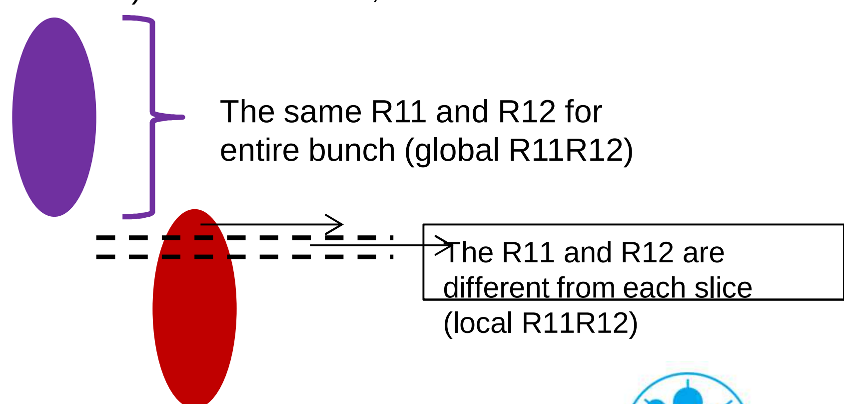
## $\sigma$, rms beam size

Get x rms size in each slice.

**Example**




A. Standard transfer matrices (quad and drift matrices)
B. Calculate by particles distribution in ASTRA which is
   1) the same value for entire bunch
      i. With space charge
      ii. Without space charge
   2) Different R11,R12 for each slice bunch



# Determination of R11 and R12

## A. Standard transfer matrices

$$Q = \begin{pmatrix} \cos\left(\sqrt{k}l\right) & \frac{\sin(\sqrt{k}l)}{\sqrt{k}} & 0 & 0 \\ -\sqrt{k}\sin\left(\sqrt{k}l\right) & \cos\left(\sqrt{k}l\right) & 0 & 0 \\ 0 & 0 & \cosh\left(\sqrt{k}l\right) & \frac{\sinh(\sqrt{k}l)}{\sqrt{k}} \\ 0 & 0 & \sqrt{k}\sinh\left(\sqrt{k}l\right) & \cosh\left(\sqrt{k}l\right) \end{pmatrix} \qquad L = \begin{pmatrix} 1 & L & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where k and l is quadrupole strength and effective length, respectively. where L is drift space.

[R] = [Q10] x [L] x [Q9] x [L] … [Q4] x [L2] x [Q3] x [L1]

## B. Particles distribution in ASTRA

@PST.Scr1 ← @High1.Scr1

$$x^1 = R_{11}x_0^1 + R_{12}{x'_0}^1$$

*N measurement* .
.
.

$$x^N = R_{11}x_0^N + R_{12}{x'_0}^N$$

$$\begin{pmatrix} x^1 \\ \vdots \\ x^N \end{pmatrix} = \begin{pmatrix} x_0^1 & {x'}_0^1 \\ \vdots & \vdots \\ x_0^N & {x'}_0^N \end{pmatrix} \cdot \begin{pmatrix} R_{11} \\ R_{12} \end{pmatrix} \Rightarrow A = B.x$$

$$x = (A^T B)^{-1}(A^T A)$$

# Sliced Emittance by Reconstruction Method (2)

Recall $\sigma_i^2 = (R_{11}^i)^2 \langle x_0^2 \rangle + (R_{12}^i)^2 \langle x_0'^2 \rangle + 2R_{11}^i R_{12}^i \langle x_0 x_0' \rangle$

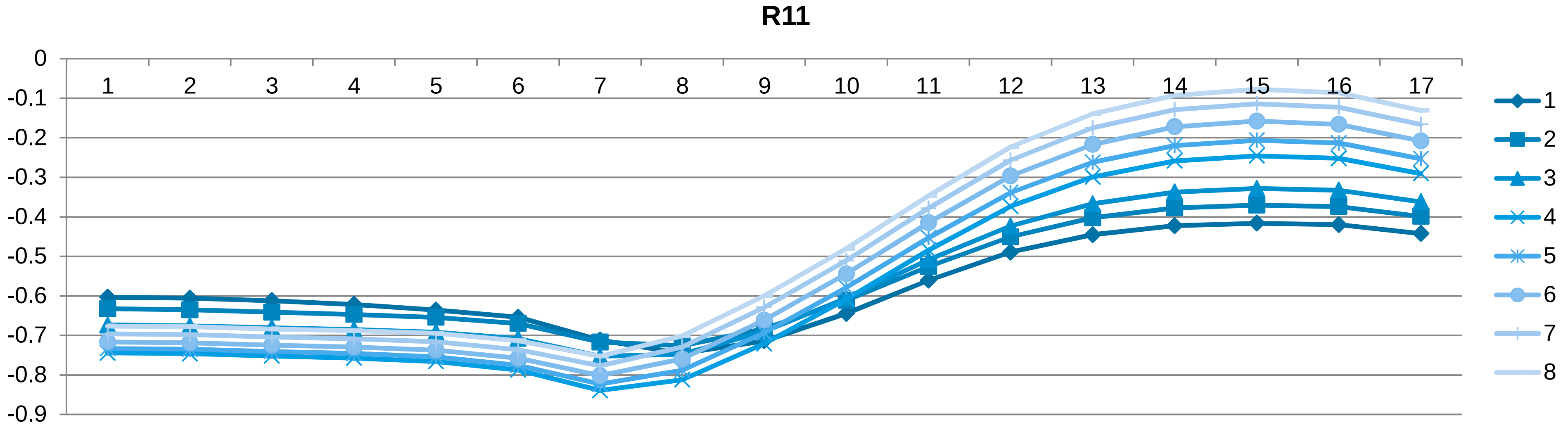
$$\left.\begin{aligned}
\sigma_1^2 &= (R_{11}^1)^2 \langle x_0^2 \rangle + (R_{12}^1)^2 \langle x_0'^2 \rangle + 2R_{11}^1 R_{11}^1 \langle x_0 x_0' \rangle \\
&\vdots \\
\sigma_i^2 &= (R_{11}^i)^2 \langle x_0^2 \rangle + (R_{12}^i)^2 \langle x_0'^2 \rangle + 2R_{11}^i R_{11}^i \langle x_0 x_0' \rangle \\
&\vdots \\
\sigma_N^2 &= (R_{11}^N)^2 \langle x_0^2 \rangle + (R_{12}^N)^2 \langle x_0'^2 \rangle + 2R_{11}^N R_{11}^N \langle x_0 x_0' \rangle
\end{aligned}\right\}$$

