## Using HEDA2 as a Bunch Compressor



- Motivation
- Transformation Matrices
$-2{ }^{\text {nd }}$ High Energy Dispersive Arm (HEDA2)
- ASTRA Simulations for the Input Beams
-Calculation of Bunch Compression by HEDA2
-Summary \& Outlook

Results of velocity bunching experiment (shifts: 30-31.10.15)

| Machine Parameters |  |
| :--- | :---: |
| Laser pulse shape | Gaussian |
| Laser temporal length | $\sim \mathbf{2 . 5} \mathbf{~ p s}$ FWHM |
| BSA | $2.0,2.5 \mathrm{~mm}$ |
| Bunch charge | $20,100,250 \mathrm{pC}$ |
| Peak power of RF in the gun | 6.3 MW |
| Peak power of RF in the booster | 2.7 MW |
| Gun phase* | 0 degree |
| Booster phase* $^{*}$ | 0 to -90 degree |

## FWHM bunch length VS booster phase



## Calculated form factors



Selected long. bunch profiles

$$
\frac{d U_{C T R}}{d \omega} \propto\left|F_{l o n g}(\omega)\right|^{2}
$$

$$
F_{\text {long }}(\omega)=\int_{-\infty}^{\infty} \rho_{\text {long }} \exp (-i \omega t) d t
$$




## Can HEDA2 be used as a bunch compressor?

Works in this presentation is trying to answer this question by using transformation matrices.

## $\mathrm{PIT}_{2}$ Transformation Matrices

Ref: A.Chao Handbook, p.56-59, 1999 S.Rimjaem, HEDA2 note, 24.9.09 D.C.Carey, SLAC-R-530,

- A charged particle is represented by a vector $\boldsymbol{X}(\boldsymbol{s})$

$$
\boldsymbol{X}^{\boldsymbol{t}}(s)=\left[\begin{array}{llllll}
x(s) & x^{\prime}(s) & y(s) & y^{\prime}(s) & \ell(s) & \Delta p / p_{0}
\end{array}\right]
$$

- The vector $\boldsymbol{X}(0)$ at position 0 is transform to another vector $\boldsymbol{X}(s)$ at position $\boldsymbol{s}$ by

$$
\boldsymbol{X}(s)=\boldsymbol{\mathcal { M }} \boldsymbol{X}(0)
$$

where $\mathcal{M}$ is a $6 x 6$ matrix characterizing lattice(s) between 0 and $s$.
It is so called "transformation matrix".

- By assuming $\Delta p$ and $p_{0}$ are constant, x - and y -motions are decoupled and system have midplane symmetry about $y=0, \boldsymbol{X}(s)=\boldsymbol{\mathcal { C }} \boldsymbol{X}(0)$ can be expanded as

$$
\left[\begin{array}{c}
x(s) \\
x^{\prime}(s) \\
y(s) \\
y^{\prime}(s) \\
\ell(s) \\
\Delta p / p_{0}
\end{array}\right]=\left[\begin{array}{cccccc}
R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\
R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\
0 & 0 & R_{33} & R_{34} & 0 & 0 \\
0 & 0 & R_{43} & R_{44} & 0 & 0 \\
R_{51} & R_{52} & 0 & 0 & R_{55} & R_{56} \\
0 & 0 & 0 & 0 & 0 & R_{66}
\end{array}\right]\left[\begin{array}{c}
x(0) \\
x^{\prime}(0) \\
y(0) \\
y^{\prime}(0) \\
\ell(0) \\
\Delta p / p_{0}
\end{array}\right]
$$

## ${ }^{\text {PITZ Transformation Matrices (2) }}$

The final vector of the electron can be written as

$$
\left[\begin{array}{c}
x(s) \\
x^{\prime}(s) \\
y(s) \\
y^{\prime}(s) \\
\ell(s) \\
\Delta p / p_{0}
\end{array}\right]=\left[\begin{array}{c}
R_{11} x(0)+R_{12} x^{\prime}(0)+R_{16}\left(\Delta p / p_{0}\right) \\
R_{21} x(0)+R_{22} x^{\prime}(0)+R_{26}\left(\Delta p / p_{0}\right) \\
R_{33} y(0)+R_{34} y^{\prime}(0) \\
R_{43} y(0)+R_{44} y^{\prime}(0) \\
R_{51} x(0)+R_{52} x^{\prime}(0)+R_{55} \ell(0)+R_{56}\left(\Delta p / p_{0}\right) \\
R_{66}\left(\Delta p / p_{0}\right)
\end{array}\right]
$$

