Estimations of the beam-plasma instability at PITZ

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(November-December 2015)

PITZ K&K seminar 08.01.2016

Beam-plasma instability: linear theory

Ref.(e.g.): Principles of Plasma Electrodynamics. By A. F. Alexandrov, L. S. Bogdankevich, A. A. Rukhadze. Springer-Verlag. Berlin, Heidelberg, New York, Tokio. 1984. Springer Series in Electrophysics, Volume 9

with $\gamma_\alpha = (1 - u_\alpha^2/c^2)^{-1/2}$; $\boldsymbol{u}_\alpha$ is the directed velocity of the particle species α and $\Omega_\alpha = e_\alpha B_0/(m_\alpha c)$ is the Larmor frequency.

Substituting (6.3.1) into (6.1.1) the oscillation spectrum of any multi-beam plasma with negligible thermal motion of the particles can be derived, especially that of a monoenergetic electron beam of small density penetrating the cold plasma. Note that the injected electron beam induces charges and currents in the plasma which neutralize the charge and the current of the beam (Exercise 6.5.1). The directed velocity of the plasma electrons forming the return current which neutralizes the beam current is small: $u_\mathrm{p} \approx u_\mathrm{b} N_\mathrm{b}/N_\mathrm{p} \ll u_\mathrm{b} \equiv u$. It can be neglected when the plasma density N_p greatly exceeds the electron beam density N_b. Therefore, (6.3.1) is applicable.

Substituting (6.3.1) into (6.1.1) and neglecting the beam contributions reproduces the dispersion equation of the cold magneto-active plasma, thoroughly analyzed in Sect. 5.2. The account of the beam terms provides small corrections to the spectra obtained there, when the frequency is far from the following resonances. It is easily seen from (6.3.1) that these contributions to the dielectric tensor become infinite under the conditions

$$\omega = k_z u \,, \quad \omega = k_z u \mp \Omega_\mathrm{e}/\gamma \,. \tag{6.3.2}$$

When the first equality called the *condition for the Cherenkov resonance* is satisfied, the beam contributes a second-order pole, and under the second condition called the *condition for the cyclotron (Doppler) resonance* it contributes a first-order pole. Thus, the interaction of a straight beam with the plasma is strongest when the condition for the Cherenkov resonance is satisfied.

6.3.1 Interaction of a Straight Electron Beam with Cold Isotropic Plasma

We begin the analysis of the interaction between a monoenergetic straight electron beam and the cold plasma with the simplest case without external magnetic field. For $\omega_\alpha' \gg \Omega_\alpha$ (6.2.1) separates into two equations:

$$k^2c^2 - \omega^2\left(1 - \frac{\omega_\mathrm{pe}^2}{\omega^2} - \frac{\omega_\mathrm{b}^2}{\gamma\omega^2}\right) = 0\,,$$

$$(k^2c^2 - \omega^2 + \omega_\mathrm{pe}^2 + \omega_\mathrm{b}^2\gamma^{-1}) \times \left(1 - \frac{\omega_\mathrm{pe}^2}{\omega^2} - \frac{\omega_\mathrm{b}^2}{\gamma^3(\omega - k_z u)^2}\right) - \frac{k_\perp^2 u^2}{\omega^2}\,\frac{\omega_\mathrm{pe}^2\omega_\mathrm{b}^2}{\gamma(\omega - k_z u)^2} = 0\,. \tag{6.3.3}$$

Here ω_pe and ω_b are the Langmuir frequencies of the bulk electrons and of the beam electrons, respectively. We have neglected the ion terms in the derivation of (6.3.3), i.e., we consider only the interaction of the electron beam with the high-frequency plasma oscillations.

The first equation of (6.3.3) describes stable oscillations with the frequency

$$\omega^2 = \omega_\mathrm{pe}^2 + \omega_\mathrm{b}^2\gamma^{-1} + k^2c^2\,. \tag{6.3.4}$$

