Chromatic effects in quadrupole scans

Emittance degradation after a scanning quadrupole

A. Mostacci et al., "*Chromatic effects in quadrupole scan emittance measurements*", PRST-AB 15, 082802 (2012)

- Theoretical overview
- Spot size variation
- Chromaticity induced emittance

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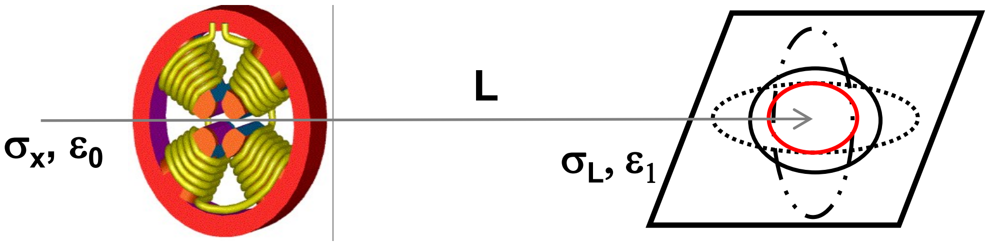
Emittance change in a chromatic single quad line



Quad: k, L_q: $f^{-1} = K(1-\delta)$ for $K = kL_q$

$\delta = \Delta p / p$, $\sigma_\gamma = \sqrt{\langle \delta^2 \rangle}$ known in advance

1. Include **correlations** between transverse coordinates and energy

$$x_1 = x_1$$
$$x_1' = x_0' + K(1-\delta)$$

Eq. 20

$$\varepsilon_1^2 = \varepsilon_0^2 + K^2\sigma_x^2\langle(x_0\delta)^2\rangle - K^2\langle x_0^2\delta\rangle^2 + 2K(\langle x_0x_0'\rangle\langle x_0^2\delta\rangle - \langle x_0x_0'\delta\rangle\sigma_x^2)$$

partial compensation of the chromatic term ε_c

$$\sigma_L^2 = \sigma_{L,\delta=0}^2 + (KL)^2\langle\delta^2 x_0^2\rangle + 2KL(1-KL)\langle\delta x_0^2\rangle + 2KL\langle\delta x_0 x_0'\rangle$$

