# Calculations of Transition Radiation from an Relativistic Electron Beam

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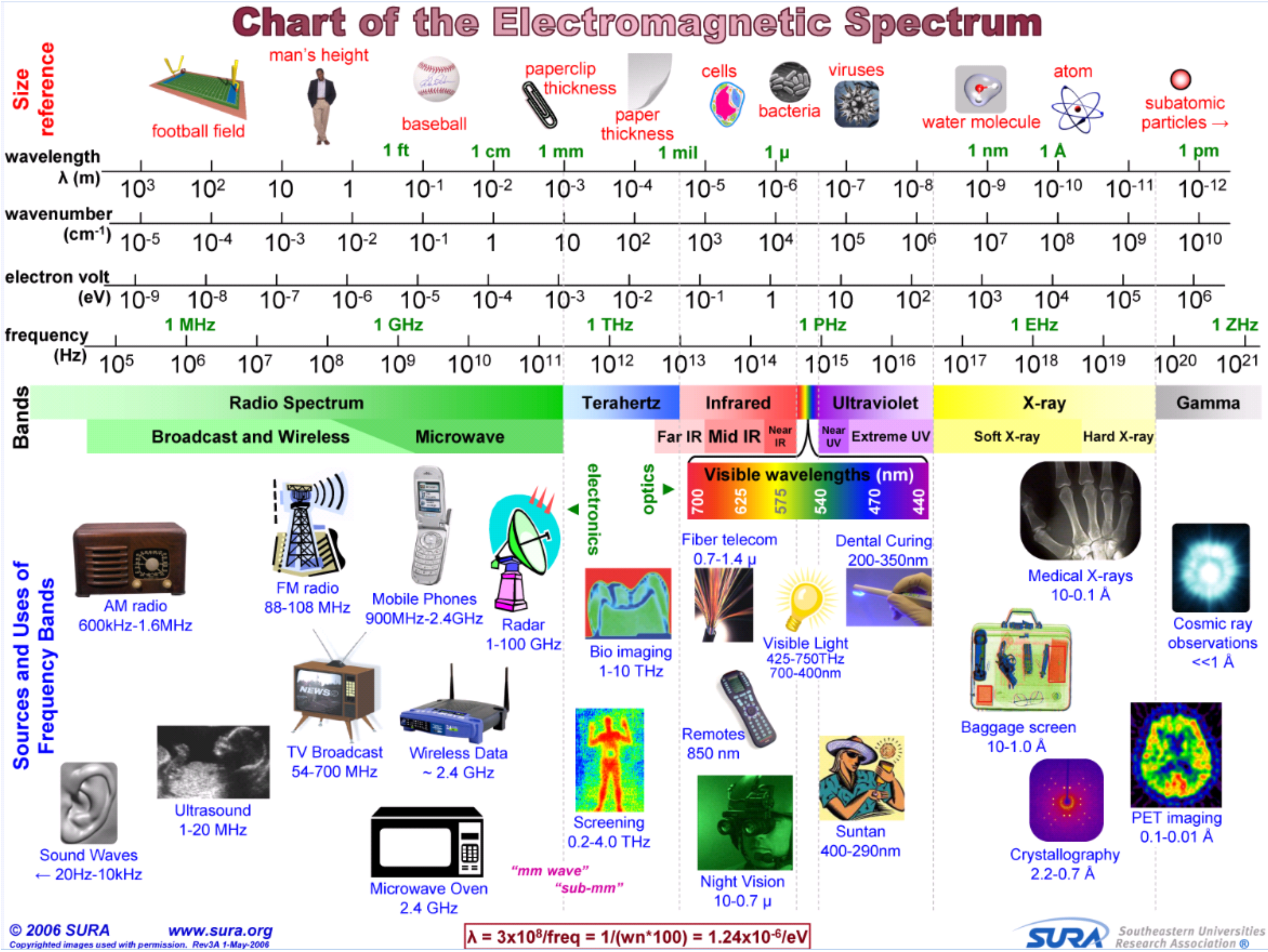
# Introduction: Where is THz spectrum?

