Using pipe with corrugated walls for a sub-terahertz FEL

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SLAC-PUB-16171, December 2014

MK, PITZ K&K seminar 24.07.2015

WAKE IN A ROUND PIPE WITH CORRUGATED WALLS

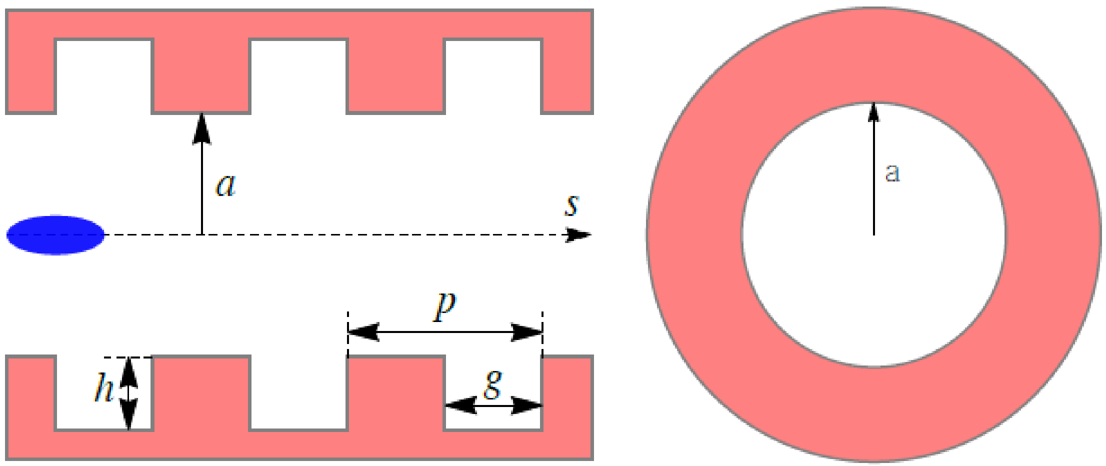


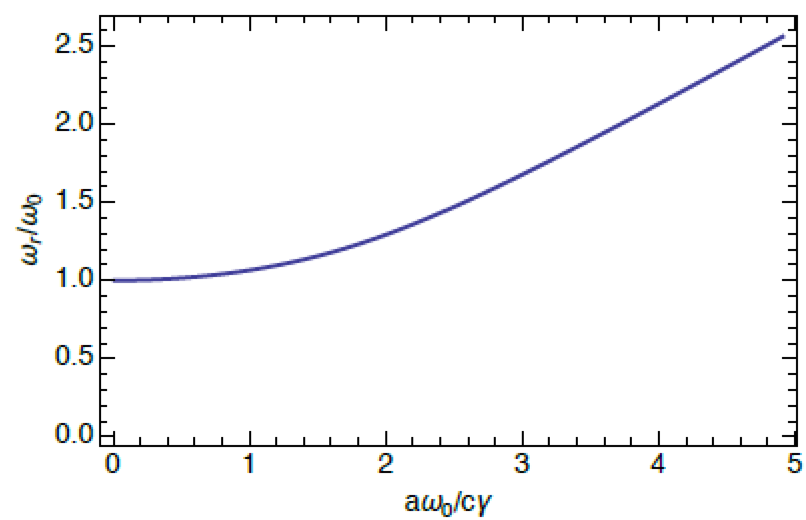
TABLE I. Corrugation and beam parameters

Pipe radius, mm	2
Depth h, μm	50
Period p, μm	40
Gap g, μm	10
Bunch charge, nC	1
Energy, MeV	5
Bunch length, ps	10

We consider a round metallic pipe with inner radius a. Small rectangular corrugations have depth h, period p and gap g, as shown in Fig. 1. In the case when $h, p \ll a$ and $h \gtrsim p$, the fundamental resonant mode with the phase velocity equal to the speed of light, $v_{ph} = c$, has the frequency $\omega_0 = ck_0$ and the group velocity v_{g0}, where [8, 9]

$$k_0 = \left(\frac{2p}{agh}\right)^{1/2}, \qquad 1 - \frac{v_{g0}}{c} = \frac{4gh}{ap}.$$

$$u\frac{I_0(uy_r)}{I_1(uy_r)} = 2y_r. \qquad u = a\omega_0/c\gamma \qquad y_r = \omega_r/\omega_0$$



