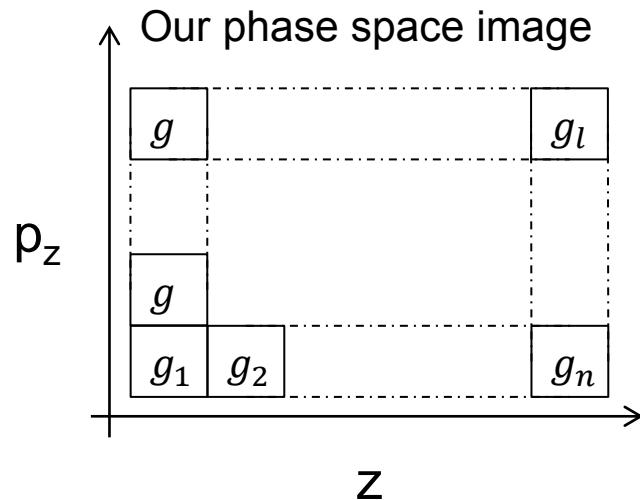


# Introduction to the longitudinal phase space tomography at PITZ, part II

1. Tomography reconstruction algorithm
2. Example
3. First experimental results
4. Conclusion

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PPS February 2013

# Reconstruction algorithm



$$p_z(z) = p_z(z) + k(\varphi) \cdot z$$

$$p_{ij} = a_{ijl} \cdot g_l$$

# Reconstruction algorithm, filling “ $a_{ijl}$ ” array

$p_{14}$	$g_{19}$			$g_{24}$	
$p_{13}$	$g_{13}$			$g_{18}$	
$p_{12}$	$g_7$			$g_{12}$	
$p_{11}$	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$
					$g_6$

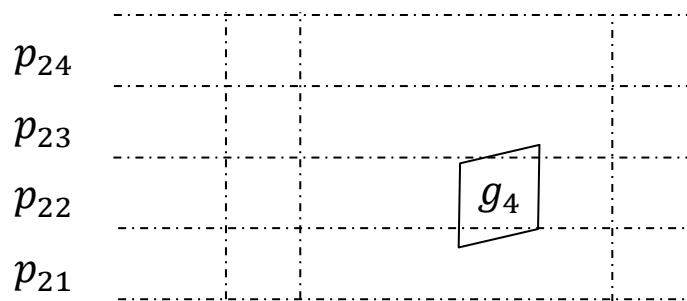
If  $\varphi_1$  mean no rotation applied then  $a_{1,1,4} = 1$ ,  $a_{1,2,4} = 0$

$p_{24}$				
$p_{23}$				
$p_{22}$			$g_4$	
$p_{21}$				

$$p_z(z) = p_z(z) + k(\varphi) \cdot z$$

If  $\varphi_2$  mean rotation applied then  $a_{2,1,4} = 0.3$ ,  $a_{2,2,4} = 0.7$

# Reconstruction algorithm, filling “ $a_{ijl}$ ” array



$$p_z(z) = p_z(z) + k(\varphi) \cdot z$$

$$a_{2,1,4} = ???, a_{2,2,4} = ???, a_{2,3,4} = ???$$

# Reconstruction algorithm, iterations

$$g_q^{(k+1)} = g_q^{(k)} + \sum_{ij} \frac{a_{ijq}(p_{ij} - \sum_l a_{ijl} \cdot g_l^{(k)})}{\sum_{nm} a_{inm}^2}$$

$i$  – phase (Nphase)

$j$  – momentum (Npz)  
– z coordinate (Nz)

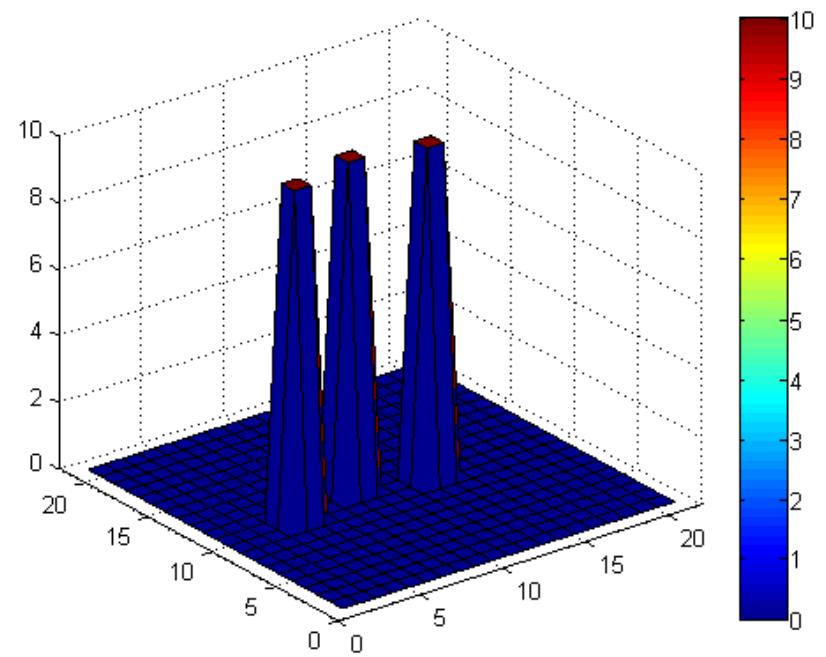
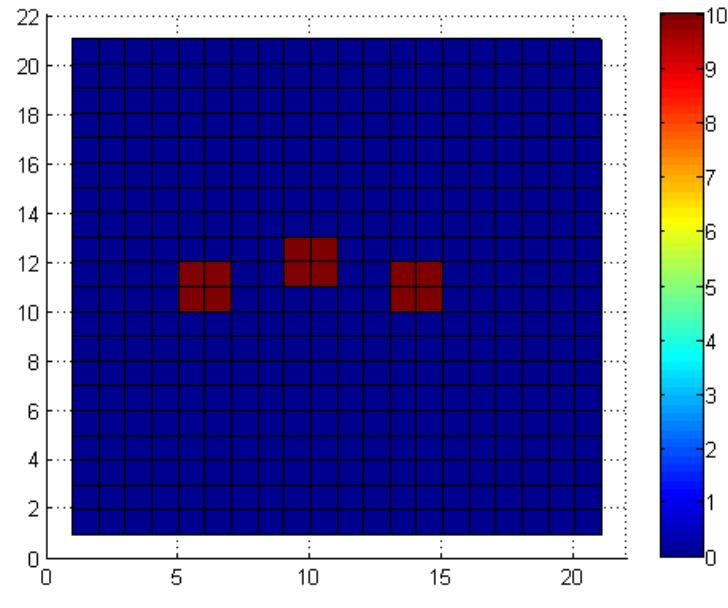
$l$  – image index (NI = Npz\*Nz)

$$\text{Npz}^* \text{Nz}^* \text{Nphase}^* \text{Npz}^* (\text{Npz}^* \text{Nz} + \text{Npz}^* \text{Npz}^* \text{Nz}) =$$