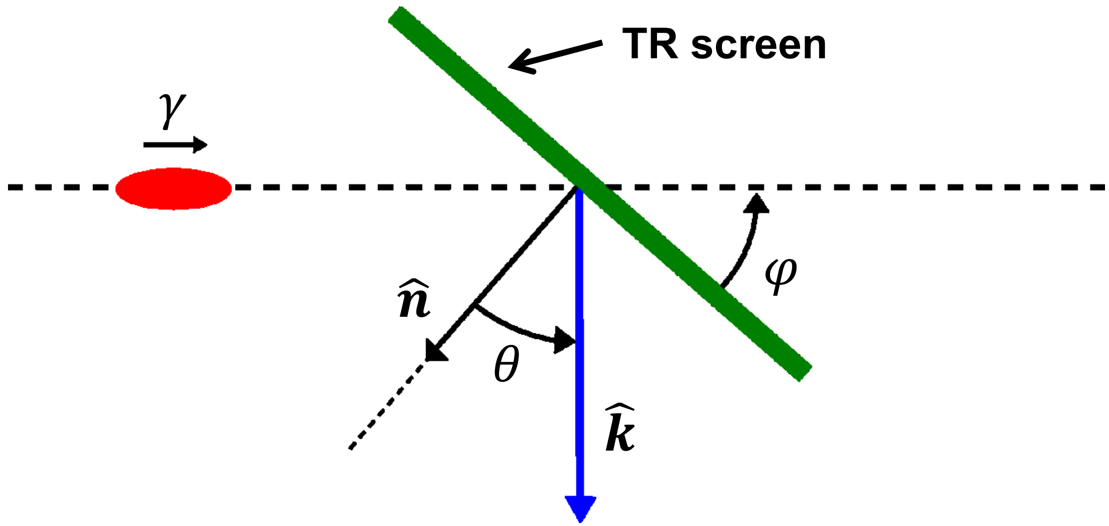




# Introduction: Transition Radiation (TR)



TR is emitted when a relativistic charged particle crosses the boundary between two media of different dielectric properties.




| | |
|---|---|
| $\gamma$ | is Lorentz factor of electron. |
| $\hat{\boldsymbol{n}}$ | is normal vector of the TR screen. |
| $\hat{\boldsymbol{k}}$ | is wave vector of outgoing radiation. |
| $\theta$ | is inclination angle between $\hat{\boldsymbol{k}}$ and $\hat{\boldsymbol{n}}$. |
| $\varphi$ | is angle between screen plane and electron beam axis. |

# TR from a Point Charge



- TR spectral energy density can be calculated from Ginzburg-Frank Formula:

$$\frac{d^2U_{\mathrm{GF}}}{d\omega d\Omega} = \frac{e^2}{4\pi^3\varepsilon_0 c}\frac{\beta^2\sin^2\theta}{(1-\beta^2\cos^2\theta)^2}$$

where

| | |
|---|---|
| $U_{\mathrm{GF}}$ | is radiation energy of backward radiation from Ginzburg-Frank formula. |
| $\omega$ | is angular frequency of radiation |
| $\Omega$ | is solid angle that subtends radiation |
| $\varepsilon_0$ | is electric constant. |
| $\beta$ | is ratio between electron velocity and speed of light. |

- The formula is valid in the following conditions:
  - Normal Incident ($\varphi = \pi/2$)
  - A single relativistic electron
  - Infinite metallic screen of perfect reflectivity
  - Far-field

# TR from a Point Charge (2)



- Generalized Ginzburg-Frank Formula:

$$\frac{d^2 U_{\text{disk}}}{d\omega d\Omega} = \frac{d^2 U_{\text{GF}}}{d\omega d\Omega} [1 - T(\gamma, \omega a, \theta)]^2$$

where $U_{\text{disk}}$ is radiation energy of backward radiation from generalized Ginzburg-Frank formula.

$a$ is radius of the circular TR screen.

$$T(\gamma, \omega a, \theta) = \frac{\omega a}{c\beta\gamma} J_0\left(\frac{\omega a \sin\theta}{c}\right) K_1\left(\frac{\omega a}{c\beta\gamma}\right) + \frac{\omega a \sin\theta}{c\beta^2\gamma^2 \sin\theta} J_1\left(\frac{\omega a \sin\theta}{c}\right) K_0\left(\frac{\omega a}{c\beta\gamma}\right)$$

$J_n, K_n$ are Bessel functions.

