Space-charge matching of the FODO lattice using the smooth-approximation theory

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Introduction

- Transverse tomography measurements require equal phase-advance steps between the PST screens
- The required optics for these desired dynamics can be calculated by MAD - neglecting space charge!

- The *smooth-approximation theory* correlates the beam dynamics **without** and **with** (linear) space charge in a *uniform focusing channel* (FODO)

Smooth approximation theory

- …there will be a special solution [of the beam envelope equation] where *R(z)* = *a* = const, *R'(z)* = 0 … and hence the beam envelope is a straight line. This special case is known as the *matched beam*…
  [Reiser M., Theory and Design of Charged Particle Beams]
- Associates the external focusing forces with the total defocusing forces including space charge → no-space-charge transport to linear-space-charge transport
- Assumptions:
  - Round beam, $\sigma_0 < 90°$, K-V distribution, …

Smooth approximation theory [matched beam in a uniform focusing lattice]

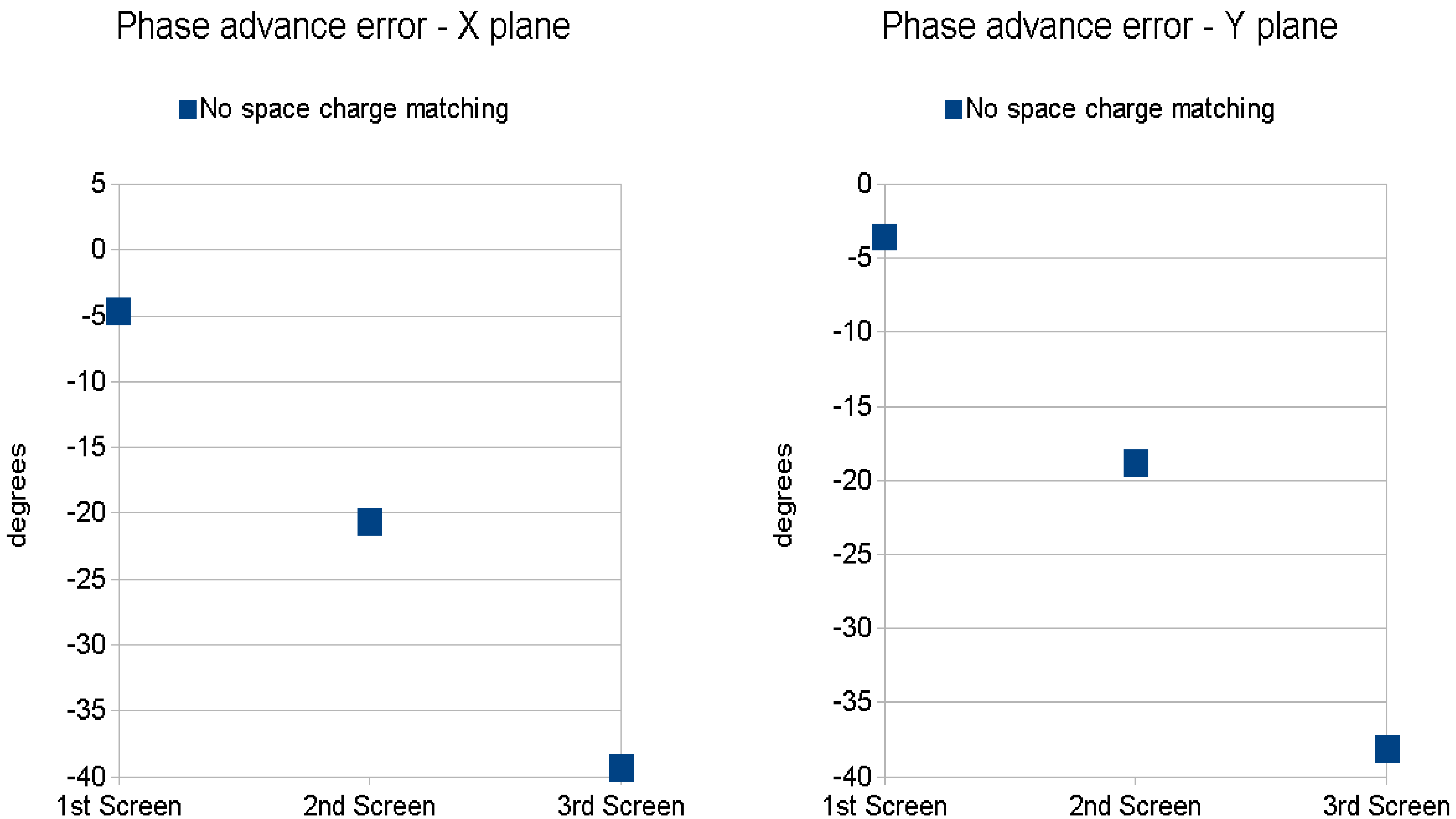
- $\sigma = \frac{S}{\bar{\beta}}$
- $\sigma = \sigma_0\left(\sqrt{1+u^2} - u\right)$
- $u = \frac{KS}{2\sigma_0 \varepsilon}$
- $\sigma_0 = \frac{S}{\bar{\beta}_0}$
- $a_{(0)} = -\frac{1}{2}\frac{d\beta_{(0)}}{dz}$
- σ : phase advance