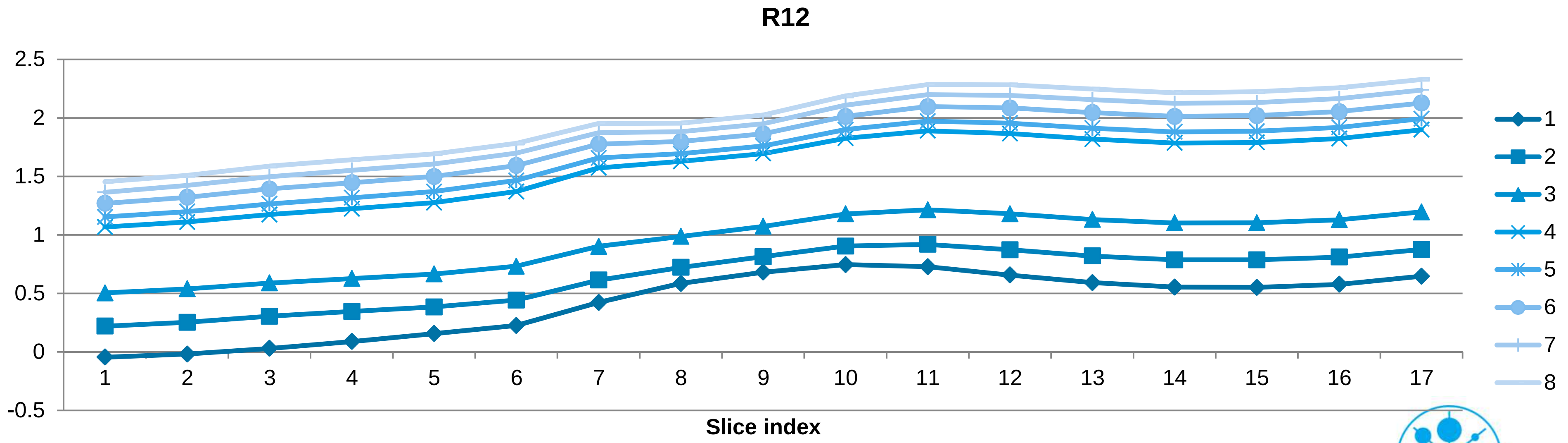
N equations

$$\begin{pmatrix} \sigma_1^2 \\ \vdots \\ \sigma_N^2 \end{pmatrix} = \begin{pmatrix} (R^1)_{11}^2 & (R^1)_{11}^2 & 2R^1{}_{11}R^1{}_{12} \\ \vdots & \vdots & \vdots \\ (R^N)_{11}^2 & (R^N)_{11}^2 & 2R^1{}_{11}R^N{}_{12} \end{pmatrix} \cdot \begin{pmatrix} \langle x_0^2 \rangle \\ \langle x_0'^2 \rangle \\ \langle x_0 x_0' \rangle \end{pmatrix} \Rightarrow A = B.x$$