- The formula is valid in the following conditions:
  - Normal Incident ($\varphi = \pi/2$)
  - A single relativistic electron
  - Finite circular TR screen with radius $a$
  - Far-field

# TR from an Electron Bunch



- TR spectral energy density from an electron bunch can be calculated from the following formula:

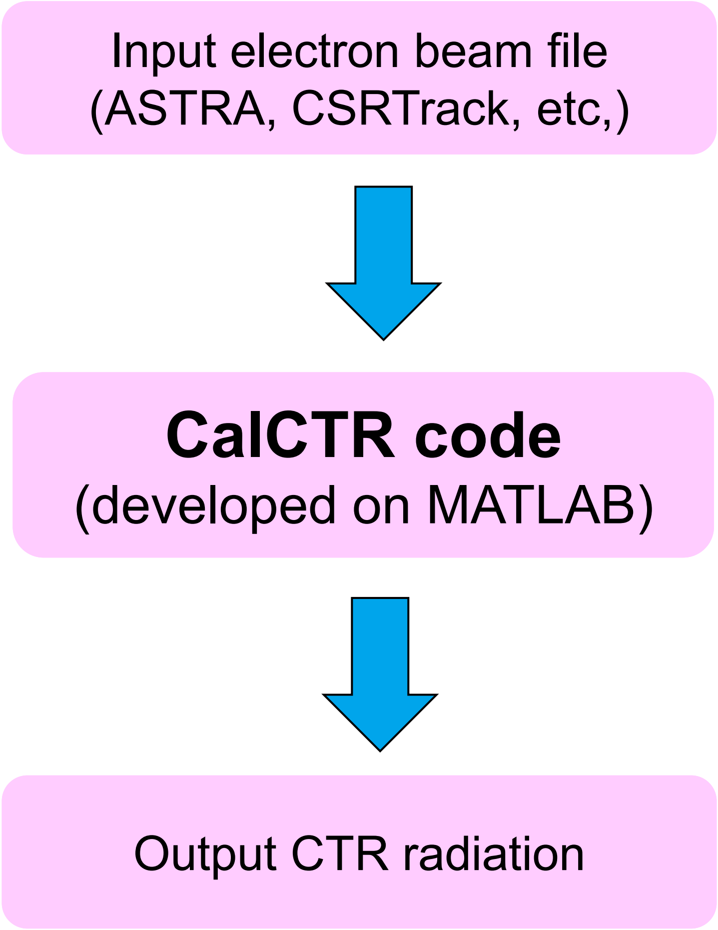
$$\frac{d^2U_{\mathrm{bunch}}}{d\omega d\Omega} = \frac{d^2U_{\mathrm{GF}}}{d\omega d\Omega} N^2 \left|F_{long}(\omega)\right|^2 \cdot \left[\frac{2c}{\omega r_b \sin\theta} J_0\left(\frac{\omega r_b \sin\theta}{c}\right) - \frac{2c\beta\gamma}{\omega r_b} I_0\left(\frac{\omega r_b}{c\beta\gamma}\right) T(\gamma, \omega a, \theta)\right]^2$$

| | | |
|---|---|---|
| where | $U_{\mathrm{disk}}$ | is radiation energy of backward radiation from generalized Ginzburg-Frank formula. |
| | $N$ | is total number of electrons in a bunch. |
| | $r_b$ | is transverse radius of electron bunch. |
| | $\rho_{long}(t)$ | is longitudinal particle density distribution of electron bunch. |
| | $F_{long}(\omega) = \int_{-\infty}^{+\infty} \rho_{long}(t) e^{-i\omega t} dt$ | ← "Form Factor" |
| | $I_n$ | is Bessel function. |

- The formula is valid in the following conditions:
  - Normal Incident ($\varphi = \pi/2$)
  - A relativistic electron bunch with $r_b$ and $\rho_{long}(t)$
  - Finite circular TR screen with radius $a$
  - Far-field