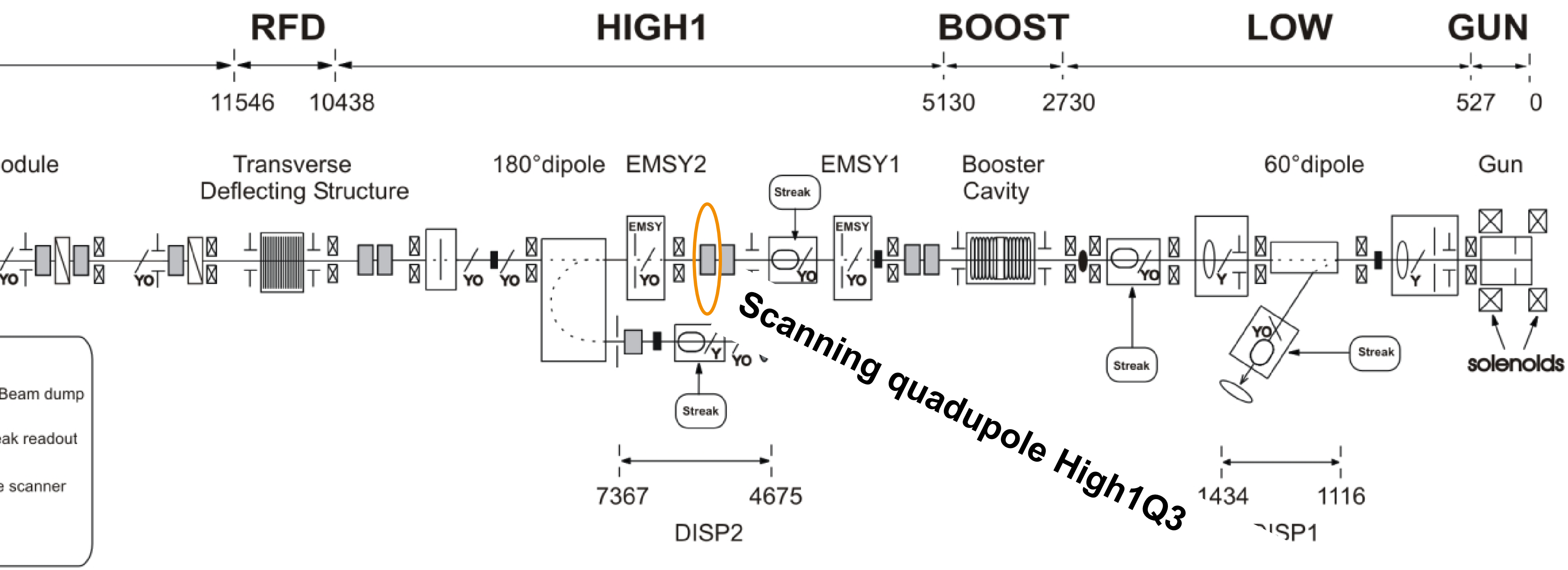
2. **Negligible correlations**

Eq. 16

$$\varepsilon_1^2 = \varepsilon_0^2 + K^2\sigma_x^4\sigma_\gamma^2 = \varepsilon_0^2 + \varepsilon_c^2$$

$$\sigma_L^2 = \sigma_{L,\delta=0}^2 + (KL)^2\sigma_\gamma^2\sigma_x^2$$

Setup at PITZ



Which screen to be used?

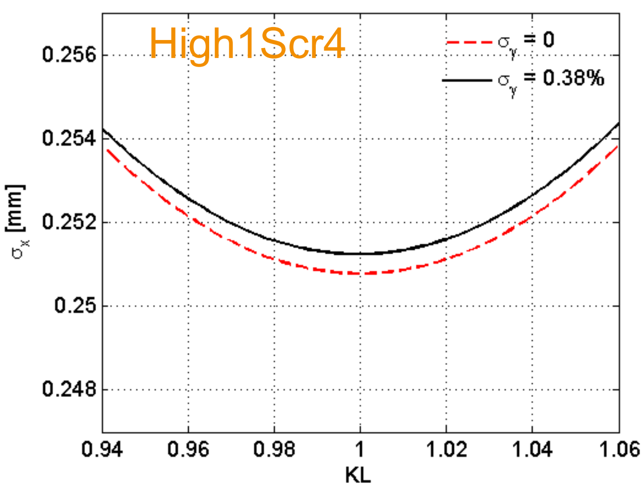
Case 1: 1 nC, ε_{EMSY1} = 0.7 mm mrad, $\sigma_{x,EMSY1}$= 0.65 mm

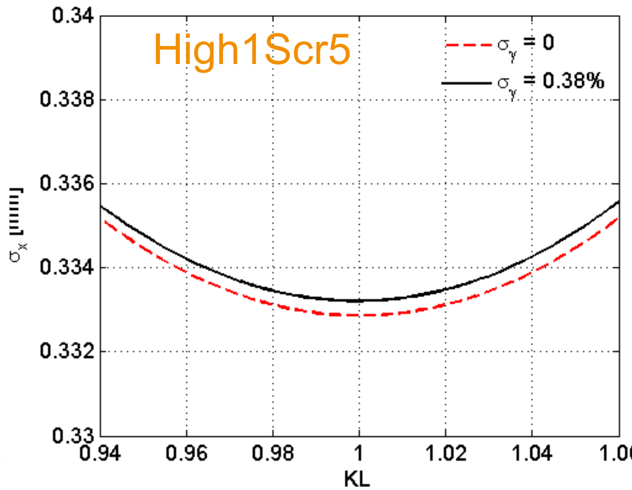
p = 24.96 MeV, σ_γ = 0.4 %

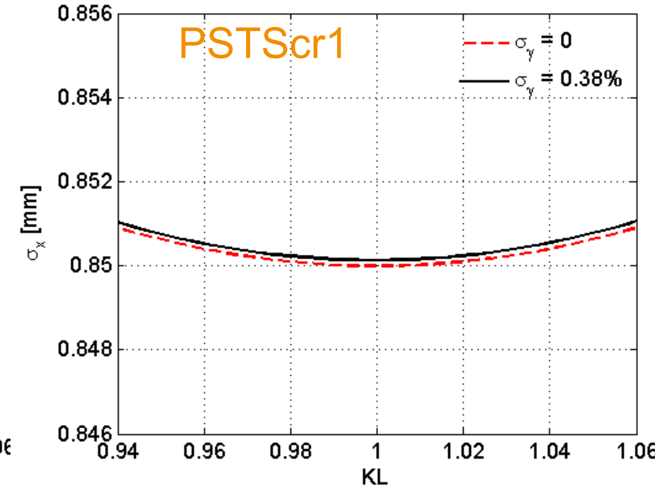
(Imain for smallest emittance on EMSY1, σ_{ini}= 0.37 mm)