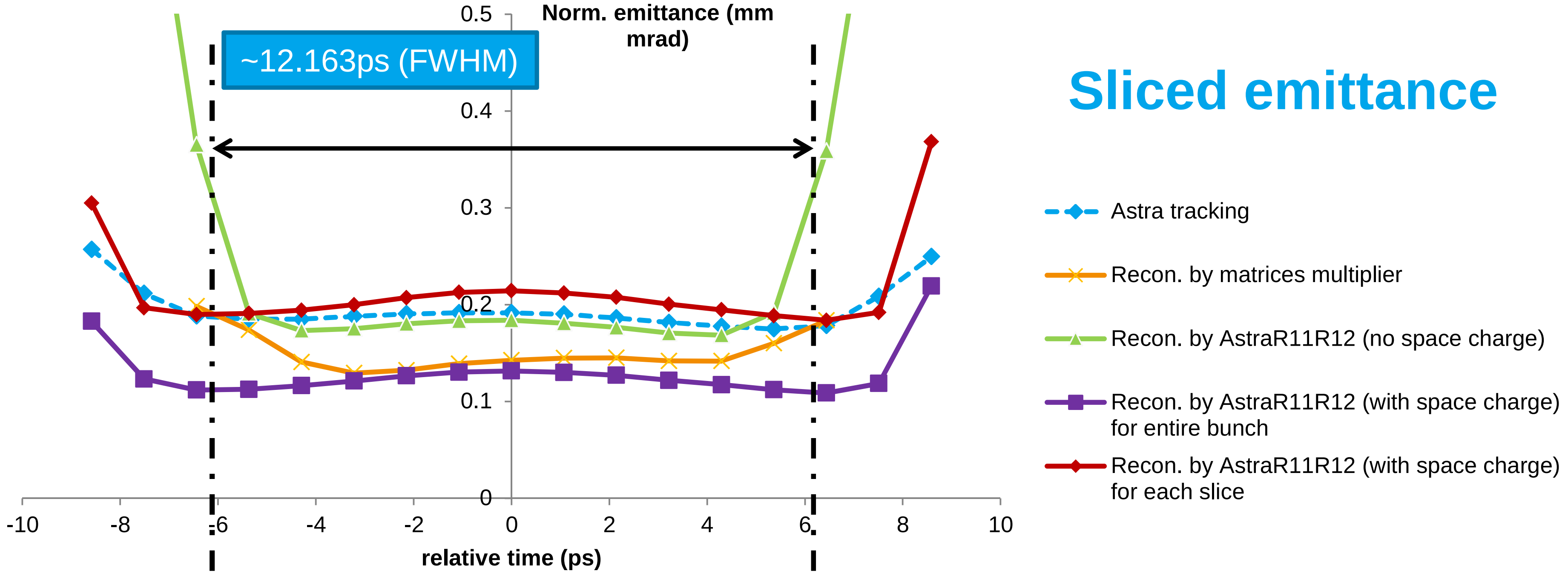
$$x = (A^T B)^{-1} (A^T A)$$

# Evolution of R11 and R12 along the longitudinal bunch



# Results




## Sliced emittance

## Projected emittance

| | Recons. by | Projected emittance (mm mrad) |
|---|---|---|
| **ASTRA tracking** | **-** | **0.4263** |
| **A** | Standard transfer matrices multiplier | **0.6225** |
| **B.1,i** | Astra with space charge | **0.4542** |
| **B.1,ii** | Astra without space charge (simulation without space charge) | **0.4186** |



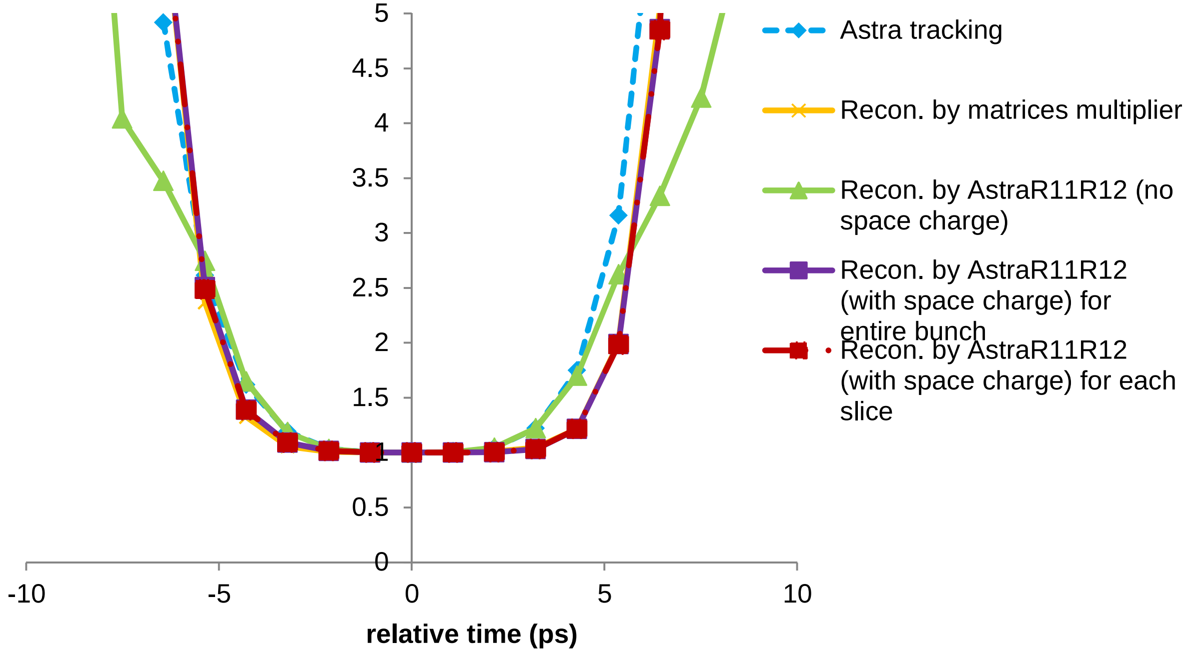
DESY

# Results (2)

## Mismatch parameter

We can calculate the mismatch parameter by $\zeta = \frac{1}{2}(\beta_0\gamma - 2\alpha\alpha_0 + \beta\gamma_0)$

where $\beta_0, \alpha_0$ and $\gamma_0$ are designed Twiss parameters. They are parameters that we want




The mismatch parameter implies to how is the difference between ellipses in phase space.