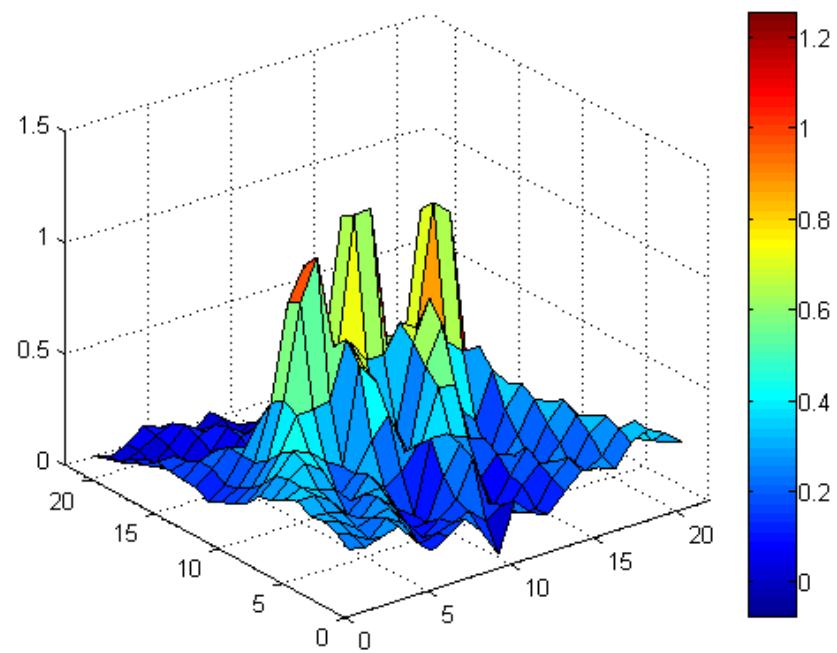
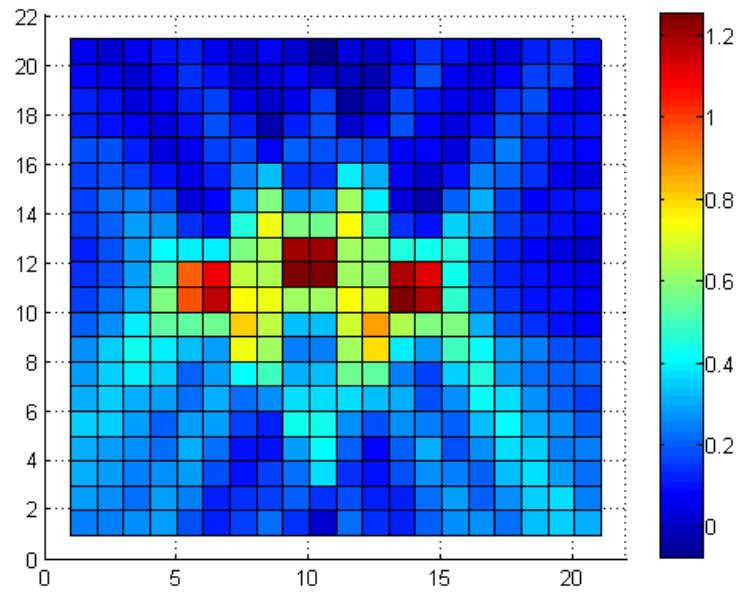
Total time is                   ~                    $\text{Npz}^3 \text{Nz}^2 \text{Nphase} (1 + \text{Npz}) =$

$$\text{Npz}^4 \text{Nz}^2 \text{Nphase}$$

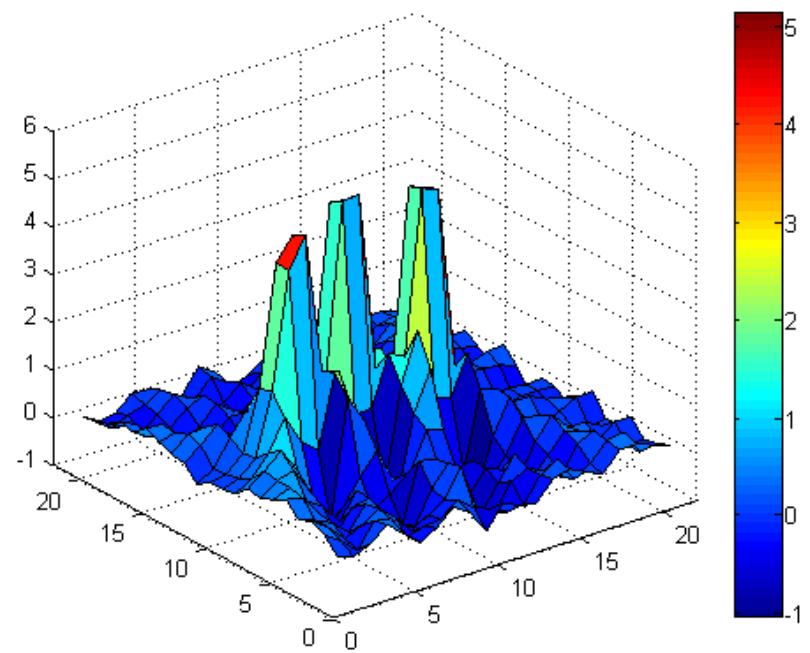
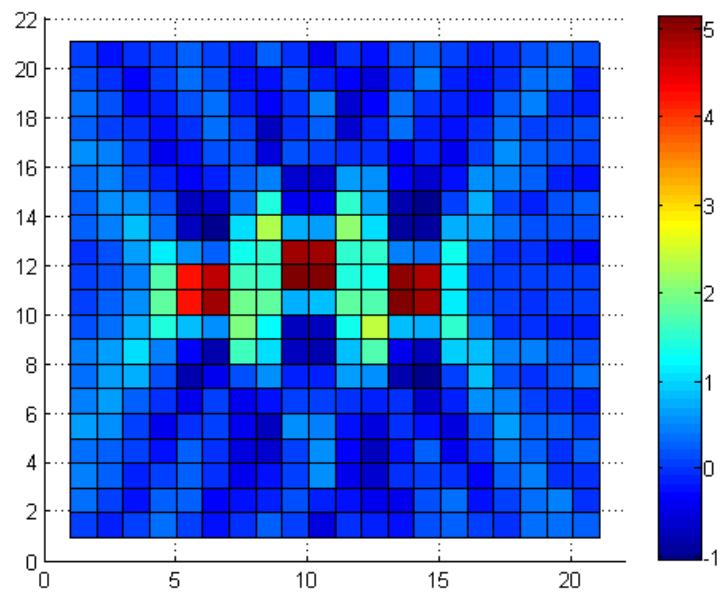
# Example