Measured beam size – influence of non-zero σ_γ

$$\sigma_L^2 = \sigma_{L,\delta=0}^2 + (KL)^2 \sigma_\gamma^2 \sigma_x^2$$







L = 1. 6284 m

max $\{\sigma_{\sigma_\gamma \neq 0} - \sigma_{\sigma_\gamma = 0}\}$ = 1.3 μm*

L = 2.1614 m

max $\{\sigma_{\sigma_\gamma \neq 0} - \sigma_{\sigma_\gamma = 0}\}$ = 1.6 μm*

L = 5.5194 m

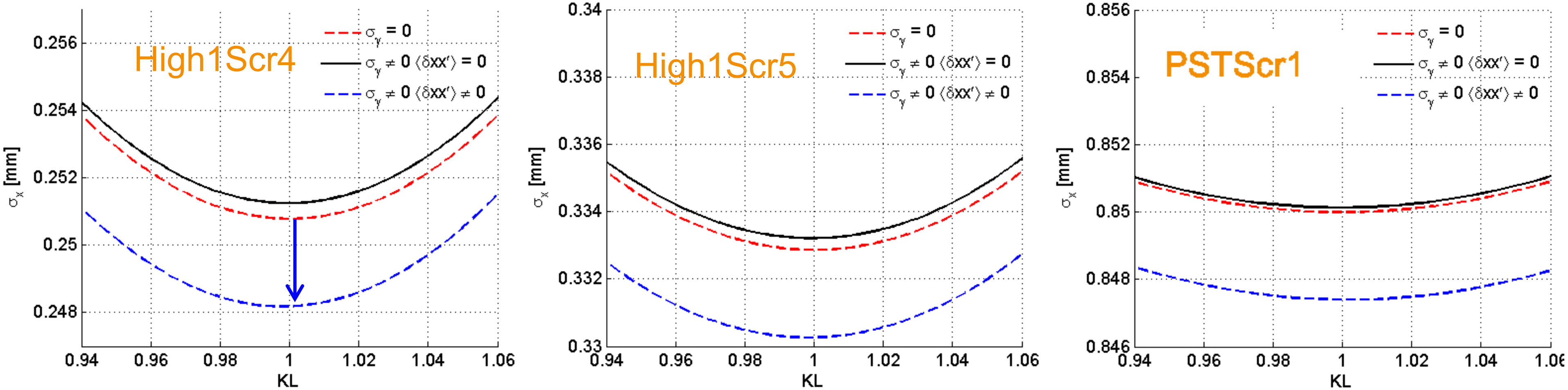
max $\{\sigma_{\sigma_\gamma \neq 0} - \sigma_{\sigma_\gamma = 0}\}$ = 3.7 μm*

- The beam size with chromaticity taken into account is always bigger.
- Such differences are hard to resolve and do not include any correlations.

* Calculated over the full gradient scanning range [-7, 7] T/m

Impact of non-zero correlations

$\Delta = \max\{\sigma_{\sigma_\gamma \neq 0} - \sigma_{\sigma_\gamma = 0}\}$



High1Scr4	High1Scr5	PSTScr1
Δ = 1.3 μm*	Δ = 1.6 μm*	Δ = 3.7 μm*
Δ = 2 μm*	Δ = 3 μm*	Δ = 2.6 μm*
from ASTRA* 1.5 μm	from ASTRA* 1.7 μm	from ASTRA* 3.2 μm

Such differences are hard to resolve, but they:

> can contribute to systematic uncertainty

> one needs to keep correlations as small as possible (a hint how to focus at the entrance of the quad)