When it equals to 1, it means that the Twiss parameters are the same as we design. (no difference)

$$\alpha_x = -< xx' >/\varepsilon_x$$
$$\beta_x = -< x^2 >/\varepsilon_x$$
$$\gamma_x = -< x'^2 >/\varepsilon_x$$

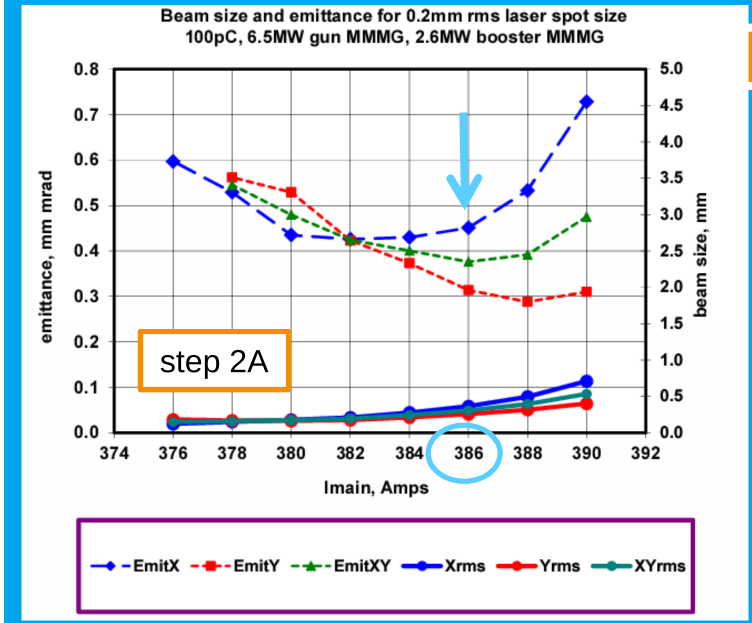
where $\varepsilon_x$ is geometrical emittance

In this case, designed Twiss parameters is at the center of the bunch.

# Part II Experiment

# Optimized Solenoid Current

We optimized solenoid current in order to get the minimized projected emittance which measured by slit-scan technique at High1.Scr1




Imain = 386A

**We also make a statistic of this measurement by 6**

## Apply quads setting by [quad gradient set] script

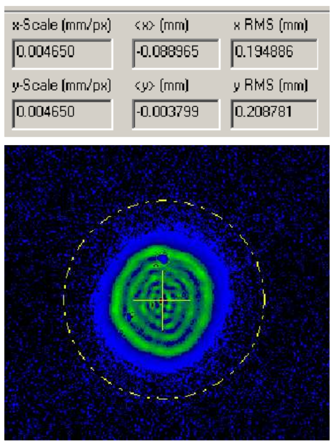
- These gradients get from the SC code. The quad gradient set script is used.
- Measured rms beam size by the video client

Example command : setQuadsH({'HIGH1.Q3'}, 10);

**Script location : ~\group\pitz\doocs\measure\Magnets\Quads\ModifiedVersion_bprach**

- There are two kind of q setting. The set A gives smaller rms beam size (~200 um) than set B (~300um).
- There are 8 and 5 settings for set A and B, respectively.
- For each quad setting, we measure the beam with and without TDS at PST.scr1

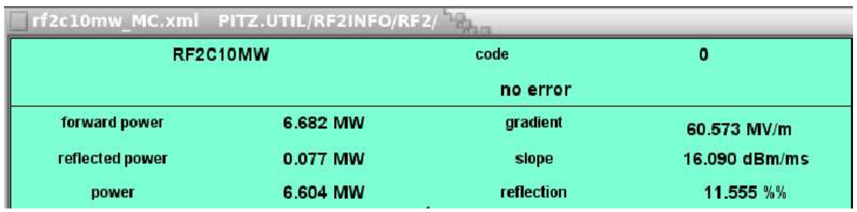
# Slice Emittance Measurement Procedure


BSA setting

## Beam preparation (according to simulation)

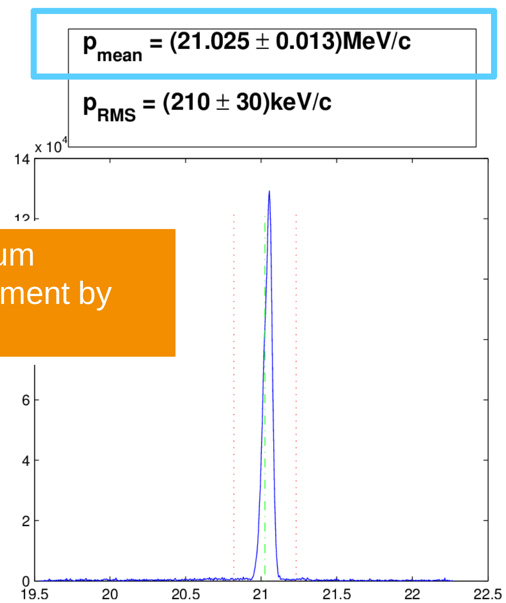
| | |
|---|---|
| BSA setting | 0.92 mm (equivalent to 0.2 rms beam size) |
| Power in gun | ~6.5MW,650us at MMMG phase |
| Power in booster | ~2.6MW |
| Charge measurement by Faraday cup (Low.FC2) | ~100pC |
| Momentum measurement by HEDA | ~21MeV/c |