# Example, 1 iteration



# Example, 10 iteration



# Experimental results, 2013.02.07 M 10:04

High1.Dipole = -79 A

I<sub>main</sub> = 377 A

LT = 1.5 %

NoP = 30

Gun RF power = 6.71 MW

Booster RF power = 3.27 MW

Gun FB ON

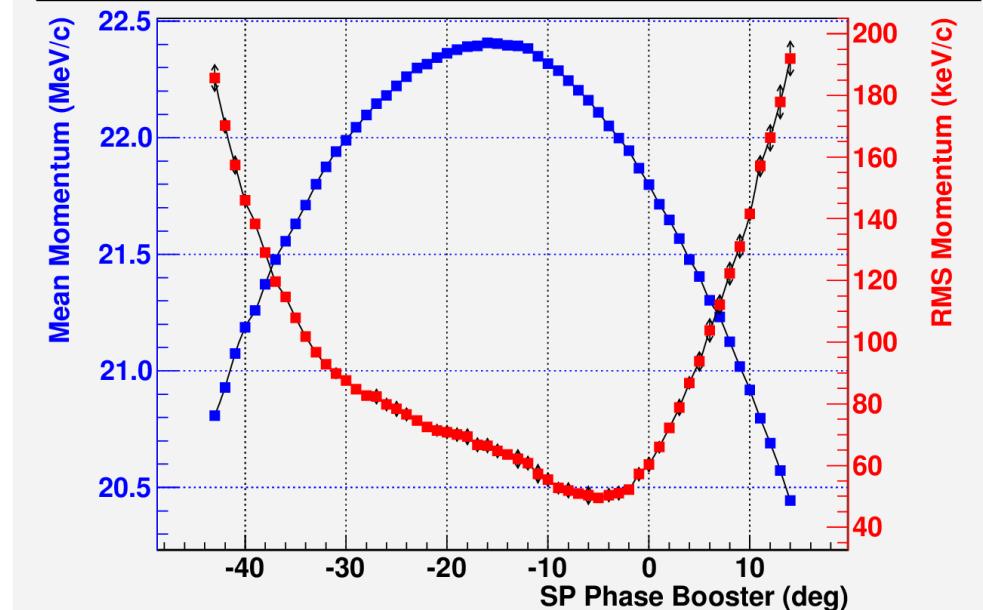
Booster FB OFF

Statistics = 10

High1.Q4 = 1.8

HEDA1 scan, 20 pC, Gauss, 2.8 ps FWHM,  
gun on-crest.

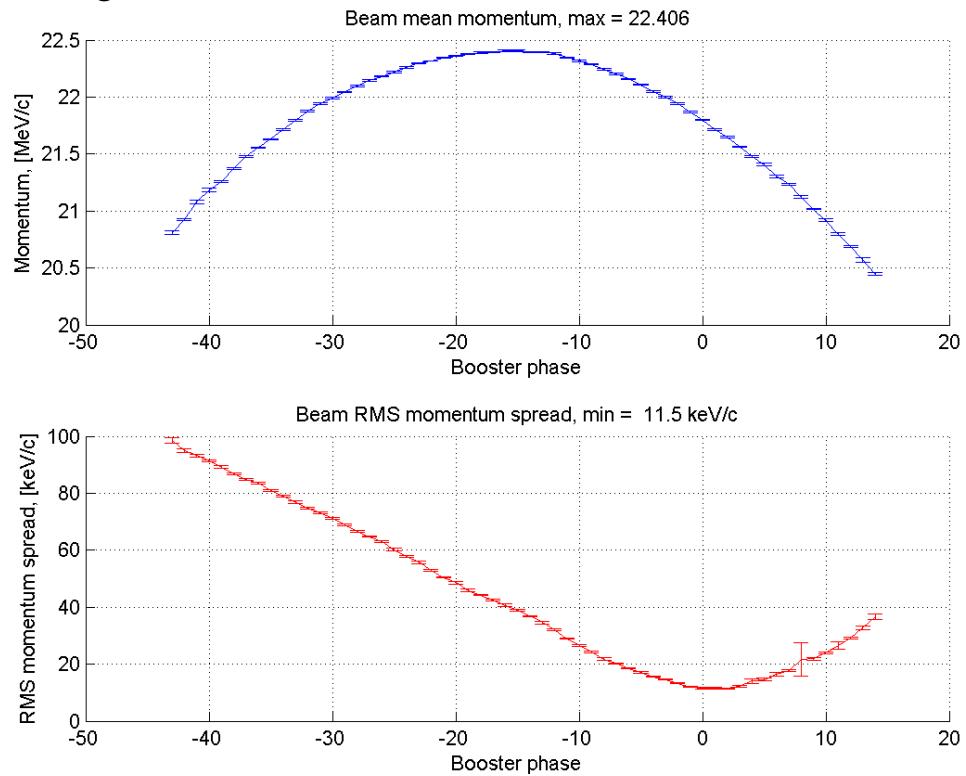
$$\langle p \rangle^{\max} \approx (22.4070 \pm 0.0061) \text{ MeV/c at } -16.0^\circ$$
$$p_{\text{RMS}}^{\min} \approx (49.53 \pm 1.45) \text{ keV/c at } -5.0^\circ$$



HEDA1 resolution expected:  $y_{\text{RMS}} = 70 \text{ um} \rightarrow 3 \text{ keV/c}$   
(see part I)

# Experimental results, 2013.02.07 M 10:04

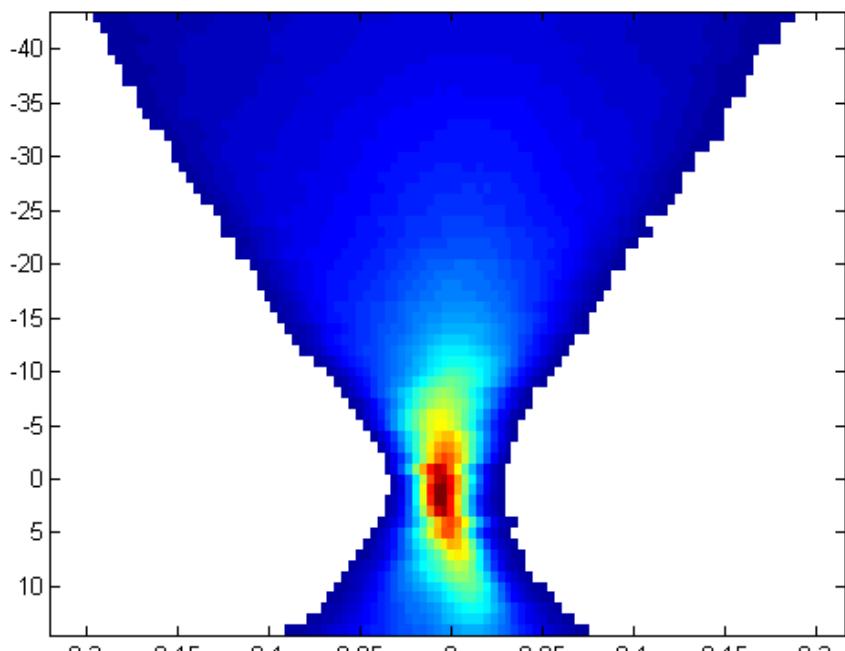
HEDA1 scan, 20 pC, Gauss, 2.8 ps FWHM,  
gun on-crest.