* Calculated over the full gradient scanning range [-7, 7] T/m

* ASTRA tracking of monochromatic beam

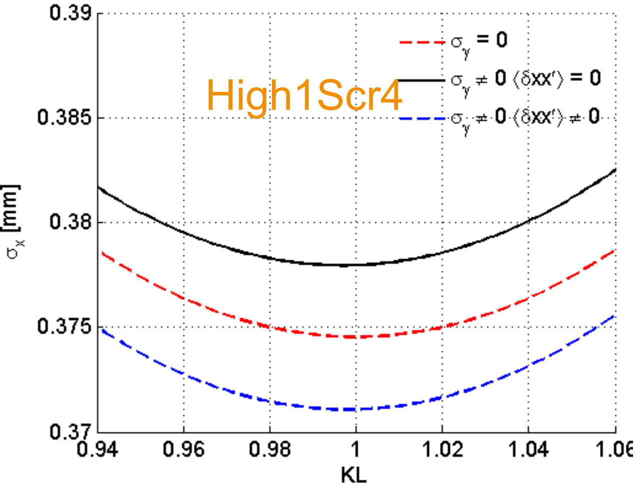
If the energy spread increases

σ_γ = 0.9% (220 keV) – Booster off-crest phase

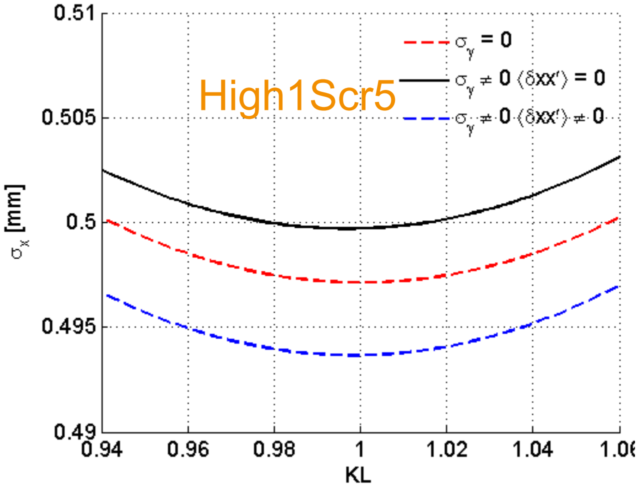
Imain for smallest emittance on EMSY1

$\sigma_{x,\,EMSY1}$ = 0.54 mm

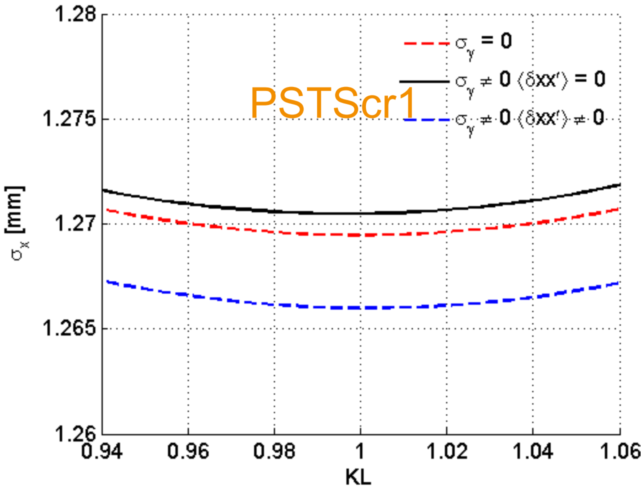
$\varepsilon_{x,\,EMSY1}$ = 0.7 mm mrad



Δ = 5 μm*



Δ = 7 μm*



Δ = 7 μm*

The spot size, incl. correlations, is further underestimated → the fit parameters smaller than the real → the emittance would be underestimated.

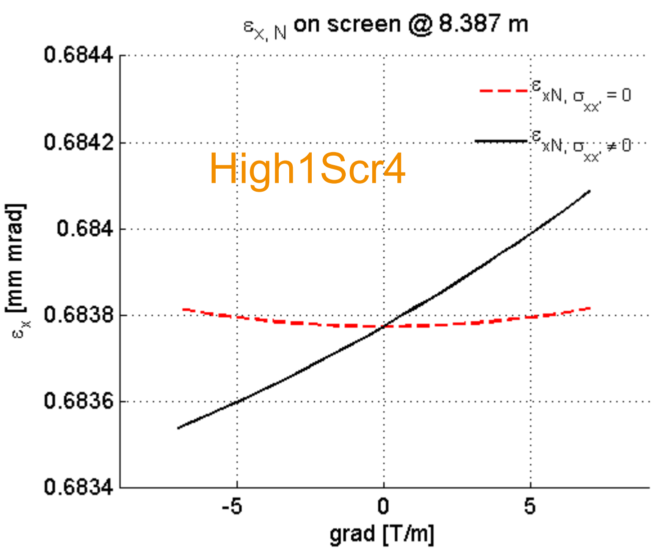
* Calculated over the full gradient scanning range [-7, 7] T/m