# Algorithm of Numerical Calculations






**Will be presented with more details later**

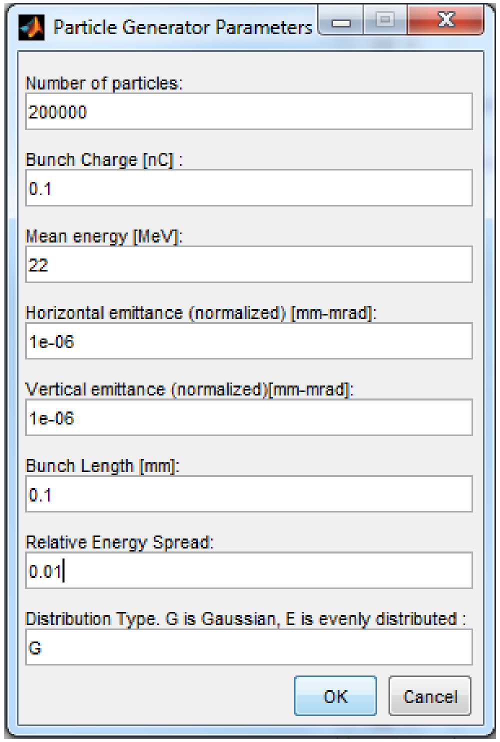
# Example Results of Numerical Calculations



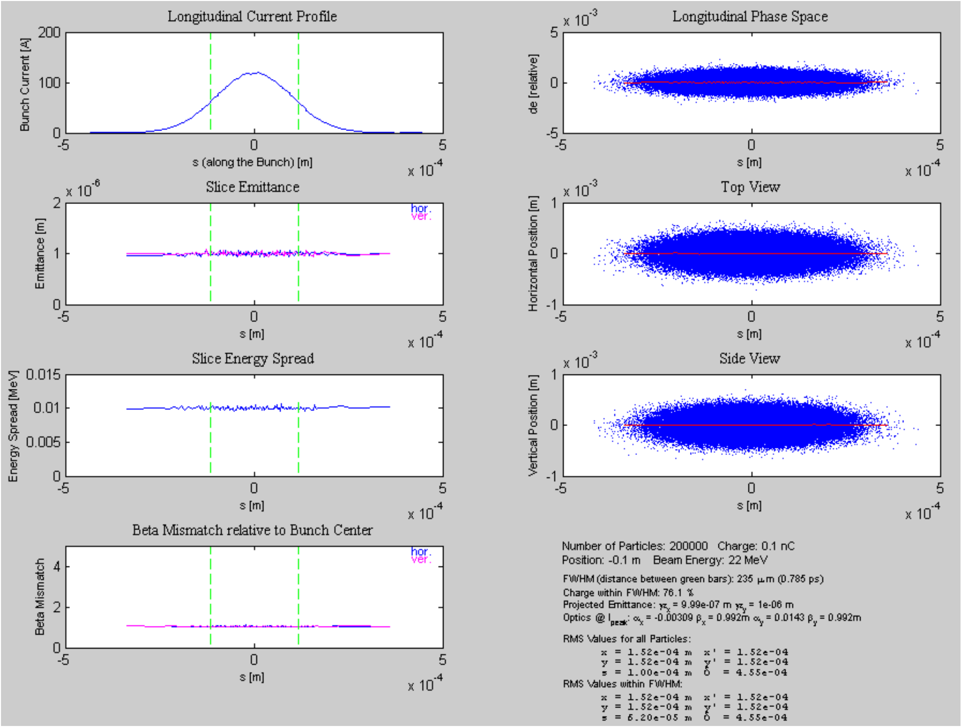
## Preparation of Input Beams

- Generated model Gaussian beams from PSViewer software with various parameters.