It is shown in Ref. [11] that small wall corrugations can be treated as a thin material layer with some effective values of the dielectric permeability ϵ and magnetic permittivity μ.

$$\mu = g/p$$

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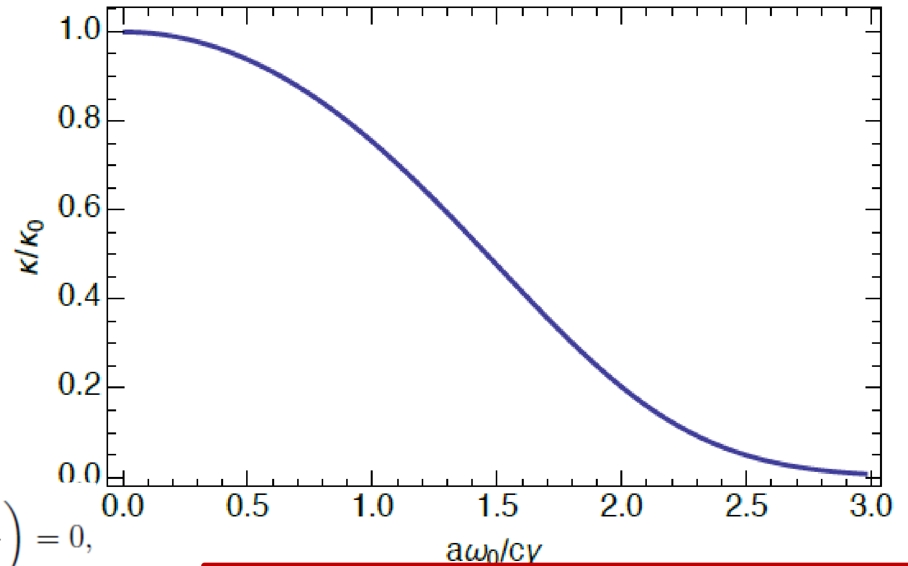
(longitudinal) wake $w(z)$

$$w(s,z) = \begin{cases} 2\varkappa \cos(\omega_r z/c), & \text{for } -s(1-v_g/v) < z < 0 \\ \varkappa, & \text{for } z = 0 \\ 0, & \text{otherwize} \end{cases}$$

$\varkappa$ is the loss factor per unit length

The loss factor $\varkappa_0$ in the limit $\gamma \to \infty$ is given by $\varkappa_0 = \dfrac{2}{a^2}$

normalized loss factor $\varkappa/\varkappa_0$ factor versus parameter $u = ak_0/\gamma$.



1D FEL EQUATIONS

$$\frac{\partial f}{\partial s} - \alpha\eta\frac{\partial f}{\partial z} - \frac{r_0}{\gamma}\frac{\partial f}{\partial \eta}\int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} d\eta' w(s, z-z') f\left(\eta', z', s - v\frac{z'-z}{v-v_g}\right) = 0,$$

dispersion relation $\dfrac{1}{2}\dfrac{(2\rho)^3}{q-i\nu}\displaystyle\int_{-\infty}^{\infty} d\eta \frac{F'(\eta)}{q - i\alpha\eta(\omega_r/ck_w)} = 1$

$$q^2(q-i\nu) = -\frac{i\alpha\omega_r}{2ck_w}(2\rho)^3 \qquad q^3 = i\frac{n_0\kappa r_0}{k_w^2\gamma^3}.$$

where the parameter ρ (an analog of the Pierce parameter [15]) is

$$(2\rho)^3 = \frac{2n_0\kappa c r_0}{k_w\gamma\omega_r}$$

power gain length $\ell = (2\mathrm{Re}\, q_1 k_w)^{-1} = \dfrac{1}{\sqrt{3}}\gamma\left(\kappa k_w \dfrac{I}{I_A}\right)^{-1/3}$

the saturation power $P_{\text{sat}} \approx \rho\gamma mc^2 \dfrac{I}{e}$

the saturation occurs at the distance equal about 10-20 gain length

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$\omega_r/2\pi = 0.34$ THz

the loss factor $\kappa = 0.6(2/a^2) = 2.7$ kV/(pC m)

$\rho = 0.013$

$\ell \approx 7$ cm

$P_{\text{sat}} \approx 6.7$ MW