Systematic uncertainty – dependence on drift length

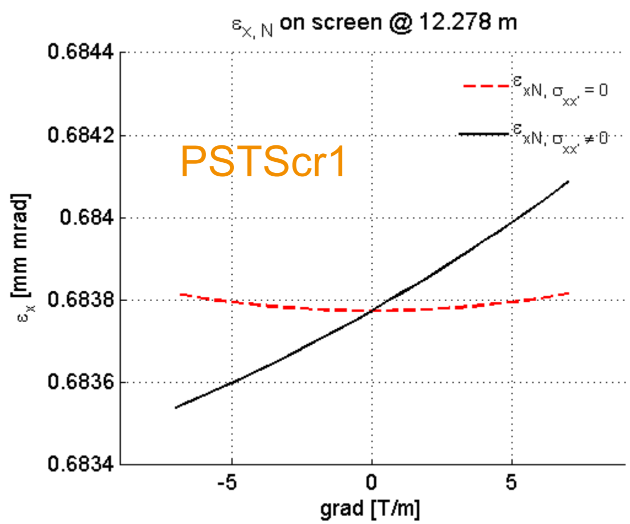
$$\sigma_L^2 = \sigma_{L,\delta=0}^2 + (KL)^2 \sigma_\gamma^2 \sigma_x^2$$

The emittance calculated from the fit: $\varepsilon_{meas}^2 = \varepsilon_{actual}^2 + \frac{\sigma_\gamma^2}{1+\sigma_\gamma^2}\frac{ac}{L^2}$ $= \Delta\varepsilon = f(\sigma_\gamma, \sigma_x, \sigma_{x'}, \sigma_{xx'})$

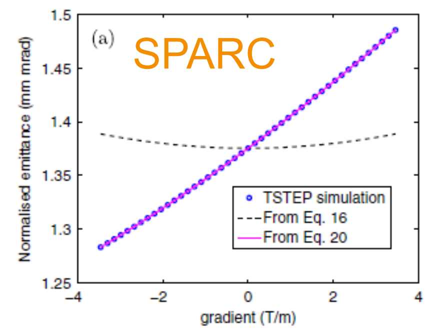
as from the fit: $a = \sigma_x^2(1+\sigma_\gamma^2), c = \sigma_x^2 + L^2\sigma_{x'}^2 + 2L\langle xx'\rangle$ ← depending directly on the parameters in front of the quadrupole



$\Delta\varepsilon \sim 0.001$ mm mrad



$\Delta\varepsilon \sim 0.0003$ mm mrad



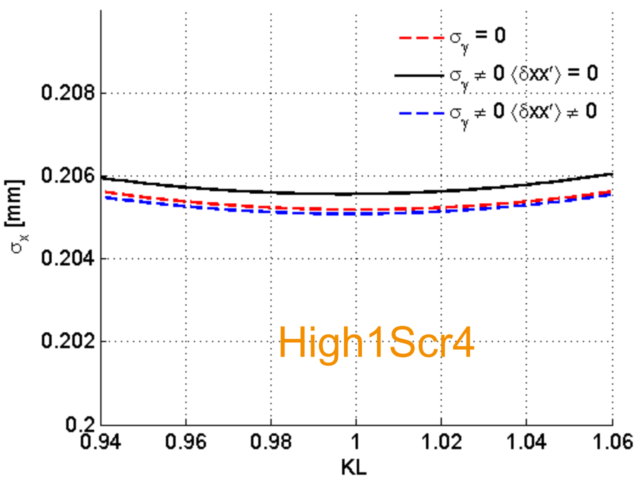
The curves agree well with the SPARC data, but not the scales.