### The Input Box




### Example of Generated Beam Profile

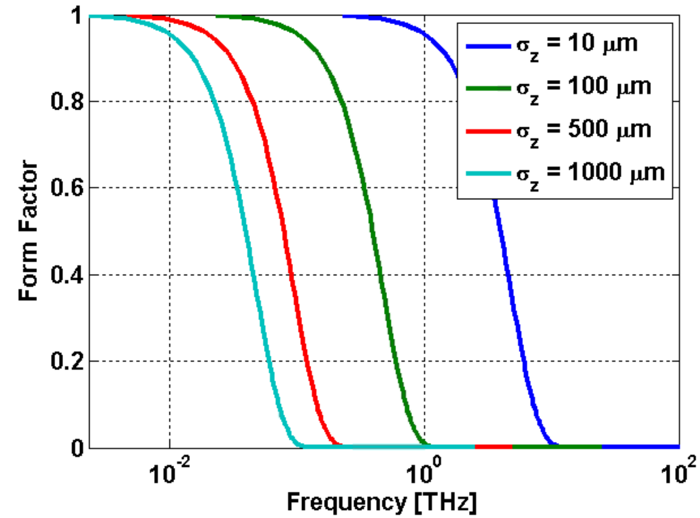


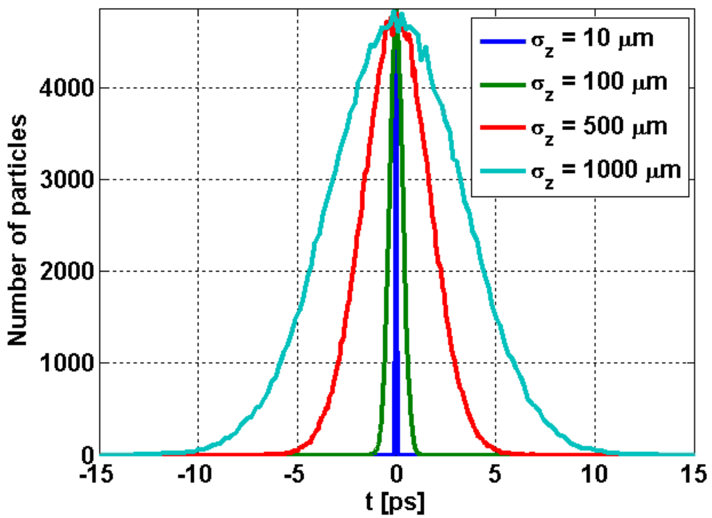
# Example Results of Numerical Calculations (2)

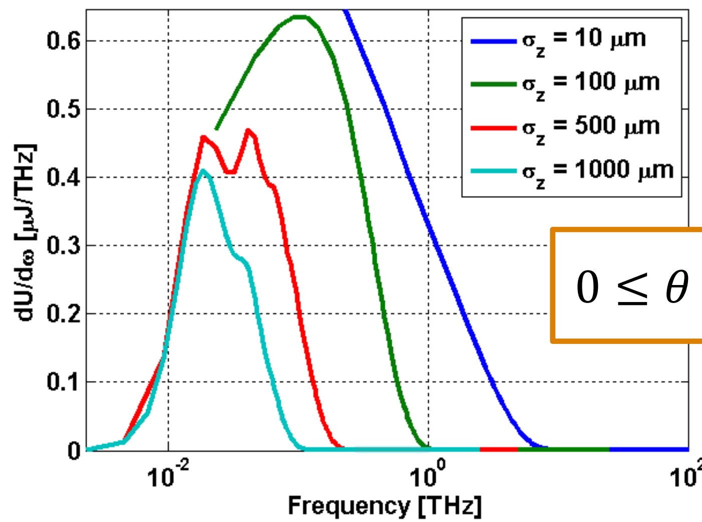


## Effects of Bunch Lengths

| Input Parameter | Value |
|---|---|
| Number of particles | 200k |
| Bunch charge | 0.1 nC |
| Mean energy | 22 MeV |
| Relative energy spread | 0.01 |
| Tr. Emittance | 1 mm mrad |
| Bunch rms length | **10,50,100,1000 μm** |
| Tr. rms size | 0.152 mm |