# Momentum projections, initial data

Booster  
phase

58 bins

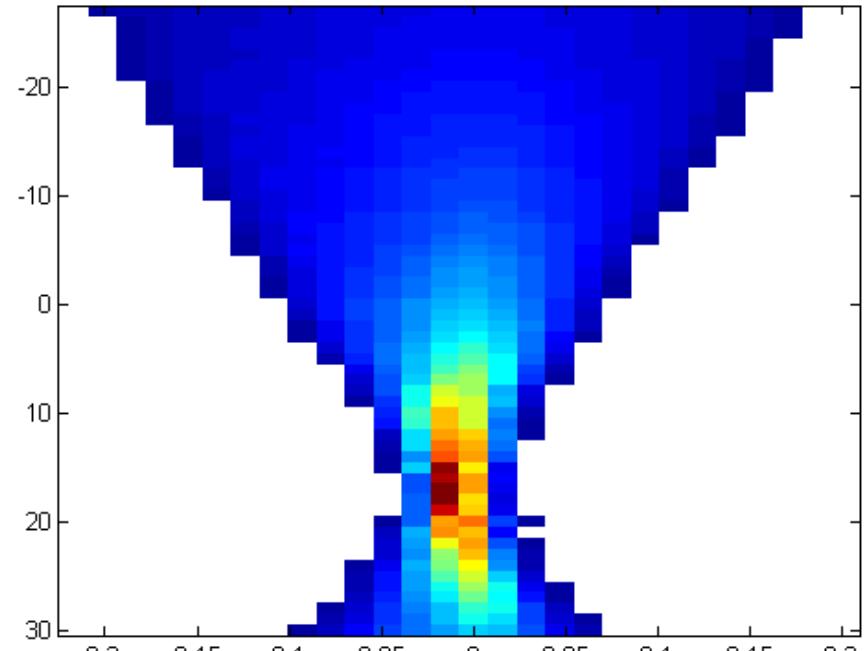


Momentum, MeV/c

112 bins

$z = [-2.0 \quad 2.0]$

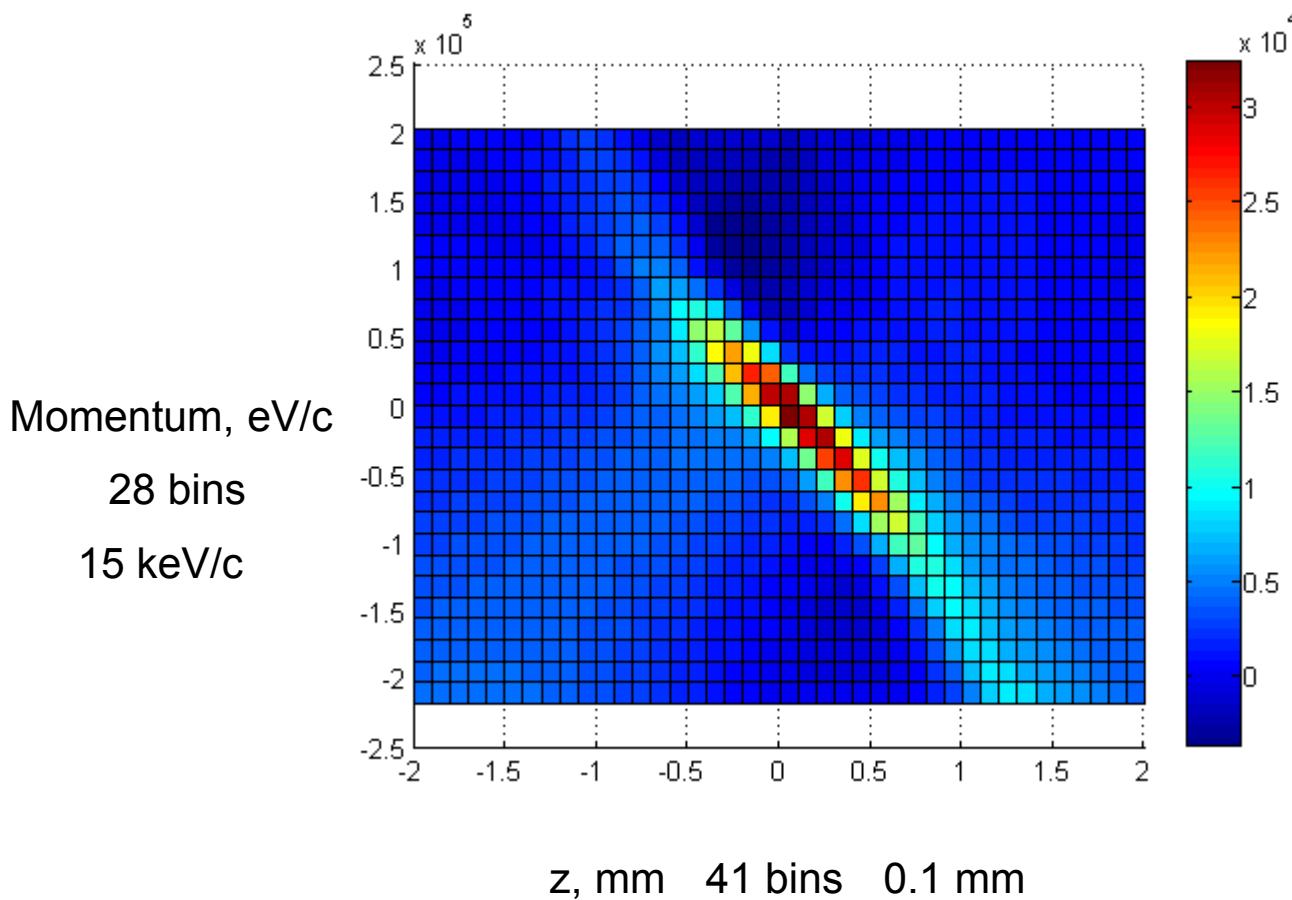