Systematic uncertainty – smaller spot size at the quad

$$\sigma_L^2 = \sigma_{L,\delta=0}^2 + (KL)^2 \sigma_\gamma^2 \sigma_x^2$$

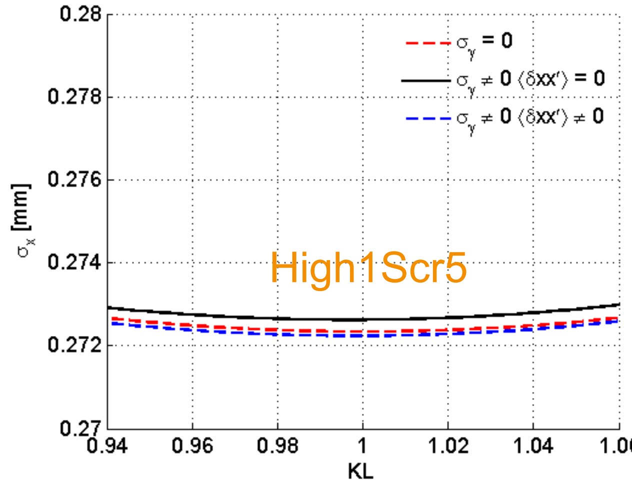
Case 2: $\sigma_{x,\,EMSY1}$ = 0.21 mm

$\varepsilon_{x,\,EMSY1}$ = 0.87 mm mrad



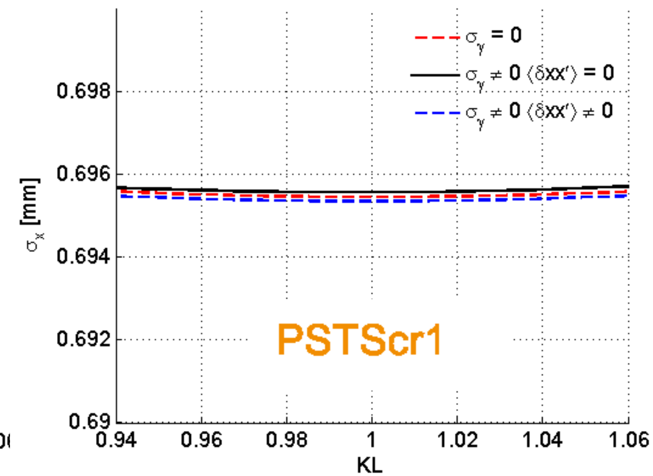
Δ = 2.4 μm*, Δ = 0.5 μm*

Δε = 3e-6 mm mrad*



Δ = 3 μm*, Δ = 0.5 μm*

Δε = 2.2e-5 mm mrad*



Δ = 7 μm*, Δ = 0.4 μm*

Δε = 1.2e-5 mm mrad*

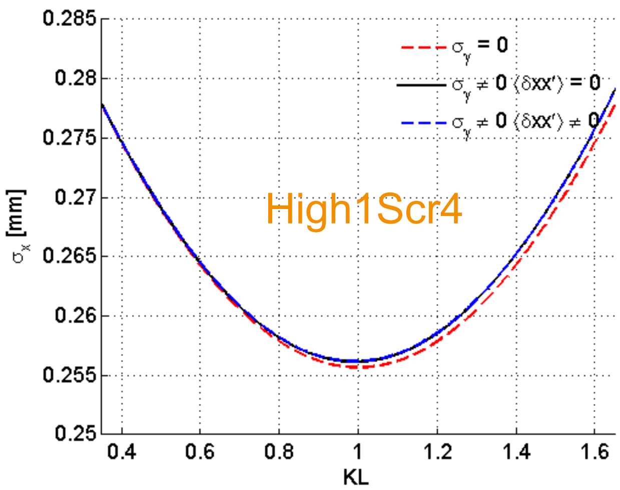
- Smaller Δ than for bigger spot sizes.
- The impact on the uncertainty is small, but the focusing should provide small correlations.

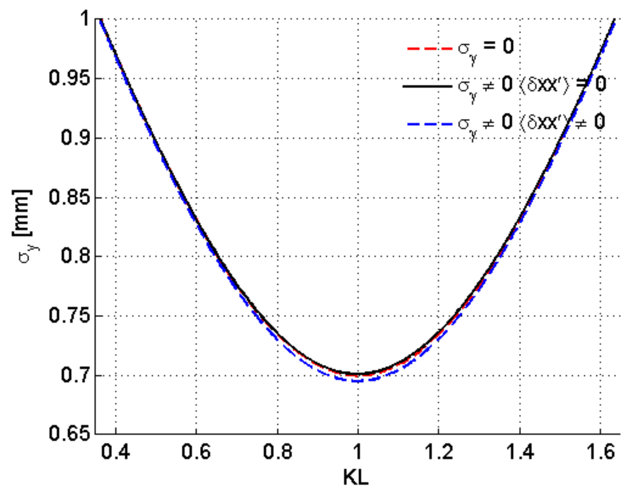
* Calculated over the full gradient scanning range [-7, 7] T/m

Quadrupole focusing including correlations

> Use High1Q1/2 to focus the beam at the entrance of the quadrupole (**Case 1**)

- Only one plane possible since small σ_x and $\sigma_{xx'}$ are needed together
  - hard since with the scan σ_y might be bigger than the screen
- Calculated gradients of High1Q1/2, then ASTRA tracking