- S : length of FODO cell
- β/α : Twiss beta/alpha
- Value with index “$_0$” : value without the effect of space charge
- K : generalized perveance
- ε : 4 rms g. emittance

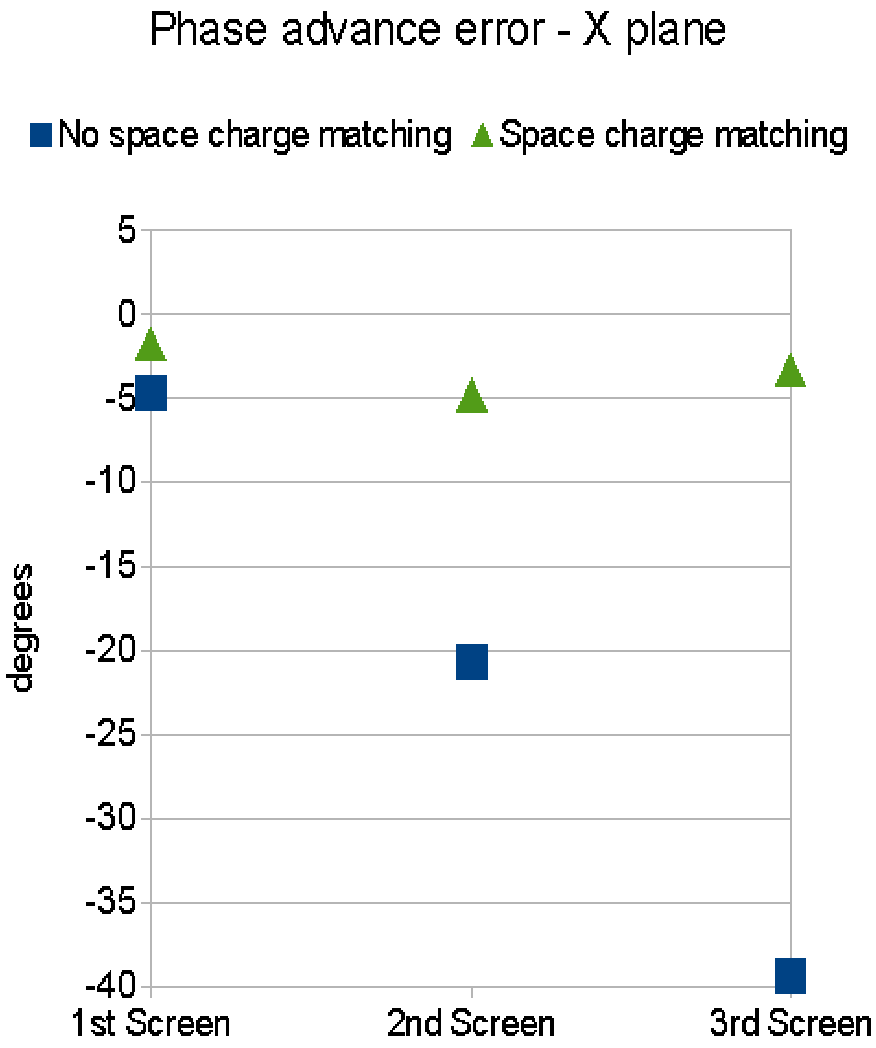
How-To

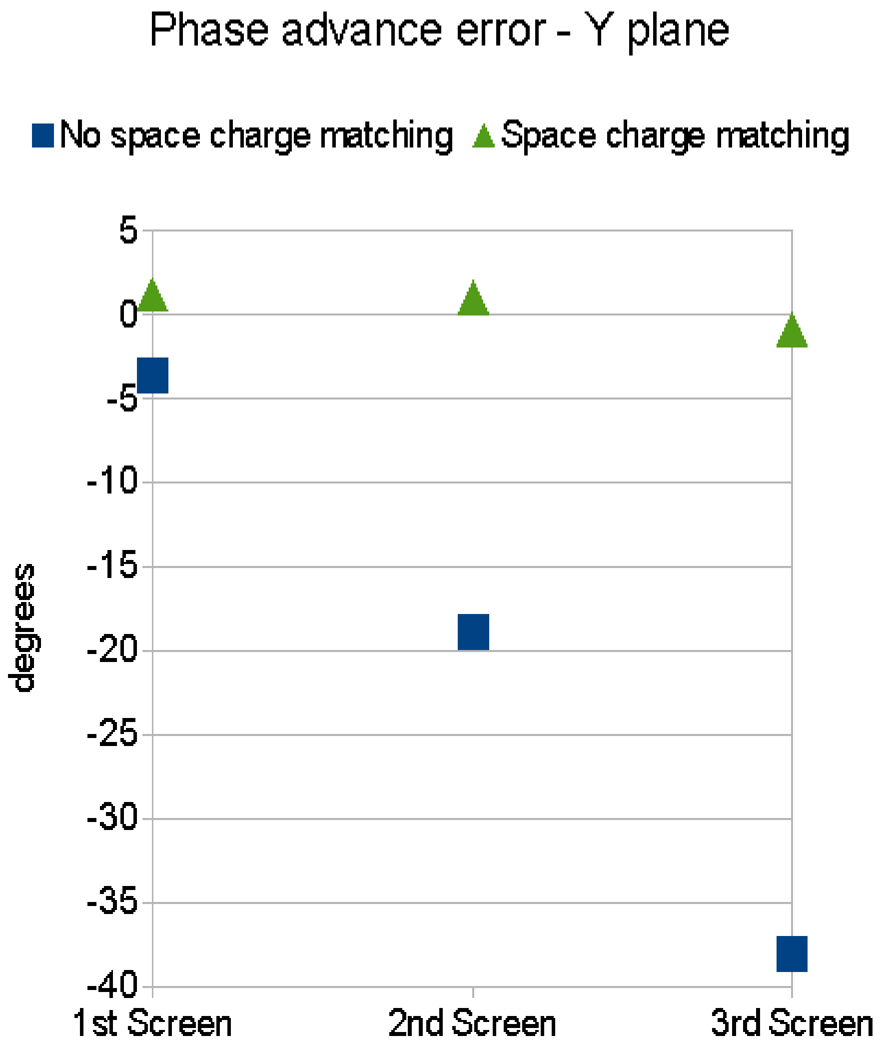
1. For a certain beam momentum and emittance σ_0^{FODO} and β_0^{FODO} are calculated
2. These two values are used by MAD for the matching of the FODO lattice
3. A uniform ASTRA input beam is generated with the **corresponding** space charge (non "$_0$") values
4. The generated input beam is tracked with ASTRA 3D-space-charge along the FODO lattice using the quadrupole strengths calculated by MAD

Case of 1.080mm*mrad (1nC,24.7MeV)