41 bins



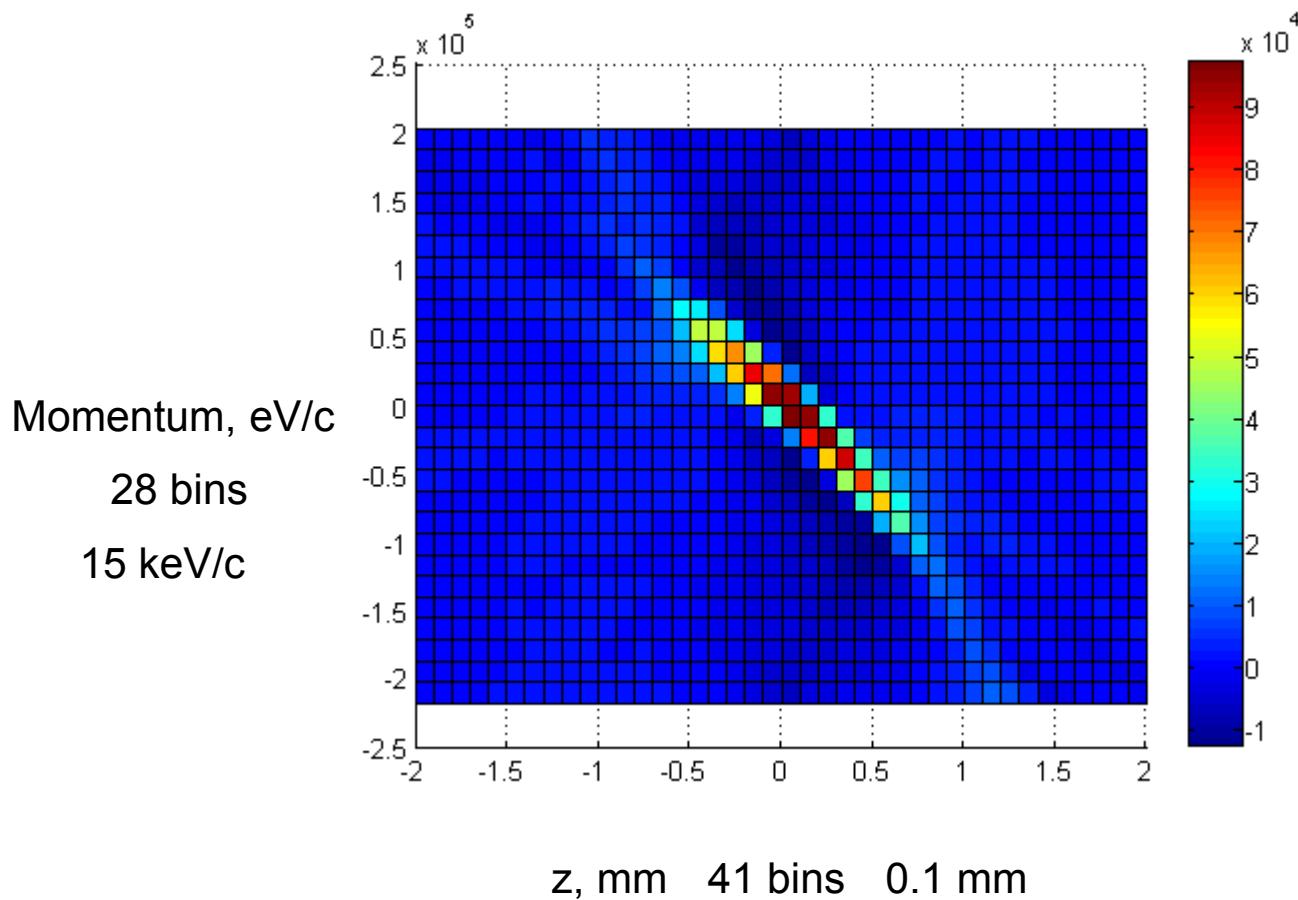
Momentum, MeV/c

28 bins

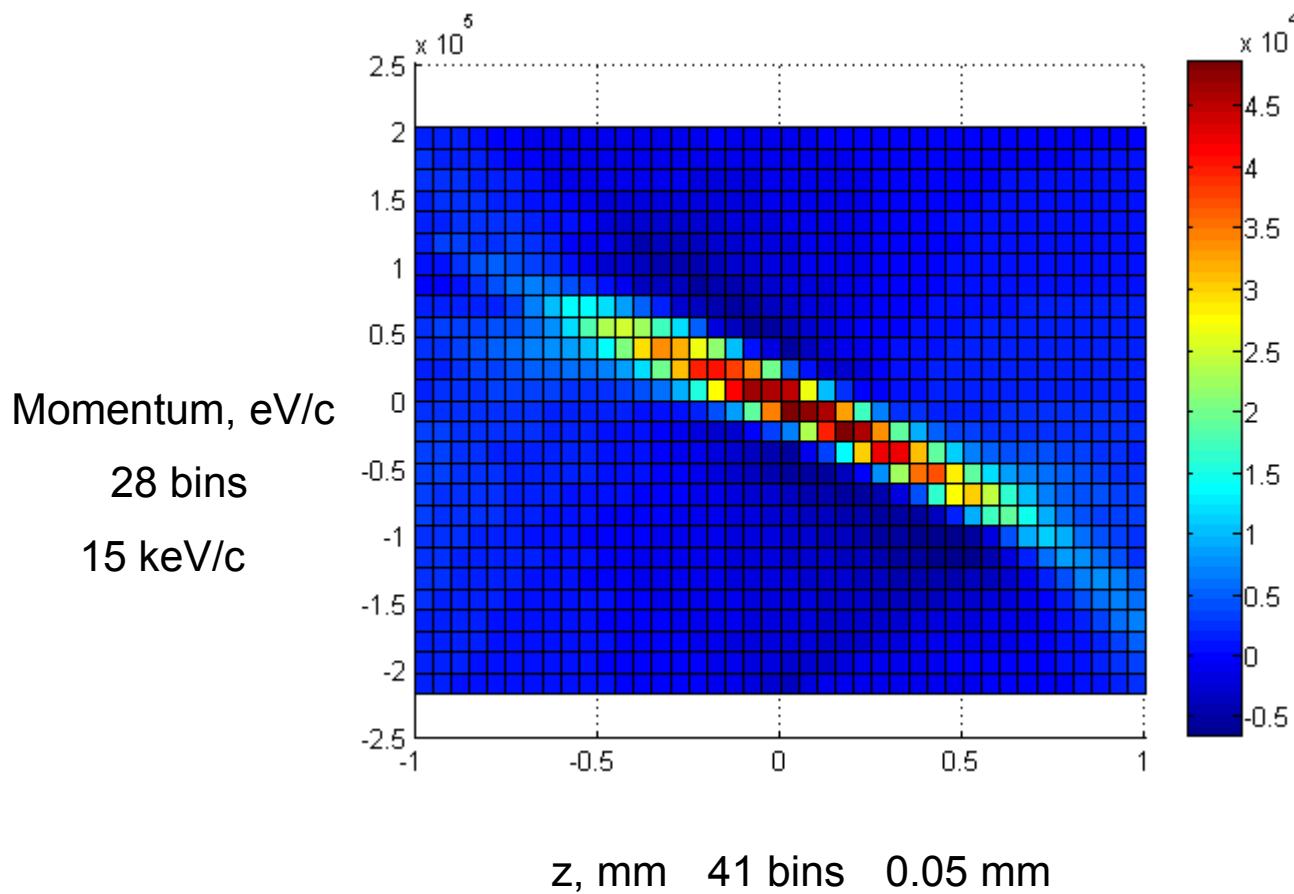
# Reconstructed phase space, 1 iteration