Δ = 0.5 μm*, Δ = 0.8 μm*

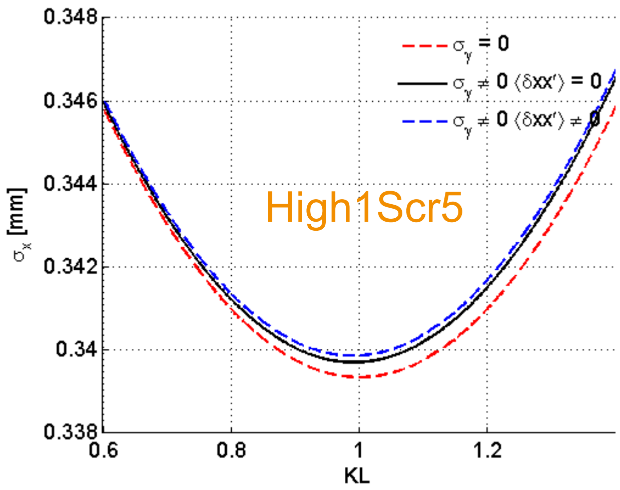
$\Delta\varepsilon$ = 3e-6 mm mrad*

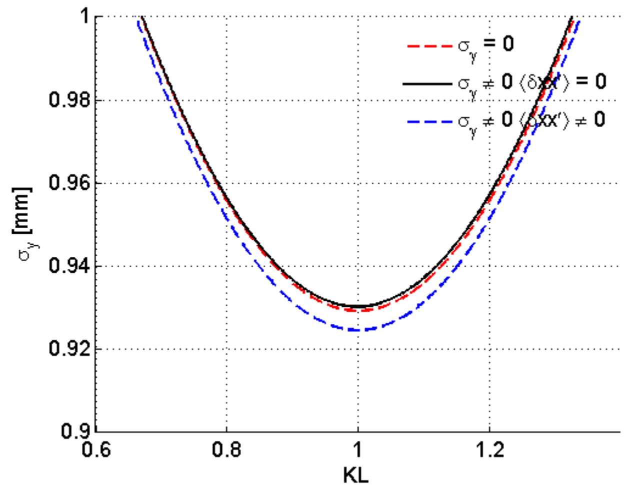
* Calculated over the full gradient scanning range [-7, 7] T/m

Quadrupole focusing including correlations

> Use High1Q1/2 to focus the beam at the entrance of the quadrupole (**Case 1**)

- Only one plane possible since small σ_x and $\sigma_{xx'}$ are needed together
  - hard since with the scan σ_y might be bigger than the screen
- Calculated gradients of High1Q1/2, then ASTRA tracking





Δ = 0.4 μm*, Δ = 1.3 μm*

$\Delta\varepsilon$ = 3e-6 mm mrad*

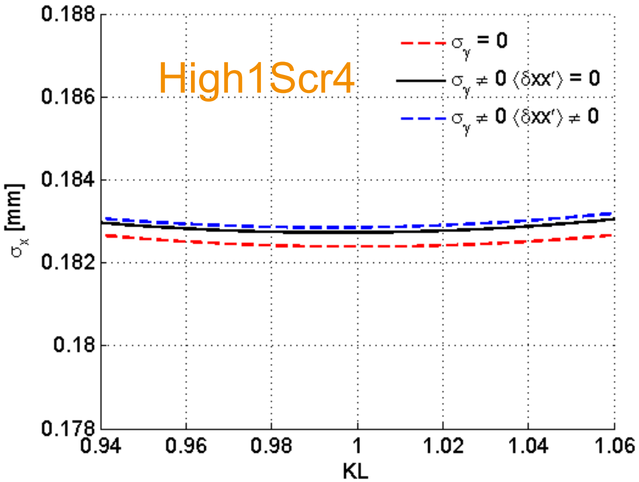
Focusing is important: keep correlations small.

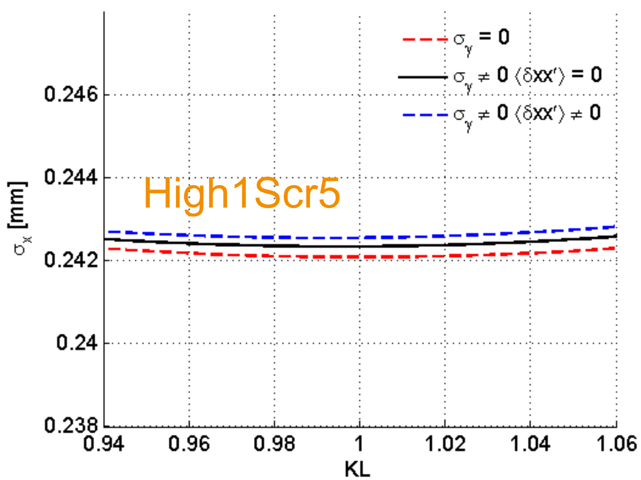
* Calculated over the full gradient scanning range [-7, 7] T/m