The contribution of the beam electrons is negligibly small due to their low density. One can easily understand why these oscillations are stable. The electric field of these longitudinal waves is oriented along the y-axis. Thus the exchange of energy with the beam electrons is impossible:

$$\boldsymbol{E}\cdot\boldsymbol{u} = 0\,. \tag{6.3.5}$$

The second equation of (6.3.3) describes a longitudinal-transverse wave with nonzero field components E_x and E_z. Here $\boldsymbol{E}\cdot\boldsymbol{u} \neq 0$ and the field can affect the beam electrons. The beam can be decelerated by the field, thus transferring part of its energy to the wave. The wave emerging from an initial fluctuation increases with time and the system appears unstable. As already noted the beam terms are most significant in the frequency range of the Cherenkov resonance. Therefore, the corresponding solution of (6.3.3) will be

$$\omega = k_z u + \delta\,,$$

with $|\delta| \ll \omega$. As a result we have for $\omega^2 \approx k_z^2u^2 \neq \omega_\mathrm{pe}^2$

$$\delta^2 = \frac{\omega_\mathrm{b}^2\gamma^{-3}}{1 - \dfrac{\omega_\mathrm{pe}^2}{k_z^2u^2}}\left(1 + \frac{k_\perp^2}{k_z^2}\,\frac{\omega_\mathrm{pe}^2\gamma^2}{k^2c^2 - k_z^2u^2 + \omega_\mathrm{pe}^2}\right) \tag{6.3.6}$$

and for $\omega^2 \approx k_z^2u^2 \approx \omega_\mathrm{pe}^2$:

$$\boxed{\delta_{1,2} = \frac{-1 \pm \mathrm{i}\sqrt{3}}{2}\,\omega_\mathrm{pe}\left(\frac{N_\mathrm{b}}{2N_\mathrm{p}}\,\frac{k_\perp^2 + k_z^2\gamma^{-2}}{k^2\gamma}\right)^{1/3}}\,,$$

$$\delta_3 = \omega_\mathrm{pe}\left(\frac{N_\mathrm{b}}{2N_\mathrm{p}}\,\frac{k_\perp^2 + k_z^2\gamma^{-2}}{k^2\gamma}\right)^{1/3}. \tag{6.3.7}$$

Equations (6.3.6, 7) show that the oscillations with the frequency $\omega \approx k_z u$ are unstable ($\mathrm{Im}\{\omega\} = \mathrm{Im}\{\delta\} > 0$). They increase with time if $k_z u \ll \omega_\mathrm{pe}$. Far from the plasma frequency, for $\omega \approx k_z u \neq \omega_\mathrm{pe}$, the increment is $\mathrm{Im}\{\delta\} \sim \omega_\mathrm{pe}(N_\mathrm{b}/N_\mathrm{p})^{1/2}$. If, however, $\omega \approx k_z u = \omega_\mathrm{pe}$ the increment is much larger: $\mathrm{Im}\{\delta\} \sim \omega_\mathrm{pe}(N_\mathrm{b}/N_\mathrm{p})^{1/3}$. This is understandable since a resonance occurs when the beam velocity coincides with the phase velocity of the plasma oscillations. Actually, the relativistic beam excites longitudinal waves

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$$\frac{\mathrm{Im}\,\delta_2}{\omega_p} = \frac{\sqrt{3}}{2}\left(\frac{N_b}{2N_p}\frac{1}{\gamma}\right)^{1/3}\cdot G(\gamma,\mu)$$

$\mu = \left(\frac{R}{\beta\delta_{sk}}\right)^2$ - plasma density parameters

$\delta_{sk} = \frac{c}{\omega_p}$ - skin-depth of the plasma

$\omega_p = \sqrt{3\cdot 10^9\cdot N_p[cm^{-3}]}$ – plasma frequency

$N_b = \frac{Q_b/e}{2\pi\sigma_{xy}^2\cdot cT}\cdot\frac{1}{2}$ – beam density

$\rightarrow \perp$ Gaussian

$G(\gamma,\mu) = \left(1-\beta^2\frac{\mu}{1+\mu}\right)^{1/3}$ – geom. pl-b factor

Growth length:

$$L \sim \frac{\beta c}{\mathrm{Im}\,\delta_2} = \frac{\beta\delta_{sk}}{\frac{\sqrt{3}}{2}\left(\frac{N_b}{2N_p}\frac{1}{\gamma}\right)^{1/3}\cdot G(\gamma,\mu)}$$

$\gamma = 41$

$T = 20ps$

$k_\perp \propto \frac{1}{R}$

$R = 2\sigma_{xy}$ -e-beam radius

xyRMS [mm]			Normalized charge density
6.3MWg	50pC	0.105	0.55
	100pC	0.110	1.00
	250pC	0.125	1.93
5MWg	50pC	0.109	0.50
	100pC	0.113	0.94
	250pC	0.126	1.90