# Reconstructed phase space, 10 iterations



# Reconstructed phase space, 5 iterations, z more detailed

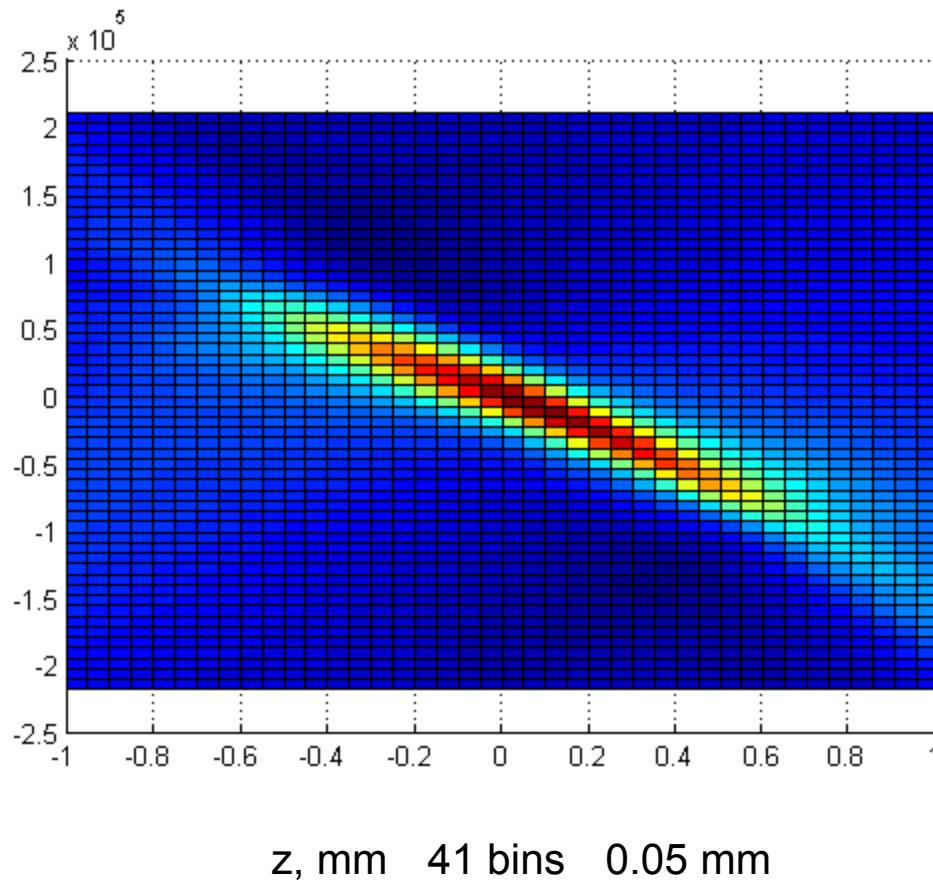


# Reconstructed phase space, 2 iterations

Momentum, eV/c

56 bins

7 keV/c



about 16 hours

# Experimental results, 2013.02.14 A 19:35

HEDA1 scan, 1 nC, flat top, 17.5 ps FWHM,  
gun on-crest.

$$\langle p \rangle^{\max} \approx (22.2357 \pm 0.0040) \text{ MeV/c at } 35.0^\circ$$
$$p_{\text{RMS}}^{\min} \approx (61.74 \pm 1.15) \text{ keV/c at } 37.0^\circ$$

High1.Dipole = -81.6 A (rdb -81.59891 A)

I<sub>main</sub> = 377 A

LT= 100 %

NoP = 1

Gun RF power = 6.8 MW

MMMG Gun phase

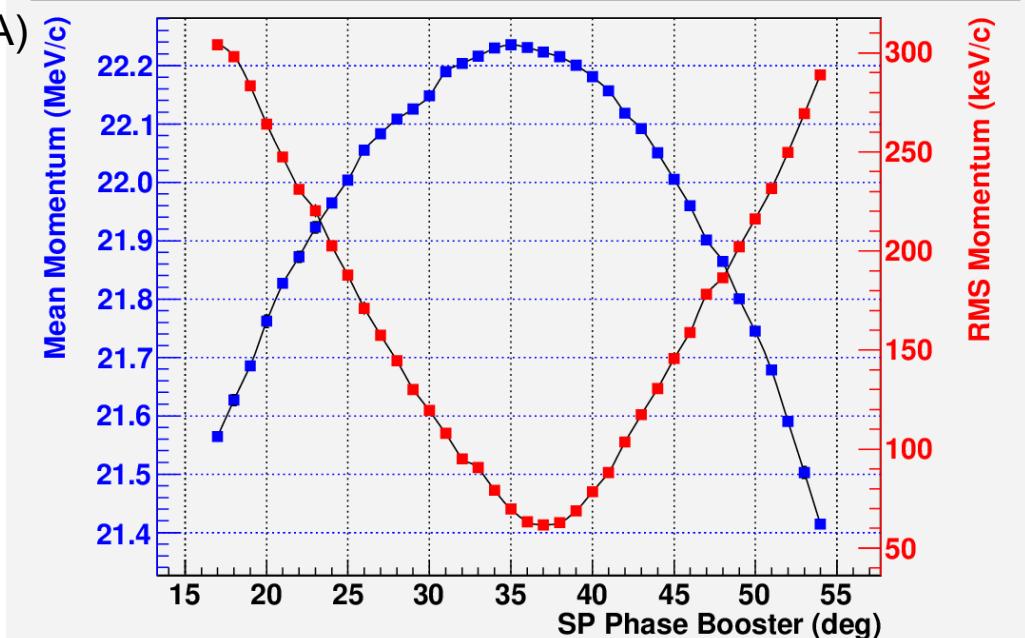
Booster RF power = 3.2 MW

Gun FB ON

Booster FB OFF

Statistics = 20

High1.Q4 = 2.1 A

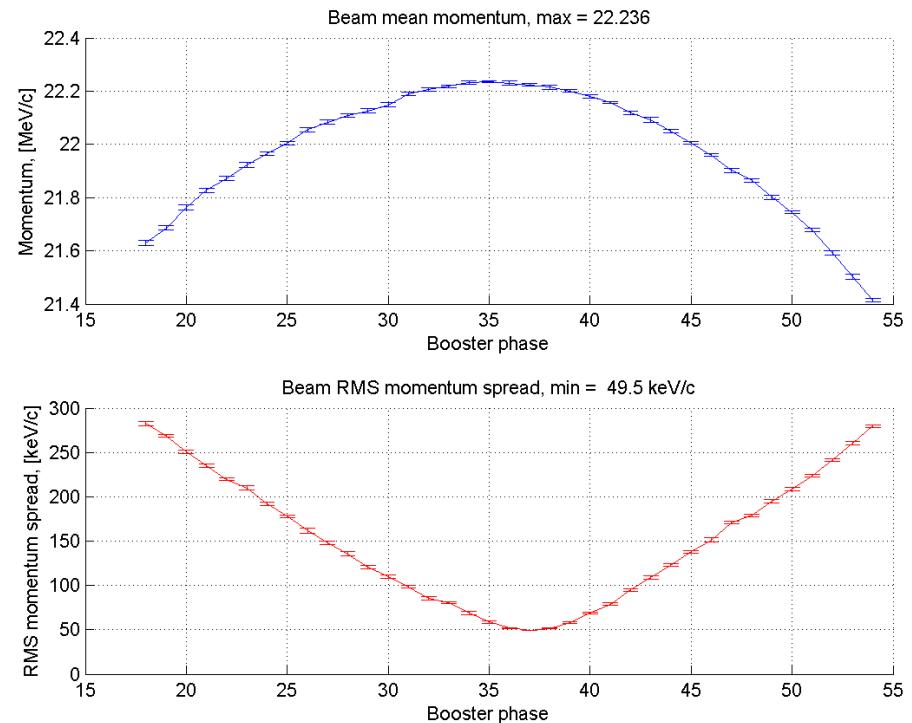


HEDA1 resolution expected: yRMS = 0.6 mm → 25 keV/c

(see part I)