Quadrupole focusing disregarding correlations

> Use High1Q1/2 to focus the beam at the entrance of the quadrupole (**Case 1**)

- Gradients of High1Q1/2: beam at the quad is converging in both planes
- ASTRA tracking





High1Scr4: Δ = 0.4 μm*; Δ = 16 μm*; $\Delta\varepsilon$ = 2e-6 mm mrad*

High1Scr5: Δ = 0.3 μm*; Δ = 20 μm*; $\Delta\varepsilon$ = 2e-6 mm mrad*

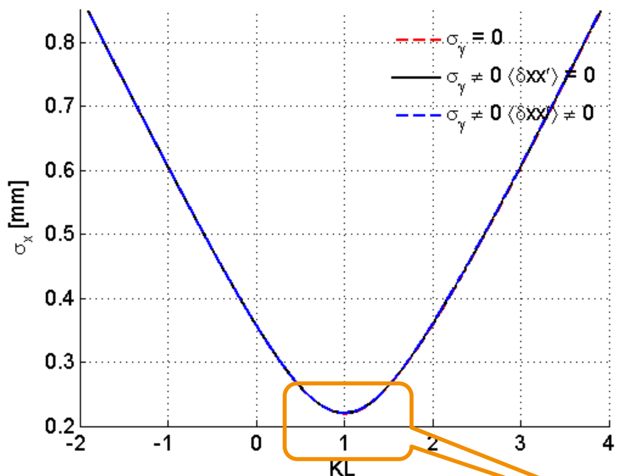
* Calculated over the full gradient scanning range [-7, 7] T/m

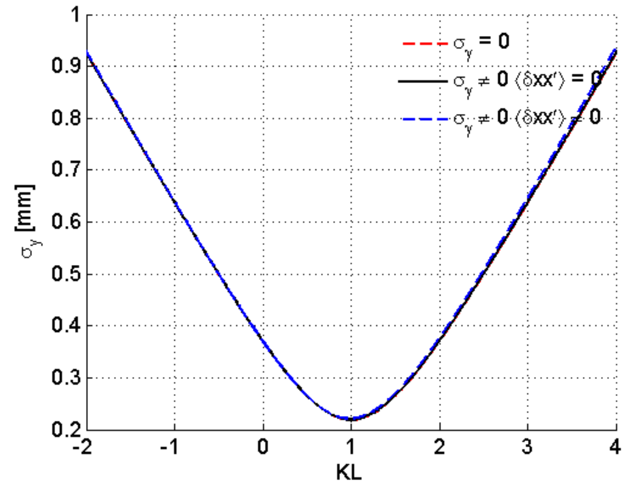
Big spot at the entrance of the quad

Case 3: EMSY1: $\varepsilon = 0.82$ mm mrad

$\sigma_{x,\,y} = 0.8$ mm, in front of High1Q3 → 1.1 mm

Focusing as in the previous slide: control $\sigma_{y'}$, $\sigma_{x'}$, $\sigma_{xx'}$

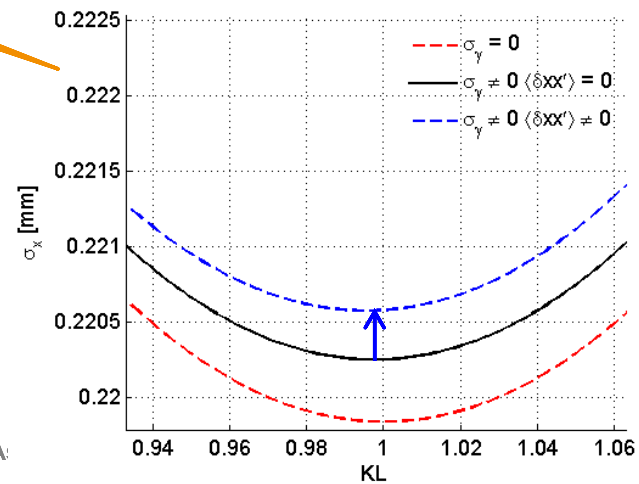




$\Delta = 0.5\ \mu$m*

$\Delta = 0.01\ \mu$m*

$\Delta\varepsilon$ = 7e-7 mm mrad*



Final remarks

- The increase in emittance after the quadrupole is negligible for
  - small beam size at the entrance of the quad
  - small energy spread
  - small correlations
  - is not really affected by the position of the observation screen
- For big spot sizes it is not sufficient to focus only
  - the focusing should include the covariance (more like matching)
  - two upstream quads needed
  - possible mostly for one plane
- Systematics can be known in advance