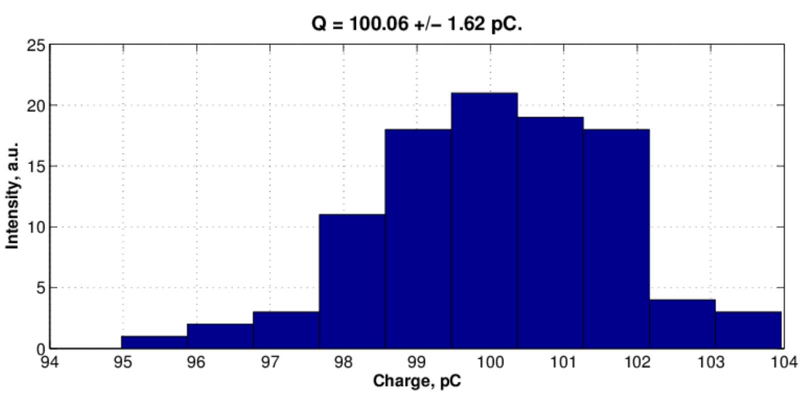

Gun setting


Booster setting

Reduce booster power until we got



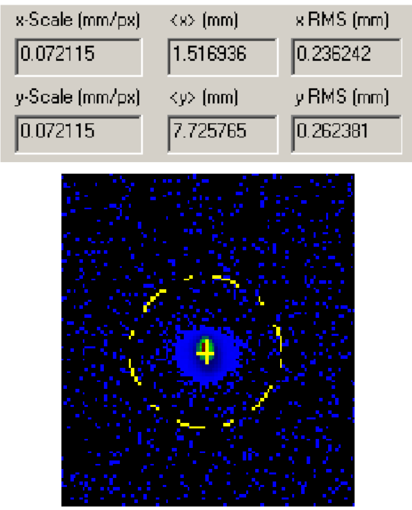

Momentum measurement by HEDA



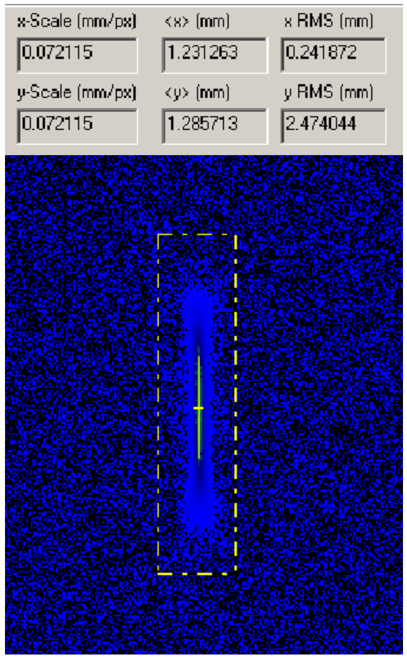

Charge measurement by Faraday cup (Low.FC2)