Matthias Gross

Plasma related experiments in KW43

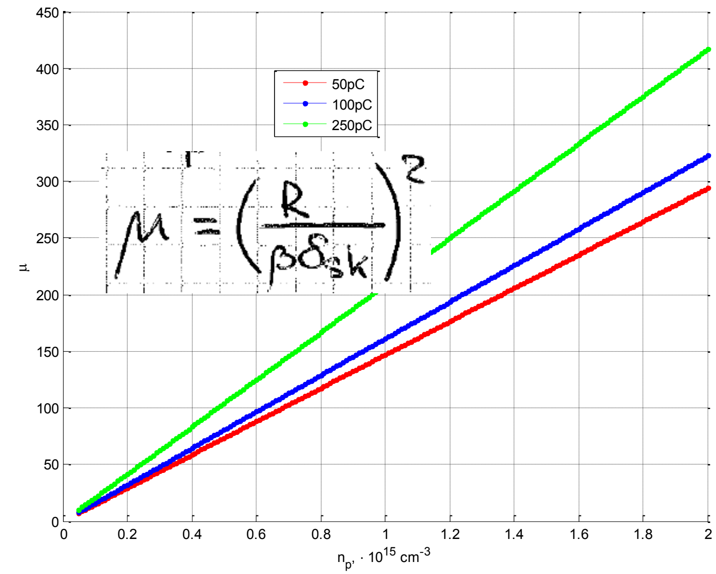
Scattering at 0.9μm PET foil and focusing onto High1.Scr2

Estimations -1

$$N_b = \frac{Q_b/e}{2\pi \sigma_{xy}^2 \cdot cT} \cdot \left(\frac{1}{2}\right)$$

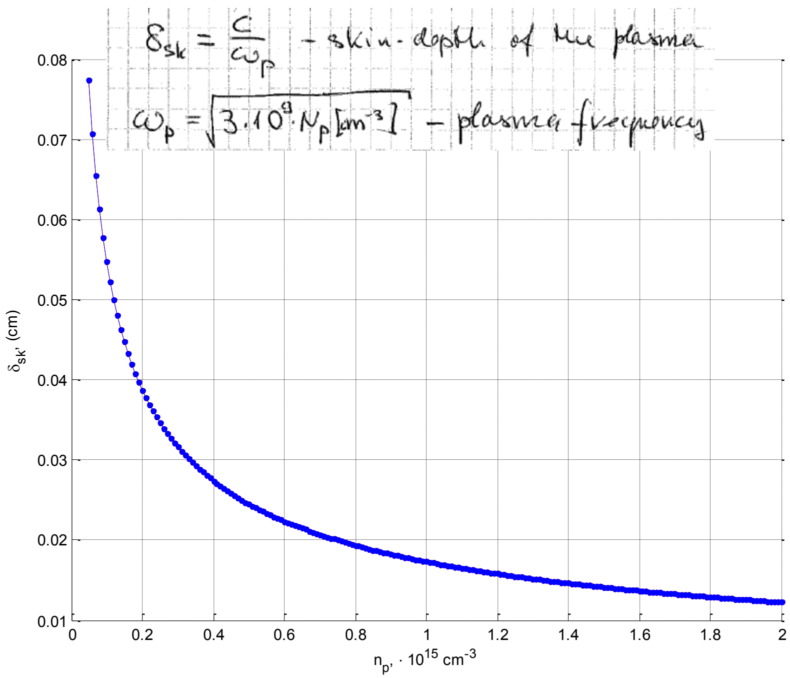
50pC	0.105	$0.38 \cdot 10^{12}$ cm^{-3}
100pC	0.110	$0.69 \cdot 10^{12}$ cm^{-3}
250pC	0.125	$1.3 \cdot 10^{12}$ cm^{-3}