# Experimental results, 2013.02.14 A 19:35

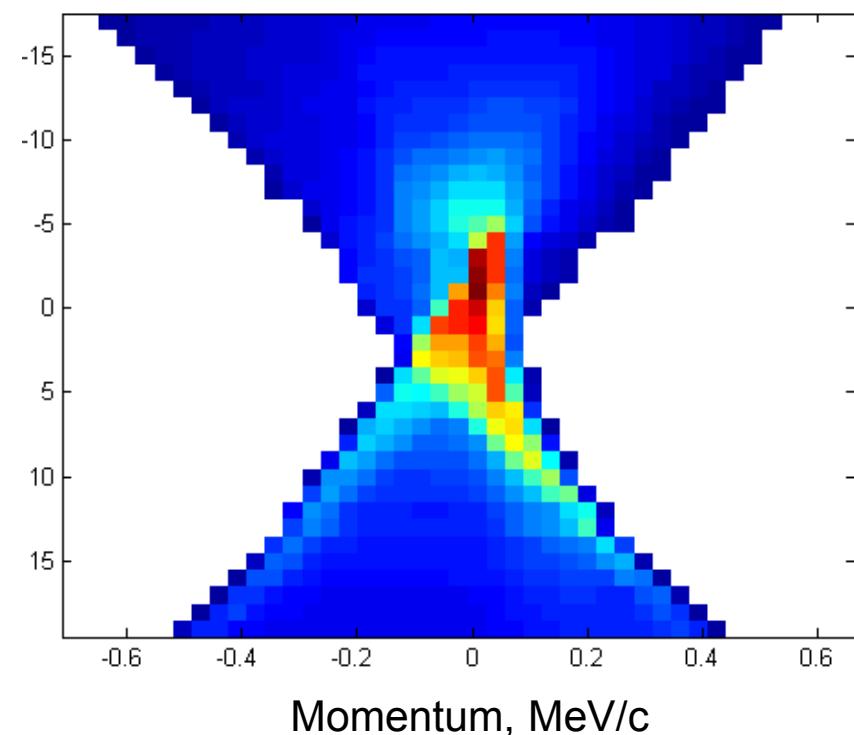
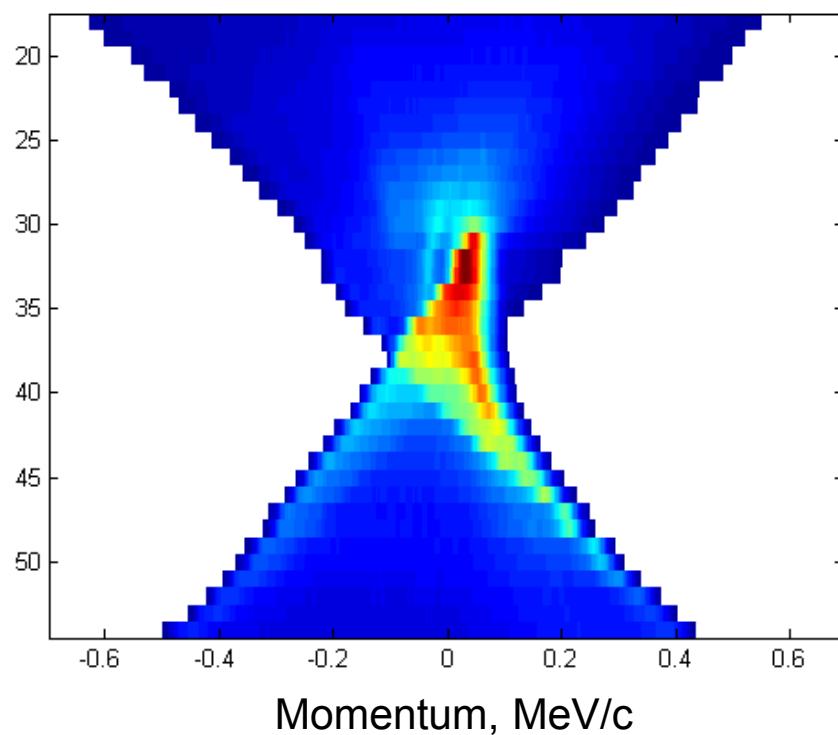
HEDA1 scan, 1 nC, flat top, 17.5 ps FWHM,  
gun on-crest.