# Measured streaked beam by 'TDS_calib.m'

Example

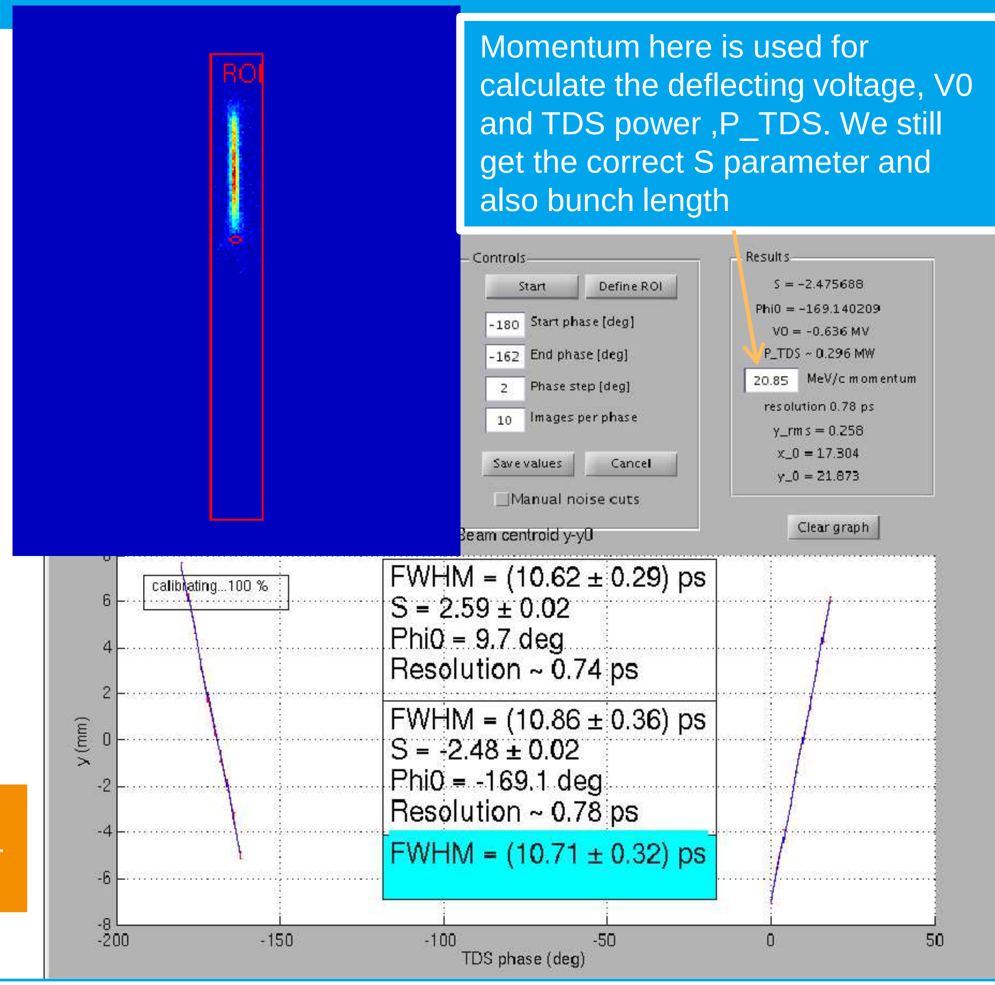



saved non-streaked beam




saved streaked beam image by using video client at zero-crossing phase of TDS

Momentum here is used for calculate the deflecting voltage, V0 and TDS power ,P_TDS. We still get the correct S parameter and also bunch length




**Script location** **Q:\group\pitz\doocs\measure\scripts\SVN\MatlabScripts\TDS**

**Saved data folder**

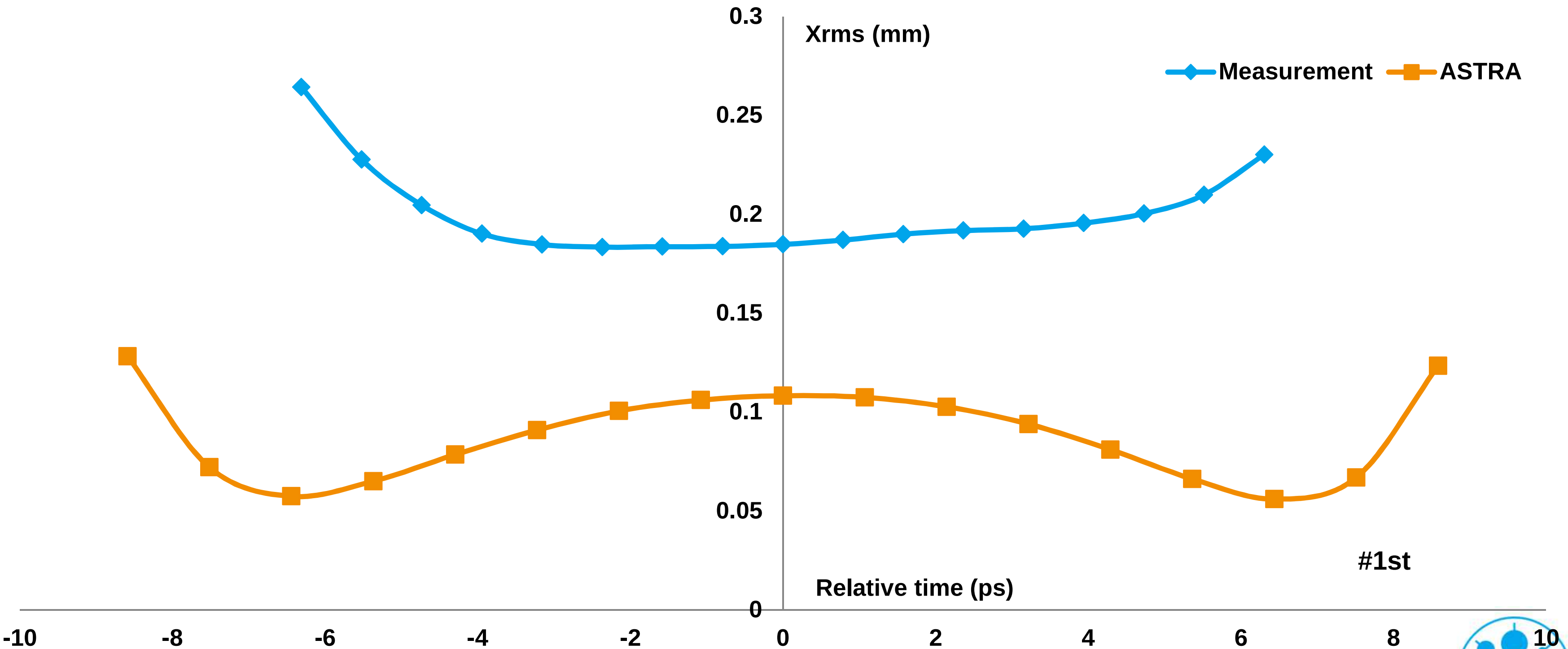
| | | |
|---|---|---|
| **Image and background** | **Q:\group\pitz\doocs\measure\LongPhSp\2016\TDS\MultiQscan** | ***.imc, *.bck** |
| **MATLAB by TDS_calib** | **Q:\group\pitz\doocs\measure\LongPhSp\2015\TDScalib\TDSscan2016** | ***.mat** |
| **Longitudinal phase space** | **Q:\group\pitz\doocs\measure\LongPhSp\2016\TDS\LPS_100pC_20160818N** | ***.imc, *.bck** |