$$\mu = \left(\frac{R}{\beta \delta_{sk}}\right)^2$$

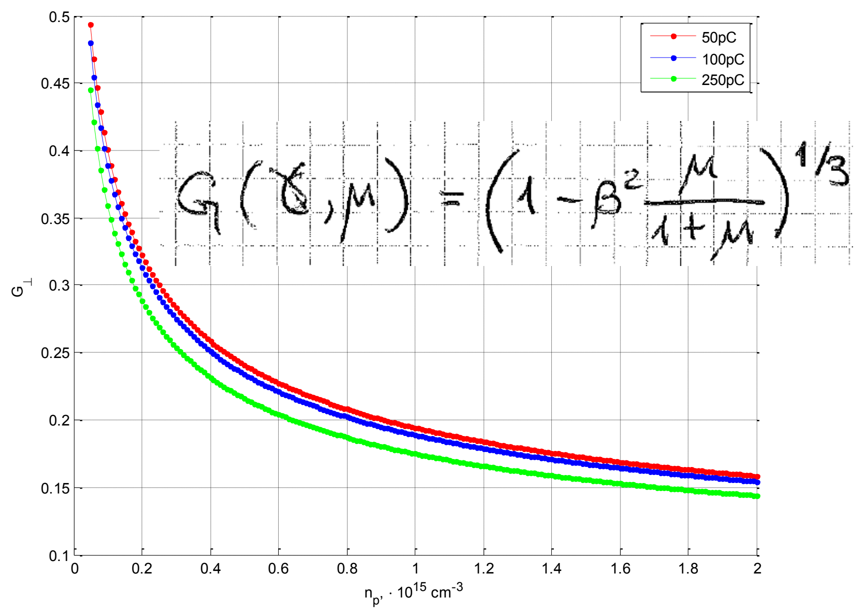


$\delta_{sk} = \frac{c}{\omega_p}$ – skin-depth of the plasma

$\omega_p = \sqrt{3 \cdot 10^9 \cdot N_p [cm^{-3}]}$ – plasma frequency



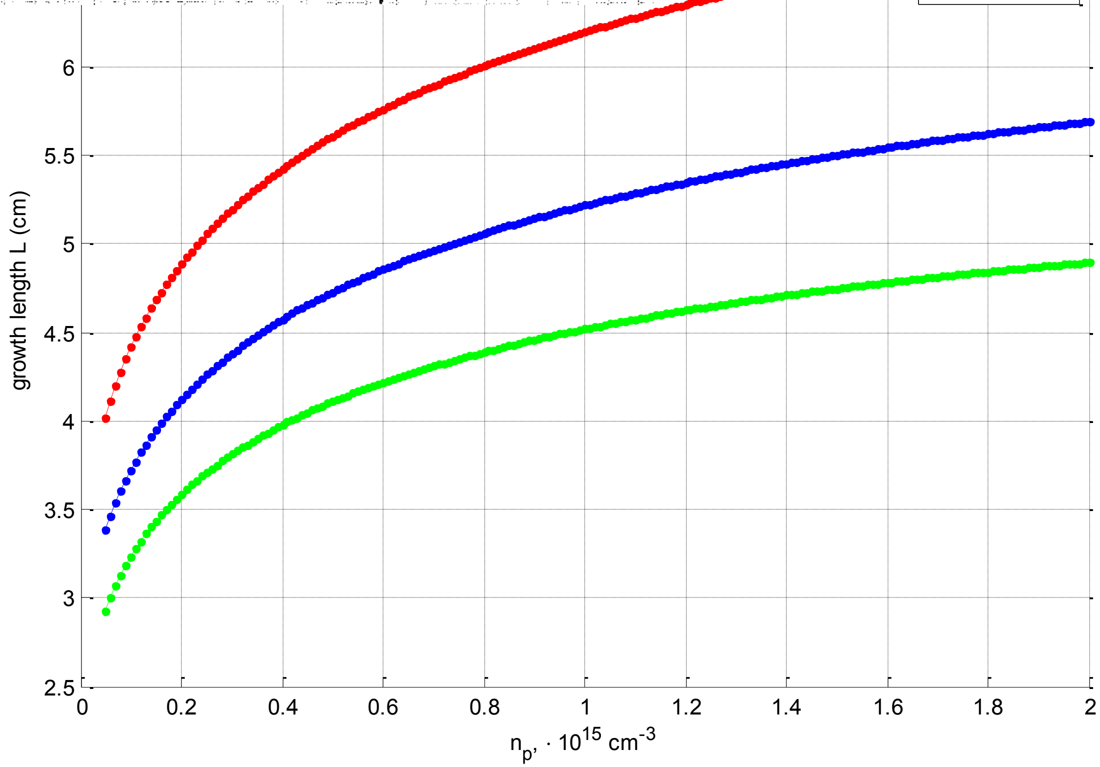
$$G(\gamma, \mu) = \left(1 - \beta^2 \frac{\mu}{1+\mu}\right)^{1/3}$$



Estimations -2

Growth length:

$$L \sim \frac{\beta c}{\Im m\, \delta_2} = \frac{\beta \delta_{sk}}{\frac{\sqrt{3}}{2}\left(\frac{N_b}{2N_p}\frac{1}{\gamma}\right)^{1/3} G(\gamma,\mu)}$$



NB:

These derivations are done under assumptions:

- Cold infinite plasma
- Monenergistic infinite electron beam with homogeneous transverse density

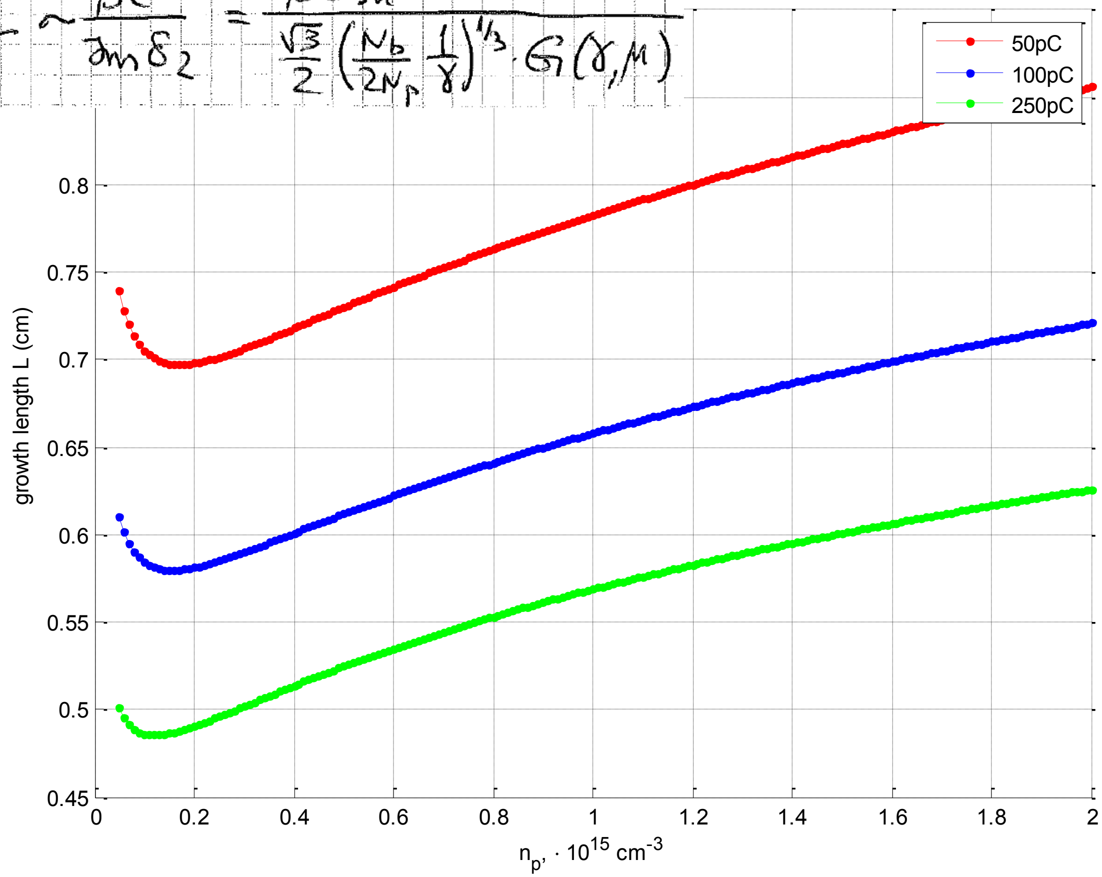
➔ More real modeling should result in L increase (weaker instability)

- Growth length dependence on Np is interesting (very strong impacted by the geometric factor G), instability increment → relative growth rate, the field strength should be larger for higher plasma densities
- L~5cm → too small (from the FEL experience -> interaction length ~10*L)
- MK: I hope I did not make mistake in the calculations (if there is – please let me know). I have not also found any theoretical contradiction

Estimations -2a: if e-beam would be smaller?

Growth length:

$$L \sim \frac{\beta c}{\operatorname{Im} \delta_2} = \frac{\beta \delta_{sk}}{\frac{\sqrt{3}}{2}\left(\frac{N_b}{2N_p}\cdot\frac{1}{\gamma}\right)^{1/3}\cdot G(\gamma,\mu)}$$



50pC	0.105	
100pC	0.110	
250pC	0.125	

}/5

$9.4*10^{12}$ cm^{-3}
$17.1*10^{12}$ cm^{-3}
$33.2*10^{12}$ cm^{-3}

$$N_b = \frac{Q_b/e}{2\pi \sigma_{xy}^2 \cdot cT} \cdot \left(\frac{1}{2}\right)$$