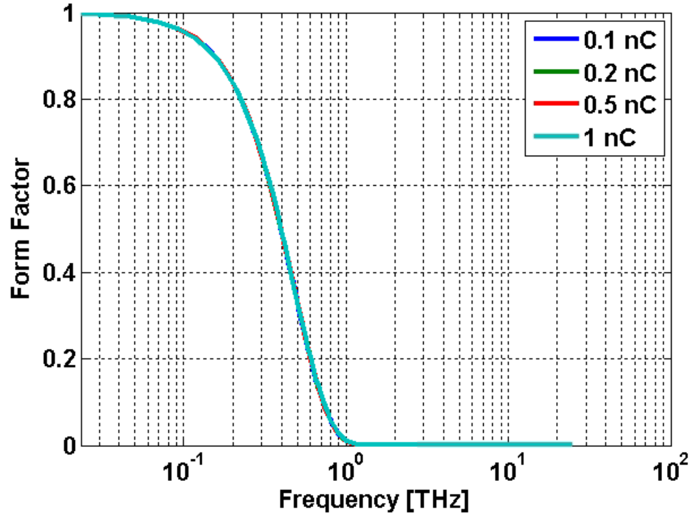

$$0 \le \theta \le \frac{\pi}{2}$$

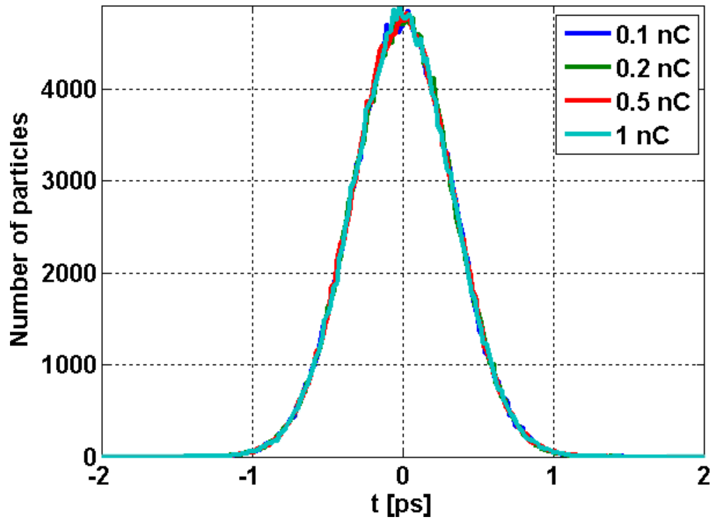
# Example Results of Numerical Calculations (3)

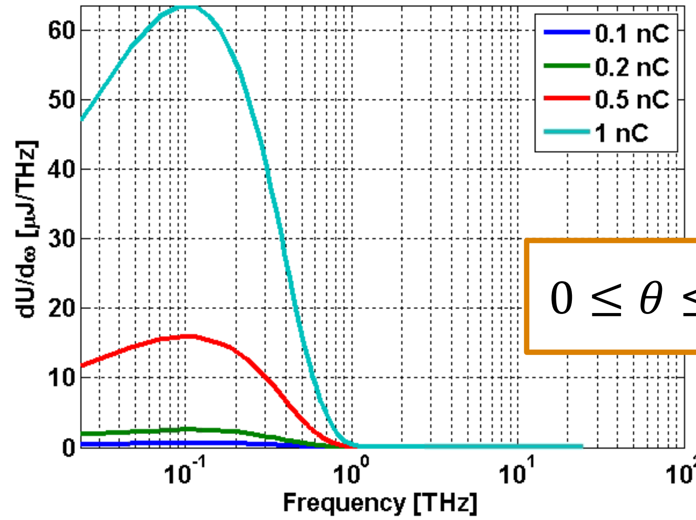


## Effects of Peak Currents

| Input Parameter | Value |
|---|---|
| Number of particles | 200k |
| Bunch charge | **0.1, 0.2, 0.5, 1 nC** |
| Mean energy | 22 MeV |
| Relative energy spread | 0.01 |
| Tr. Emittance | 1 mm mrad |
| Bunch rms length | 100 µm |
| Tr. rms size | 0.152 mm |