# Simulation and Measurement Comparison

## Twiss Parameters and projected emittance comparison at High1.Scr1

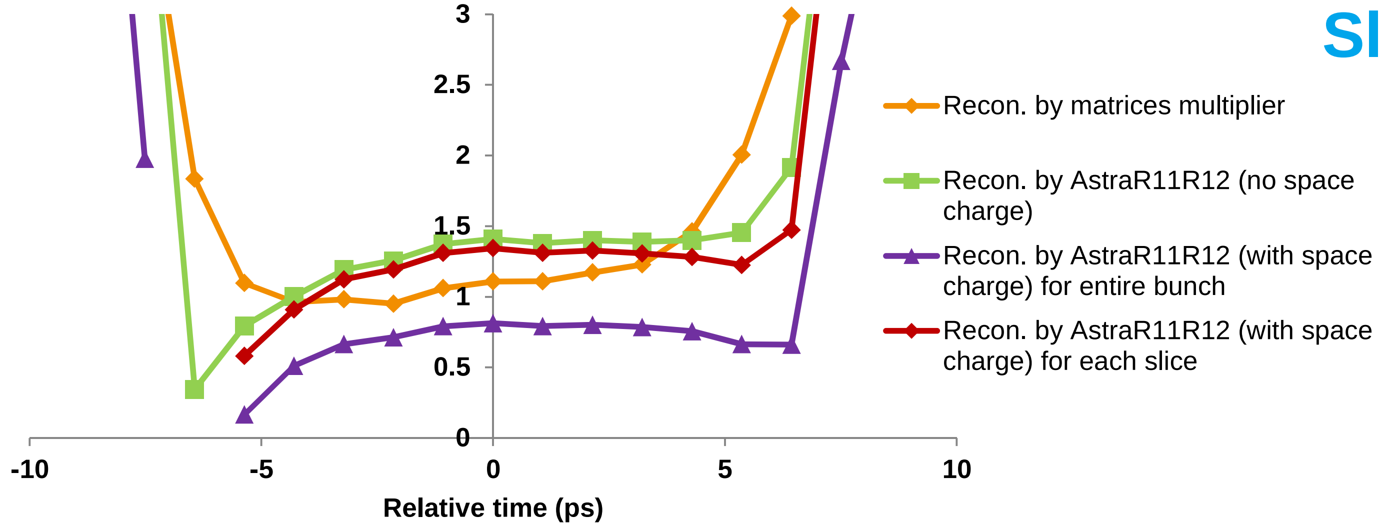
| | Projected emittance (mm mrad) | Solenoid current (A) |
|---|---|---|
| ASTRA tracking | 0.4263 | 385 |
| Measurement | 0.431(0.012) | 386 |

| horizontal | Astra tracking | Measurement@ EMSY1 |
|---|---|---|
| $\beta$(mm) | 6.0624 | 9.759(0.59) |
| $\alpha$ (mrad) | -1.9881 | -3.922(0.19) |
| $\gamma$ | 0.8169 | 1.676(0.06) |



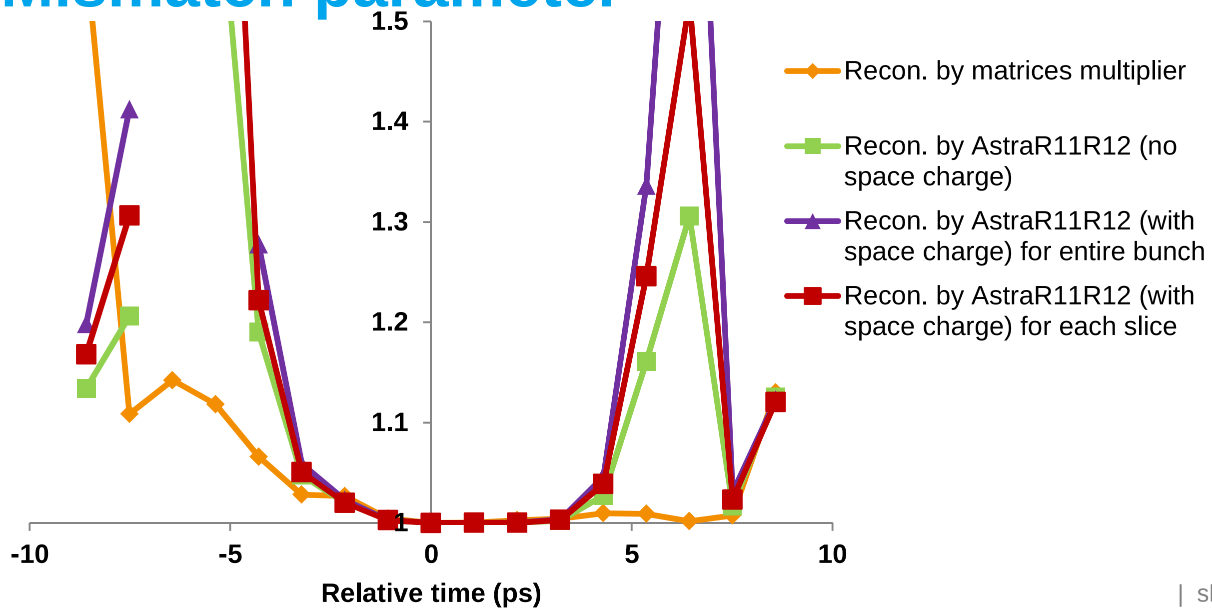
# Result

## Sliced emittance




**Calculate sliced by using xrms from measurement and still use the same R11 and R12 from previous results**

## Mismatch parameter




## Projected emittance

| Recons. by | Projected emittance (mm mrad) |
|---|---|
| Standard transfer matrices multiplier | **1.2950** |
| R11,R12 Astra with space charge | **0.8255** |
| R11,R12 Astra without space charge (simulation without space charge) | **1.4791** |