- Transformation matrix of a drift space with length of $L$

$$
\boldsymbol{M}_{\boldsymbol{D}}(L)=\left[\begin{array}{cccccc}
1 & L & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & L & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & \frac{L}{\gamma^{2}} \\
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
$$

## ${ }^{\text {PITZ Transformation Matrices (3) }}$

## - Transformation matrix of a sector magnet



## ${ }^{\text {PITZ Transformation Matrices (4) }}$

-Pole face rotation (wedge angle) matrix

$$
\boldsymbol{\mathcal { M }}_{\beta}=\left[\begin{array}{cccccc}
1 & 0 & 0 & 0 & 0 & 0 \\
\frac{\tan \beta}{\rho} & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & -\frac{\tan \beta}{\rho} & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
$$

- Transformation matrix of a sector magnet including wedge angles

$$
\mathcal{M}_{\text {SecD }}=\mathcal{M}_{\beta_{\text {out }}} \mathcal{M}_{\text {RecD }} \mathcal{M}_{\beta_{\text {in }}}
$$



Disp3.D1



Disp3.D2
Ref: PITZ3-Koordinaten_15-10-15.xIsx HEDA2-note, PITZ-wiki

Specifications of Dipole Magnets in the Second High Energy Dispersive Arm (HEDA2)

| Parameter | Dipole 1 | Dipole 2 | Dipole 3 |
| :--- | :---: | :---: | :---: |
| Tvne | sector | sector with exit wedge_angle | sector |
| Bending angle, $\alpha$ (degree) | 60 | 120 | 60 |
| Entrance wedge angle, $\beta_{\text {in }}$ (degree) | 0 | 0 | 0 |
| Exit wedge angle, $\beta_{\text {out }}$ (degree) | 0 | 9 | 0 |
| Bending radius, $\rho(\mathrm{mm})$ | 600 | 400 | 400 |
| Maximum magnetic field (T) | 0.23 | 0.34 | 0.34 |
| Effective length (mm) | 628.3 | 837.8 | 418.9 |
| Pole gab (mm) | 60 | 60 | 60 |
| Vertical field homogeneity (dB/B) | $\pm 5 \times 10^{-4}$ | $\pm 5 \times 10^{-4}$ | $\pm 5 \times 10^{-4}$ |
| Good field region in vertical <br> direction (mm) | $\pm 25$ | $\pm 25$ | $\pm 25$ |
| Good field region in radial <br> direction (mm) | $\pm 70$ | $\pm 50$ | $\pm 50$ |

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## PITZ HEDA2: Tr. Matrices Used in This Work


$X_{\text {final }}=\mathcal{M}_{\text {SecD3 }} \mathcal{M}_{\text {D23 }} \mathcal{M}_{\text {SecD } 2} \mathcal{M}_{D 12} \mathcal{M}_{\text {SecD } 1} \boldsymbol{X}_{\text {initial }}$

## ${ }^{\text {PIT }}$ ASTRA Simulations for the Input Beams

Examples of Longitudinal Phase Space of the Output Beams with Imain = 330A

| Input for ASTRA Simulation |  |
| :--- | :---: |
| Zstart $\boldsymbol{\rightarrow}$ Zstop | $0 \rightarrow 6 \mathrm{~m}$ |
| \# macroparticles | $\mathbf{2 0 k}$ |
| Laser pulse shape | Gaussian |
| Laser temporal length | $\mathbf{2 . 4 3} \mathbf{~ p s ~ F W H M ~}$ |
| Bunch charge | $\mathbf{1 0 0} \mathrm{pC}$ |
| Laser BSA size | $\mathbf{2 ~ m m}$ |
| Main solenoid current | $\mathbf{( 2 3 0 : 1 0 : 3 3 0 ) ~ A}$ |
| Peak field in gun | $60.5 \mathrm{MV} / \mathrm{m}$ |
| Peak field in booster | $17.2 \mathrm{MV} / \mathrm{m}$ |
| Gun RF phase* | 0 degree |
| Booster RF phase* | $\mathbf{( - 9 0 : 3 0 : 9 0 ) ~ d e g r e e ~}$ |

*With respect to the Maximum Mean
Momentum Gain (MMMG) phase

## Boo. Phase $=0^{\circ}$




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## ${ }^{\text {PITZ }}$ Calculation of Bunch Compression by HEDA2