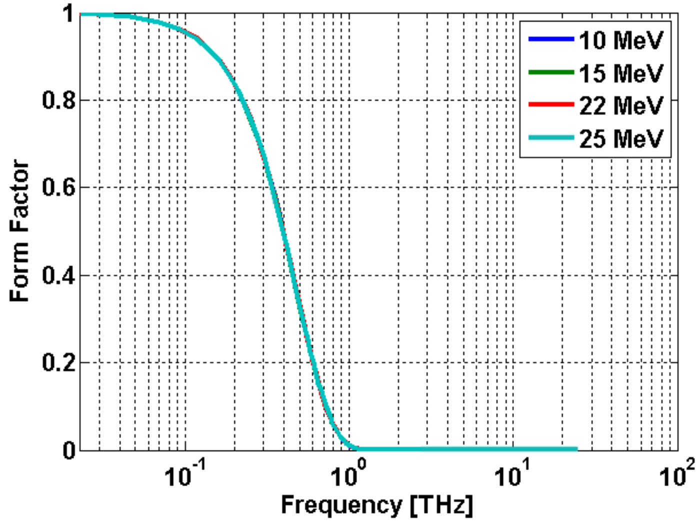

$$0 \leq \theta \leq \frac{\pi}{2}$$

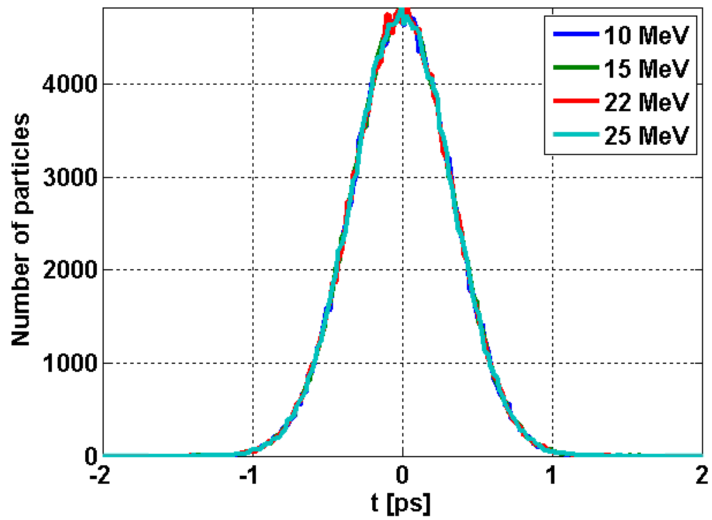
# Example Results of Numerical Calculations (4)

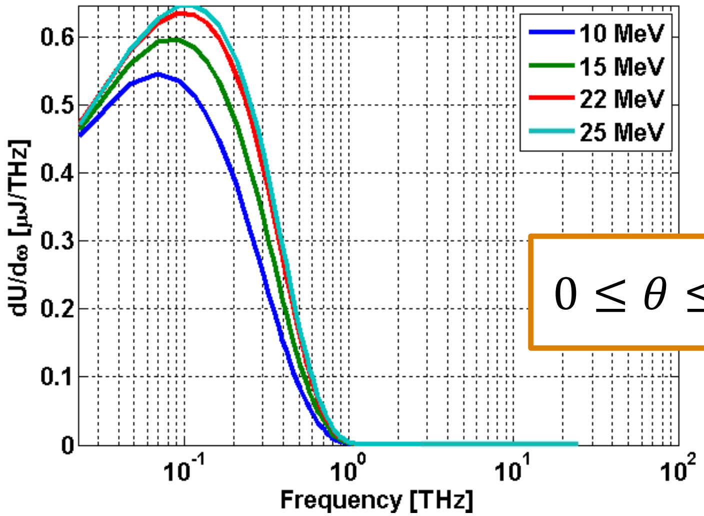


## Effects of Beam Energies

| Input Parameter | Value |
|---|---|
| Number of particles | 200k |
| Bunch charge | 0.1 nC |
| Mean energy | **10, 15, 22, 25 MeV** |
| Relative energy spread | 0.01 |
| Tr. Emittance | 1 mm mrad |
| Bunch rms length | 100 µm |
| Tr. rms size | 0.152 mm |