# Conclusions

- Simulation of measurement for slice emittance is done by ASTRA. The reconstruction method for emittance requires R11 and R12.
- For space charge dominant beam (charge of 100pc, momentum of 21MeV/c), the standard matrices multiplier gives underestimated slice emittance due to no space charge including.
- The R11 and R12 are constructed by numerically (ASTRA distribution) performing the slice emittance and mismatch parameter provide the reasonable result compared to the ASTRA tracking at High1.Scr1. However, the R11 and R12 should be calculated for each beam slice individually.
- Quadrupole gradient for beam matching from SC code can be reliable in experiment.
- Since we have the larger rms beam size (also different Twiss parameters) from measurement and use R11 and R12 from previous simulations, the slice emittance and projected emittance consequently is higher than simulation of measurement.

# Outlook

- We must construct Astra distribution at High1.Scr1 to get the same Twiss parameters as measurement and then hopefully we can get real R11 R12 !! (ps viewer tool)
- Reconstruction with less slices (e.g. 7 instead of 17)
- Measurement of 500 pC beam (and simulation…)
- Error estimation…

# Outlook (2)

- The simulation of measurement for slice emittance measurement by slit scan technique (@EMSY3) and TDS.

  Advantages

  - No reconstruction required
  - Directly measure the slice emittance at High2.Scr1

  Difficulties

  - Since the TDS is far away from EMSY3 (~5 m), the streak beam is need to be transported by quadrupoles in PST-section.
  - Steering free for quads is necessary.
  - Avoid beam waist during the transportation.
  - In measurement, if the intensity of beam let at High2.Scr2 is very low, Need LYSO screen. [Or increase charge density?] or focus by Q
  - Is slit length long enough for streak beam ?? (or move it vertically)
  - Fast scan tool is ready ?

- Multi-screen method?

Thank you for your attention

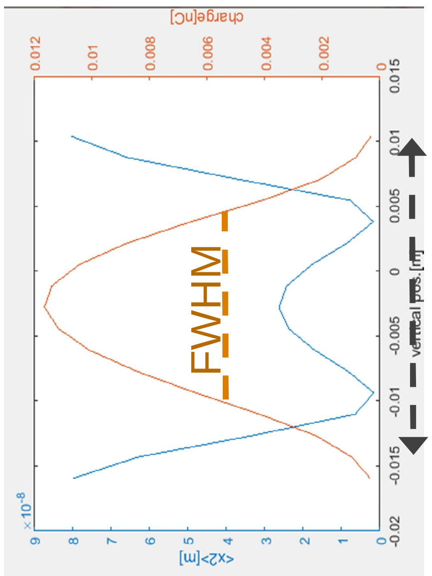
# Backup Slide

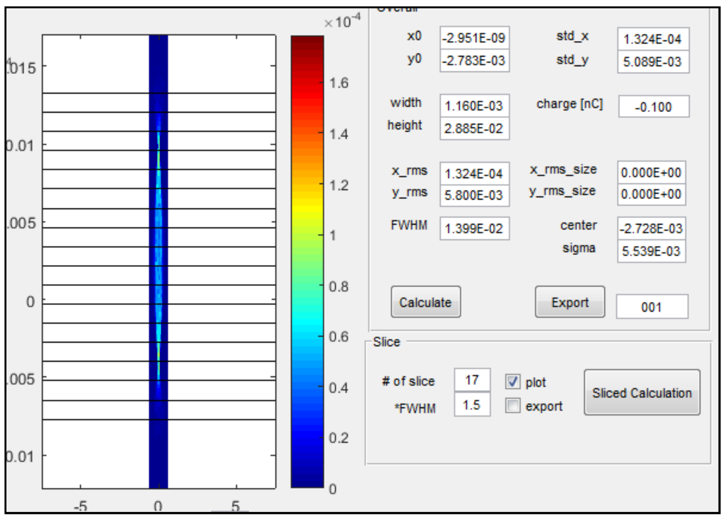
Why don't I use parabola fitting?

The parabola fitting appropriates for single quad scan, if we have double-quad scan it can use surface fitting. In this case we have more than 2 so we instead use the  matrix algebra fitting.

In ASTRA simulation, the xrms is calculated by statistical calculation in each slice. But In measurement, the xrms is calculated by Gaussian fitting because the data is too noisy.

Emittance calculation for N  measurement points [https://cds.cern.ch/record/321551/files/clic-note-326.pdf]
At least square for fitting curve, see theorem [http://www.ms.uky.edu/~ma138/Spring16/Curve_fitting.pdf]




| | S parameter | Bunch length |
|---|---|---|
| **Astra** | 3.182 | ~12.16 |
| **Measurement** | 2.68 | ~10.0 |