$\rightarrow$ Energy-chirp with positive slope $\left(\Phi_{\text {booster }}=30^{\circ}\right), I_{\text {main }}=330 \mathrm{~A}$



$P_{\text {mean }}=19.67 \mathrm{MeV} / \mathrm{c}$
R56 $=\mathbf{-} 0.0232 \mathrm{~m}$




## PITZ Calculation of Bunch Compression by HEDA2

- Energy-chirp with negative slope $\left(\Phi_{\text {booster }}=-30^{\circ}\right), I_{\text {main }}=330 \mathrm{~A}$



## PITZ Calculation of Bunch Compression by HEDA2

- Energy-chirp with negative slope $\left(\Phi_{\text {booster }}=-60^{\circ}\right), I_{\text {main }}=330 \mathrm{~A}$



$P_{\text {mean }}=14.37 \mathrm{MeV} / \mathrm{c}$
$R 56=-0.0213 \mathrm{~m}$





## Pitz Calculation of Bunch Compression by HEDA2

- Energy-chirp with negative slope $\left(\Phi_{\text {booster }}=-90^{\circ}\right), I_{\text {main }}=330 \mathrm{~A}$



$P_{\text {mean }}=7.27 \mathrm{MeV} / \mathrm{c}$
R56 $=\mathbf{- 0 . 0 1 0 3 m}$





## ${ }^{\text {PITIT C Calculation of Bunch Compression by HEDA2 }}$

-Summary Plot: bunch length VS booster phase


## ${ }^{\text {PITL }}$ Calculation of Bunch Compression by HEDA2

- Summary Plot: bunch length VS main solenoid current

- Bunch compression by HEDA2 was calculated by using transportation matrices.
- Can we use HEDA2 as a bunch compressor?
- Yes, we can! Some results show the compression but...
- Beam size and booster phase should be optimized further.
- Correction of the calculation script still has to be checked.
- Check correction of the calculation script:
- Different transformation matrices for HEDA2

■ Check with the Zeuthen Chicane

- Find the optimum transverse beam size and booster phase (investigation of the parameter space).
- More macro-particles for the initial beams
- HEDA2 transport simulations with ASTRA and CSRTrack
-S2E simulation, CTR Calculations
-Play with other bunch charges


## BACKUP

## A simple case of 4-bending magnet chicane

- Zeuthen Chicane : a benchmark layout used for CSR calculation comparisons at 2002 ICFA beam dynamics workshop

- Bend magnet length $: L_{B}=0.5 m$
- Drift length B1-B2 and B3-B4(projected) : $\Delta \mathrm{L}=5 \mathrm{~m}$
- Drift length B2-B3 : $\Delta L_{c}=1 \mathrm{~m}$
- Bend radius $\quad: \rho=10.3 \mathrm{~m}$
- Effective total chicane length $\left(L_{T}-\Delta L_{C}\right)=12 \mathrm{~m}$
- Bending angle : $\theta_{0}=2.77 \mathrm{deg}$

Bunch charge : $q=1 \mathrm{nC}$

- Momentum compaction: $\mathrm{R}_{56}=\mathbf{- 2 5} \mathrm{mm}$

Electron energy : $\mathrm{E}=\mathbf{5} \mathrm{GeV}$

- $2^{\text {nd }}$ order momentum compaction : $\mathrm{T}_{566}=38 \mathrm{~mm}$

Initial bunch length : 0.2 mm

- Total projected length of chicane : $L_{T}=13 \mathrm{~m}$

Final bunch length : 0.02 mm

## PITZ Matrix Transportation for Dipole Magnet and Drift Spaces

### 1.3 Matrix Transportation for Dipole Magnet and Drift Spaces

Matrix transportation for the particle travels through the dipole magnet from the initial position $\left(S_{i}\right)$ to the final position $\left(S_{f}\right)$ as shown in Fig. 1 can be written as

$$
\begin{equation*}
M=M_{L_{\text {out }}} M_{D} M_{L_{\text {tn }}} \tag{14}
\end{equation*}
$$

where $M_{L_{\text {tn }}}$ and $M_{L_{\text {out }}}$ are the transport matrices for the drift spaces before and after the dipole magnet. The non-zero matrix elements in Eq. (14) are

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$R_{11}=\cos \alpha\left[1+\frac{L_{\text {out }}}{\rho}\left(\tan \beta_{\text {in }}+\tan \beta_{\text {out }}\right)\right]+\sin \alpha\left[\tan \beta_{\text {in }}+\frac{L_{\text {oun }}}{\rho}\left(\tan \beta_{\text {in }} \tan \beta_{\text {out }}-1\right)\right]$,