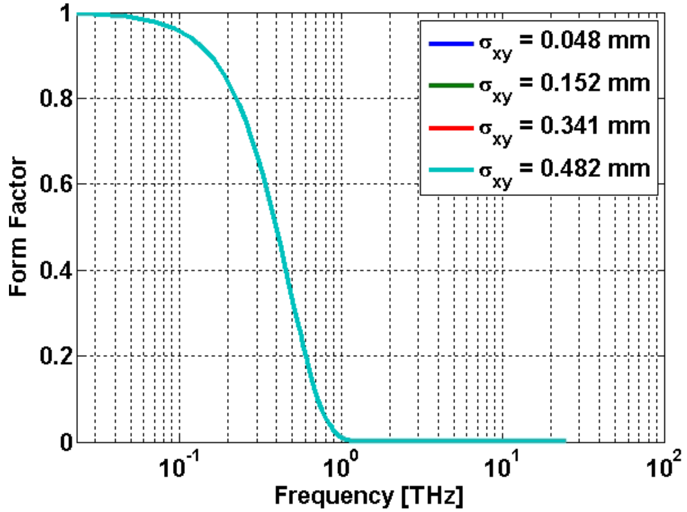

$$0 \leq \theta \leq \frac{\pi}{2}$$

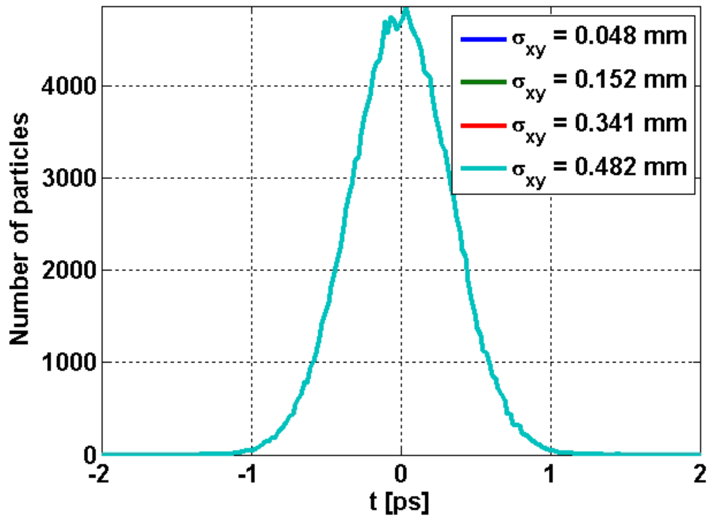
# Example Results of Numerical Calculations (5)

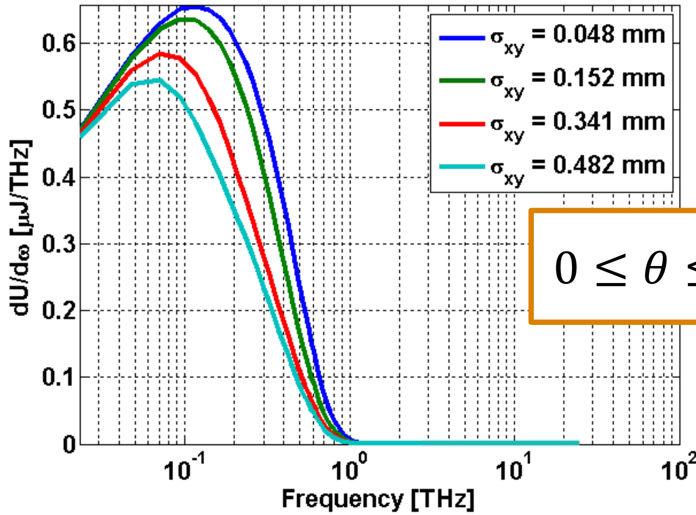


## Effects of Beam Sizes

| Input Parameter | Value |
|---|---|
| Number of particles | 200k |
| Bunch charge | 0.1 nC |
| Mean energy | 22 MeV |
| Relative energy spread | 0.01 |
| Tr. Emittance | 1 mm mrad |
| Bunch rms length | 100 µm |
| Tr. rms size | **0.048, 0.152, 0.341, 0.482 mm** |




$$0 \leq \theta \leq \frac{\pi}{2}$$

# Summary and Outlook



## Summary

- Beta version of CalCTR has been developed.
- Effects of electron beam parameters to the CTR enegy spectrum density were studied.

## Outlook

- Implement in CalCTR:
  - sliced energy
  - slice beam size
  - oblique target screen
  - near-field calculations
  - electric field of THz pulse
- Simulation of velocity bunching (PPS beginning of May)
- Calculation of TR from PITZ Beam (PPS beginning of May)
- Consideration of TR diagnostics (PPS end of May)

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**Thanks for your attentions!**