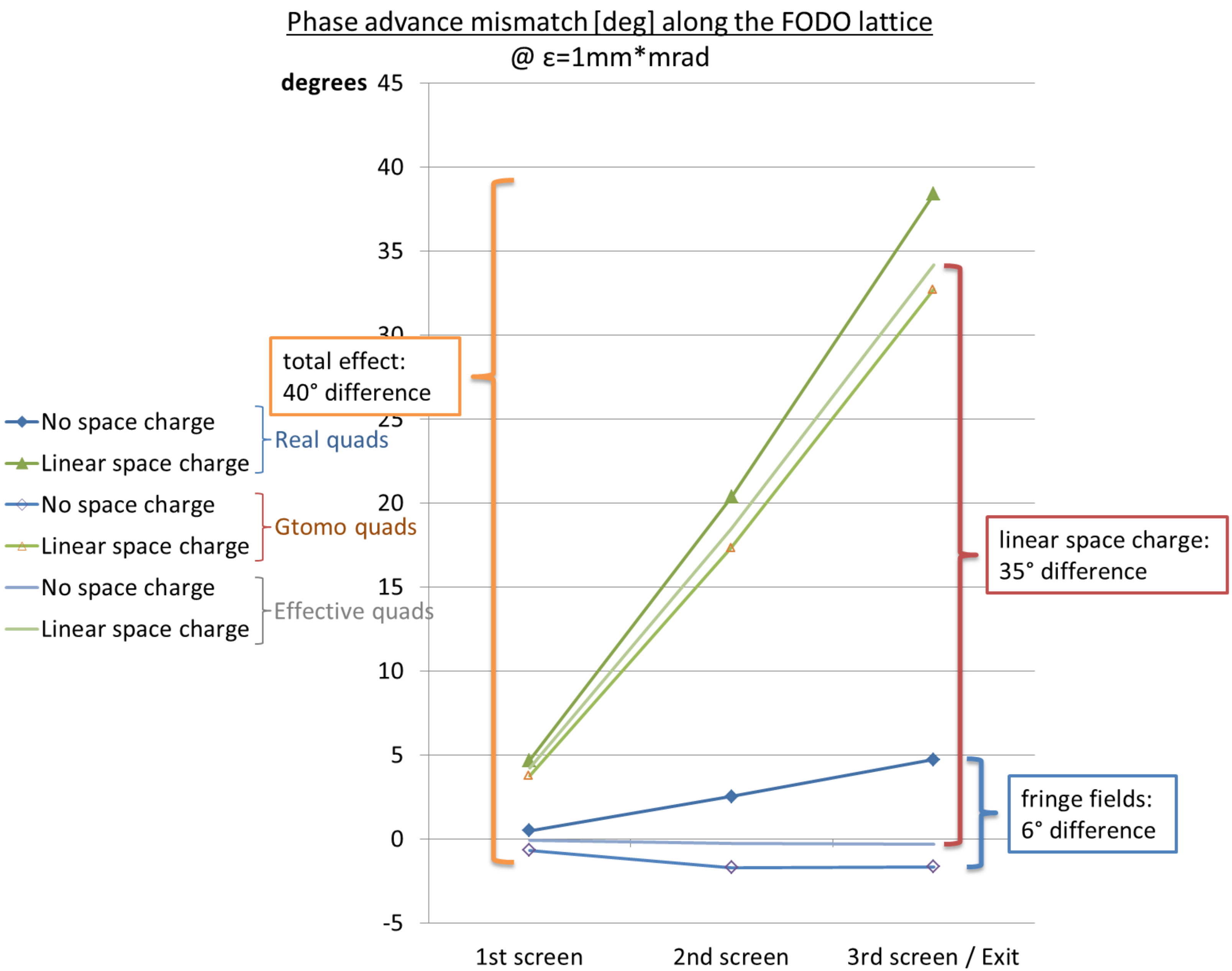


Case of 1.080mm*mrad (1nC,24.7MeV)