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$R_{11}=\cos \alpha\left[1+\frac{L_{\text {out }}}{\rho}\left(\tan \beta_{\text {in }}+\tan \beta_{\text {out }}\right)\right]+\sin \alpha\left[\tan \beta_{\text {in }}+\frac{L_{\text {oun }}}{\rho}\left(\tan \beta_{\text {in }} \tan \beta_{\text {out }}-1\right)\right]$,
$R_{12}=L_{\text {in }} \tan \beta_{\text {in }}\left[1+\frac{L_{\text {out }} \tan \beta_{\text {out }}}{\rho}\right]+\cos \alpha\left[L_{\text {in }}+L_{\text {out }}+\frac{L_{\text {in }} L_{\text {out }}^{p}}{\rho}\left(\tan \beta_{\text {in }}+\tan \beta_{\text {out }}\right)\right]+$
$R_{12}=L_{\text {in }} \tan \beta_{\text {in }}\left[1+\frac{L_{\text {out }} \tan \beta_{\text {out }}}{\rho}\right]+\cos \alpha\left[L_{\text {in }}+L_{\text {out }}+\frac{L_{\text {in }} L_{\text {out }}^{p}}{\rho}\left(\tan \beta_{\text {in }}+\tan \beta_{\text {out }}\right)\right]+$
$\sin \left[\alpha+L_{\text {out }}\left(\tan \beta_{\text {out }}-\frac{L_{\text {tu }}}{\rho}\right)\right]$,
$\sin \left[\alpha+L_{\text {out }}\left(\tan \beta_{\text {out }}-\frac{L_{\text {tu }}}{\rho}\right)\right]$,
$R_{16}=\rho(1-\cos \alpha)+L_{\text {out }}\left[\sin \alpha+\tan \beta_{\text {out }}(1-\cos \alpha)\right]$,
$R_{16}=\rho(1-\cos \alpha)+L_{\text {out }}\left[\sin \alpha+\tan \beta_{\text {out }}(1-\cos \alpha)\right]$,
$R_{21}=\frac{\cos \alpha}{\rho}\left(\tan \beta_{\text {in }}+\tan \beta_{\text {out }}\right)+\frac{\sin \alpha}{\rho}\left(\tan \beta_{\text {in }} \tan \beta_{\text {out }}-1\right)$,
$R_{21}=\frac{\cos \alpha}{\rho}\left(\tan \beta_{\text {in }}+\tan \beta_{\text {out }}\right)+\frac{\sin \alpha}{\rho}\left(\tan \beta_{\text {in }} \tan \beta_{\text {out }}-1\right)$,
$R_{22}=\stackrel{\rho}{\rho} \alpha\left[1+\frac{L_{\text {in }}}{\rho}\left(\tan \beta_{\text {in }}+\tan \stackrel{\beta}{\beta}_{\text {out }}\right)\right]+\sin \alpha\left[\tan \beta_{\text {out }}\left(1+\frac{L_{\text {in }} \tan \beta_{\text {in }}}{\rho}\right)-\frac{L_{\text {in }}}{\rho}\right]$,
$R_{22}=\stackrel{\rho}{\rho} \alpha\left[1+\frac{L_{\text {in }}}{\rho}\left(\tan \beta_{\text {in }}+\tan \stackrel{\beta}{\beta}_{\text {out }}\right)\right]+\sin \alpha\left[\tan \beta_{\text {out }}\left(1+\frac{L_{\text {in }} \tan \beta_{\text {in }}}{\rho}\right)-\frac{L_{\text {in }}}{\rho}\right]$,
$R_{26}=\sin \alpha+\tan \beta_{\text {out }}(1-\cos \alpha)$,
$R_{26}=\sin \alpha+\tan \beta_{\text {out }}(1-\cos \alpha)$,
$R_{33}=1-\alpha \tan \left(\beta_{\text {in }}-\psi_{\text {in }}\right)+\frac{L_{\text {oun }}}{\rho}\left[-\tan \left(\beta_{\text {in }}-\psi_{\text {in }}\right)-\tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)\right]+$
$R_{33}=1-\alpha \tan \left(\beta_{\text {in }}-\psi_{\text {in }}\right)+\frac{L_{\text {oun }}}{\rho}\left[-\tan \left(\beta_{\text {in }}-\psi_{\text {in }}\right)-\tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)\right]+$
$\frac{L_{\text {out }} \alpha}{\rho} \tan \left(\beta_{\text {in }}-\psi_{\text {in }}\right) \tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)$,
$\frac{L_{\text {out }} \alpha}{\rho} \tan \left(\beta_{\text {in }}-\psi_{\text {in }}\right) \tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)$,