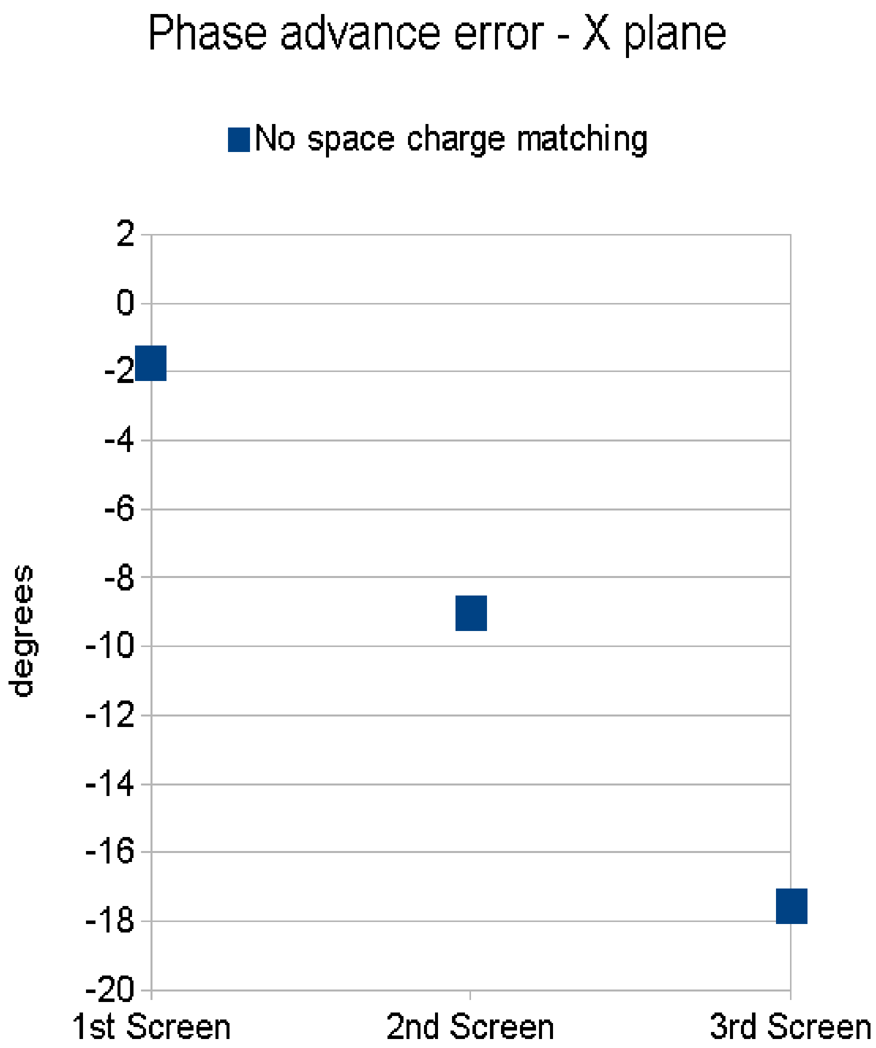


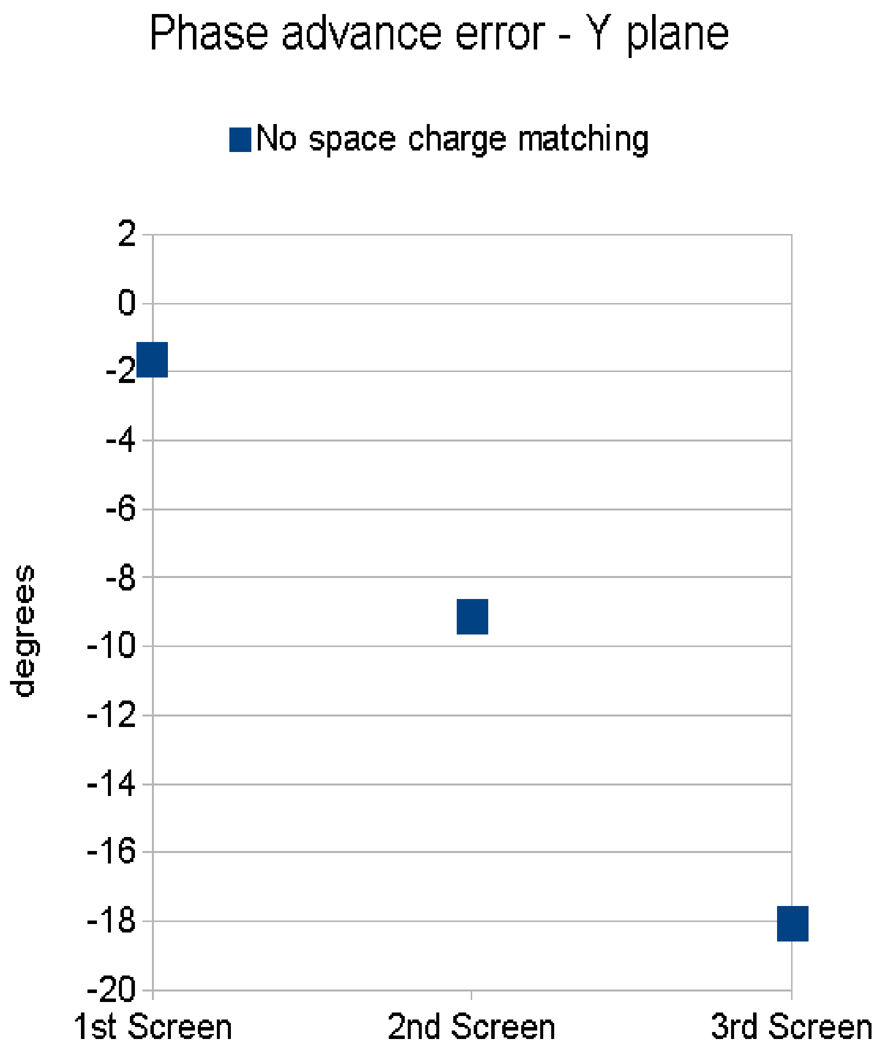


Investigation on quad fringe fields ~1 year ago…

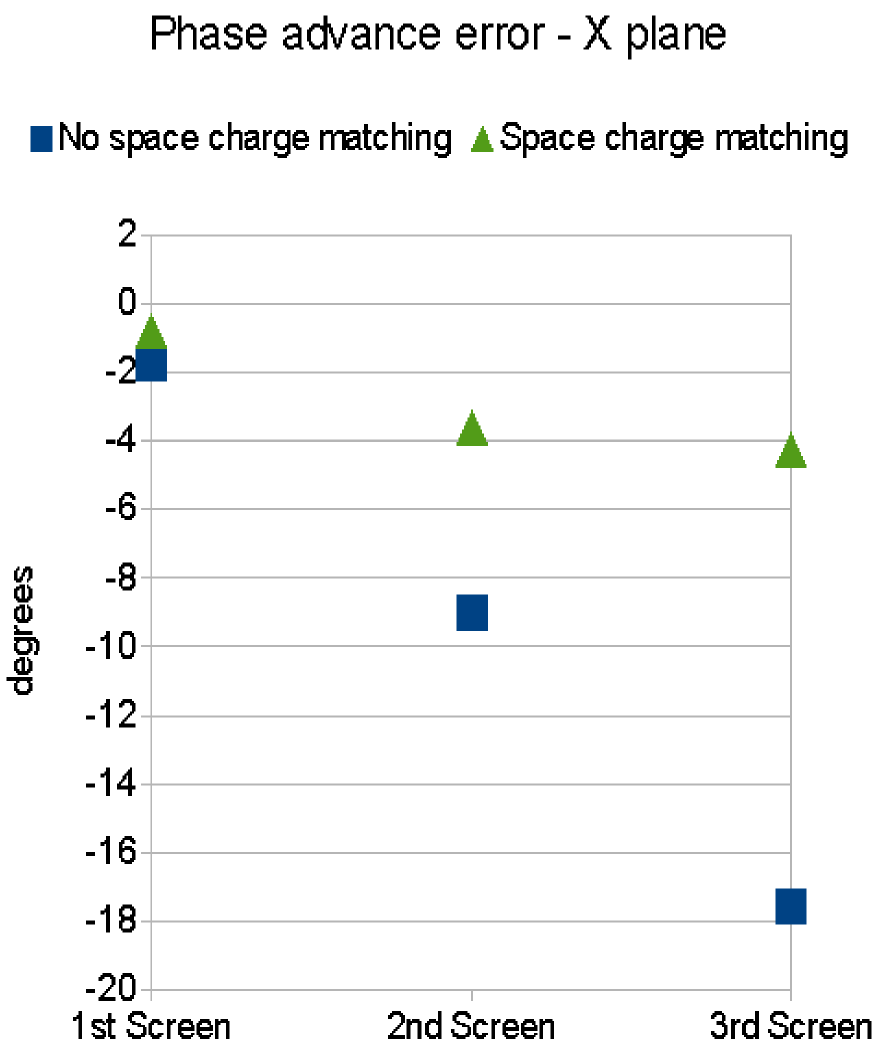


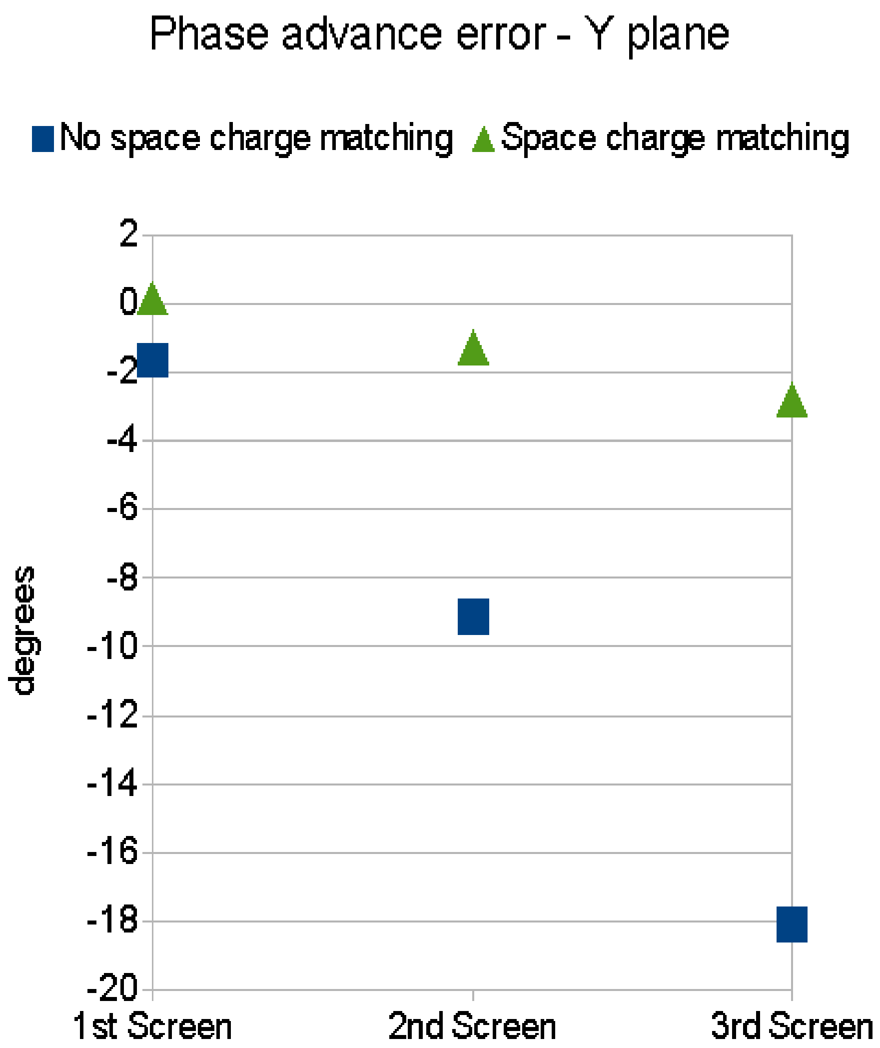
Case of 3.321mm*mrad (1nC,24.7MeV)





Case of 3.321mm*mrad (1nC,24.7MeV)





Outcome

- The smooth-approximation theory can be combined with MAD to get a space-charge compensated matching of the FODO lattice
- As expected, the correction is bigger for beams with stronger space charge
- The remaining (small) matching error seems to be an effect of the quad representation:
  - small MAD mismatches due to the exact effective length of the different magnets (deviation from 43.0mm)
  - fringe fields
- Small differences between different beam distributions are expected according to past studies