$R_{34}={ }_{L_{\text {in }}}^{\rho}+L_{\text {out }}+\rho\left[\alpha-L_{\text {in }} \tan \left(\beta_{\text {in }}-\psi_{\text {in }}\right)-L_{\text {out }} \tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)\right]+\frac{L_{\text {in }} L_{\text {out }}}{\rho}$
$R_{34}={ }_{L_{\text {in }}}^{\rho}+L_{\text {out }}+\rho\left[\alpha-L_{\text {in }} \tan \left(\beta_{\text {in }}-\psi_{\text {in }}\right)-L_{\text {out }} \tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)\right]+\frac{L_{\text {in }} L_{\text {out }}}{\rho}$
$\left[-\tan \left(\beta_{\text {in }}-\psi_{\text {in }}\right)+\tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)+\alpha L_{\text {out }} \tan \left(\beta_{\text {in }}-\psi_{\text {in }}\right) \tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)\right]$,
$\left[-\tan \left(\beta_{\text {in }}-\psi_{\text {in }}\right)+\tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)+\alpha L_{\text {out }} \tan \left(\beta_{\text {in }}-\psi_{\text {in }}\right) \tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)\right]$,
$R_{43}=-\frac{\tan \left(\beta_{t n}-\psi_{t n}\right)}{\rho}-\frac{\tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)}{\rho}+\frac{\alpha \tan \left(\beta_{i n}-\psi_{\text {in }}\right) \tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)}{\rho}$,
$R_{43}=-\frac{\tan \left(\beta_{t n}-\psi_{t n}\right)}{\rho}-\frac{\tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)}{\rho}+\frac{\alpha \tan \left(\beta_{i n}-\psi_{\text {in }}\right) \tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)}{\rho}$,
$R_{44}=1-\alpha \tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)^{\rho}+\frac{L_{\text {in }}}{\rho}\left[-\tan \left(\beta_{\text {in }}-\psi_{\text {in }}\right)^{\rho}-\tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)\right]+$
$R_{44}=1-\alpha \tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)^{\rho}+\frac{L_{\text {in }}}{\rho}\left[-\tan \left(\beta_{\text {in }}-\psi_{\text {in }}\right)^{\rho}-\tan \left(\beta_{\text {out }}-\psi_{\text {out }}\right)\right]+$
$\frac{L_{\text {in }} \alpha}{\rho} \tan \left(\beta_{\text {in }}-\psi_{\text {in }}\right) \tan \left(\beta_{\text {out }}^{\rho}-\psi_{\text {out }}\right)$,
$\frac{L_{\text {in }} \alpha}{\rho} \tan \left(\beta_{\text {in }}-\psi_{\text {in }}\right) \tan \left(\beta_{\text {out }}^{\rho}-\psi_{\text {out }}\right)$,
$R_{51}=\sin \alpha+\tan \beta_{\text {in }}(1-\cos \alpha)$,
$R_{51}=\sin \alpha+\tan \beta_{\text {in }}(1-\cos \alpha)$,
$R_{52}=L_{\text {in }}\left[\sin \alpha+\tan \beta_{\text {in }}(1-\cos \alpha)\right]+\rho(1-\cos \alpha)$,
$R_{52}=L_{\text {in }}\left[\sin \alpha+\tan \beta_{\text {in }}(1-\cos \alpha)\right]+\rho(1-\cos \alpha)$,
$R_{55}=1$,
$R_{55}=1$,
$R_{56}=\frac{L_{i n}+L_{\text {gut }}+\rho \alpha}{\gamma^{2}}-\rho(\alpha-\sin \alpha)$,
$R_{56}=\frac{L_{i n}+L_{\text {gut }}+\rho \alpha}{\gamma^{2}}-\rho(\alpha-\sin \alpha)$,
$R_{66}=1$.

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$R_{66}=1$.

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Ref: S.Rimjaem, "Optimization of HEDA2 Spectrometer Using Matrix Transportation", PITZ Note, 24.04.2009

$$
\ell(s)=R_{51} x(0)+R_{52} x^{\prime}(0)+R_{55} \ell(0)+R_{56}\left(\Delta p / p_{0}\right)
$$