# Momentum projections, initial data

Booster  
phase

37 bins



348 bins

43 bins

$z = [-6.0 \quad 8.0]$

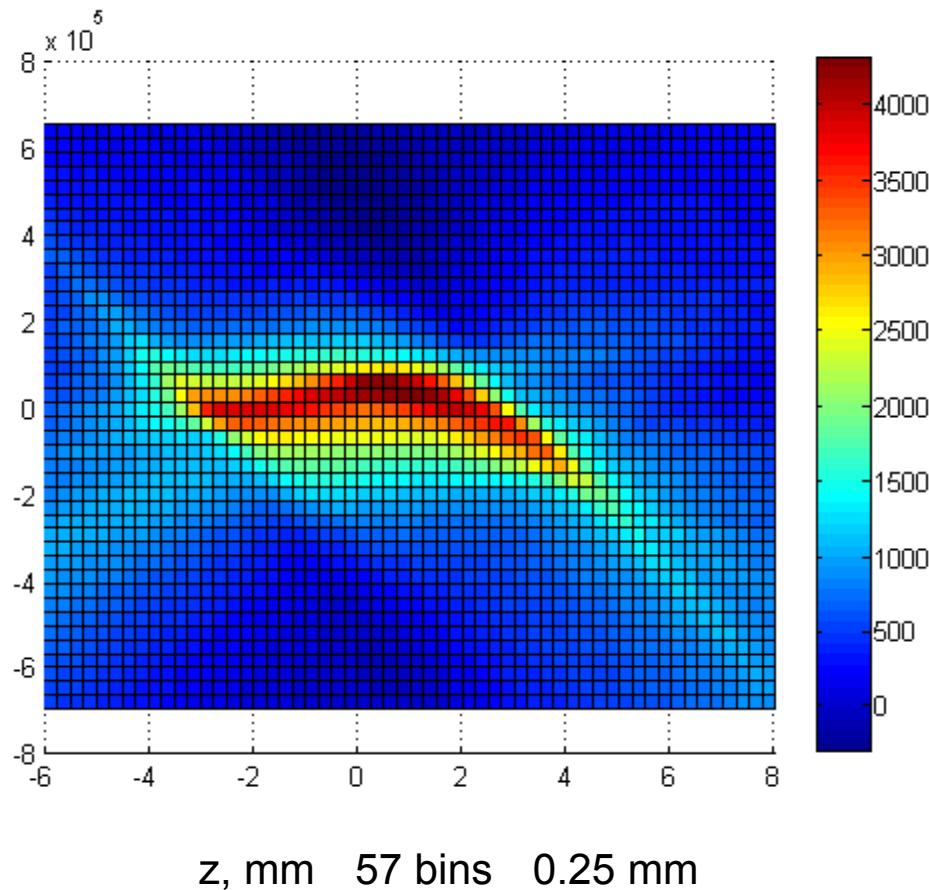
57 bins

# Reconstructed phase space, 1 iteration

Momentum, eV/c

43 bins

30 keV/c

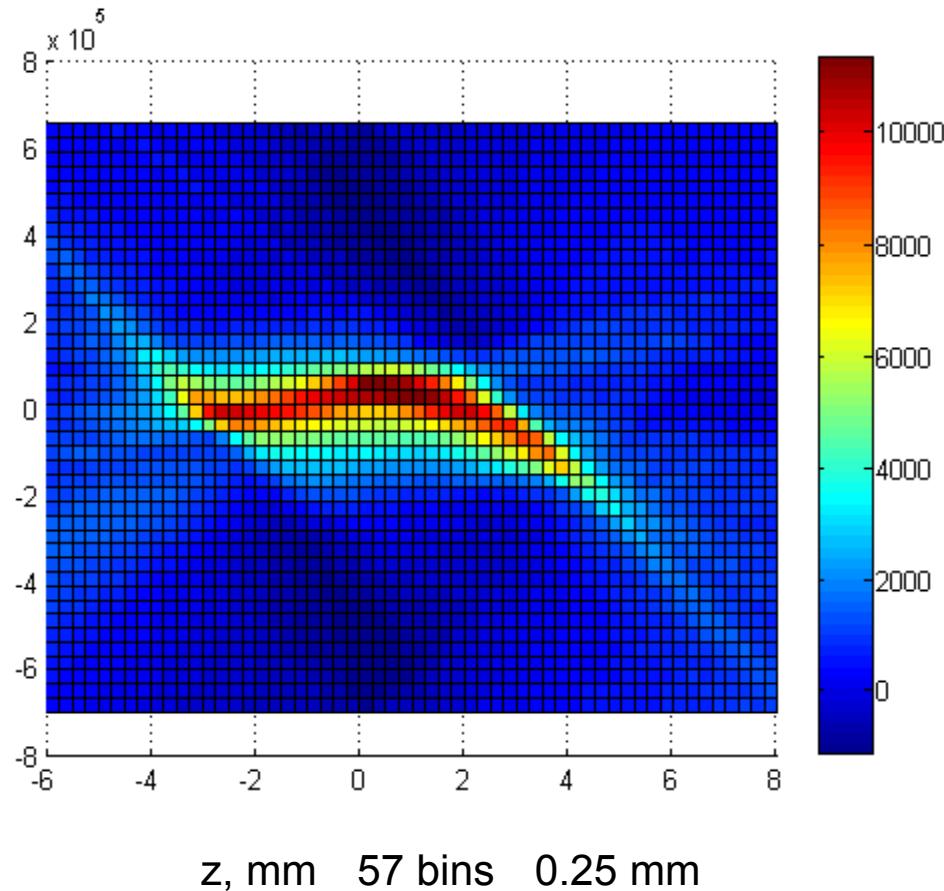


# Reconstructed phase space, 4 iterations

Momentum, eV/c

43 bins

30 keV/c

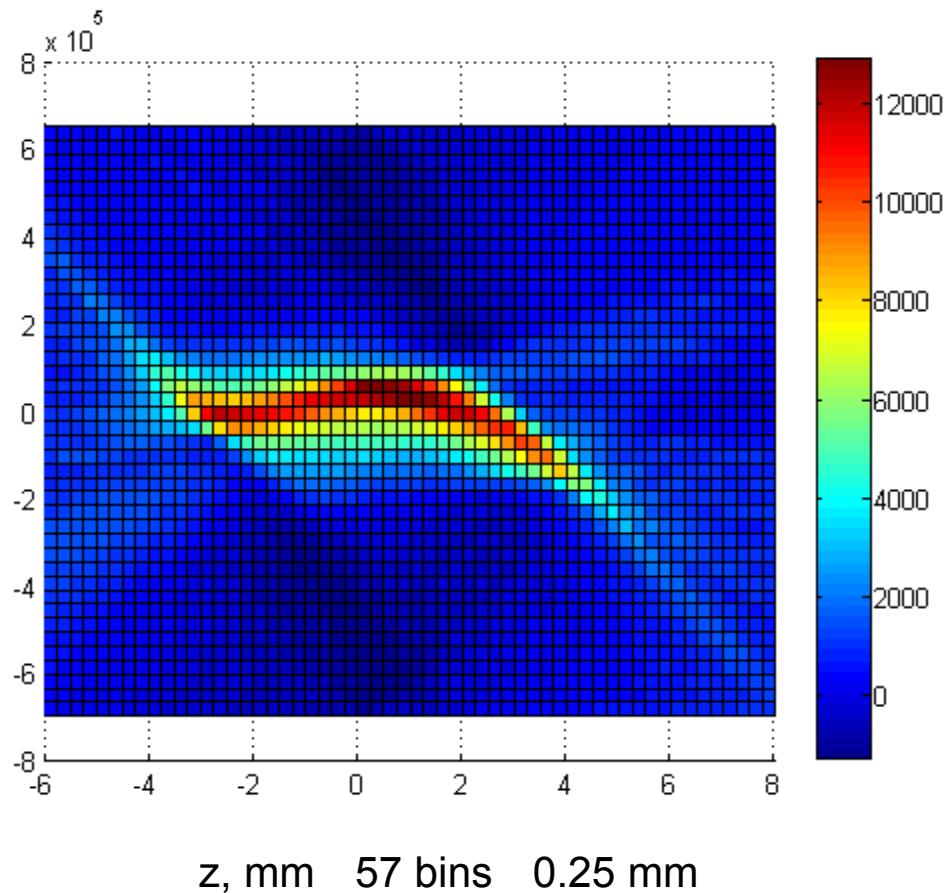


# Reconstructed phase space, 5 iterations

Momentum, eV/c

43 bins

30 keV/c



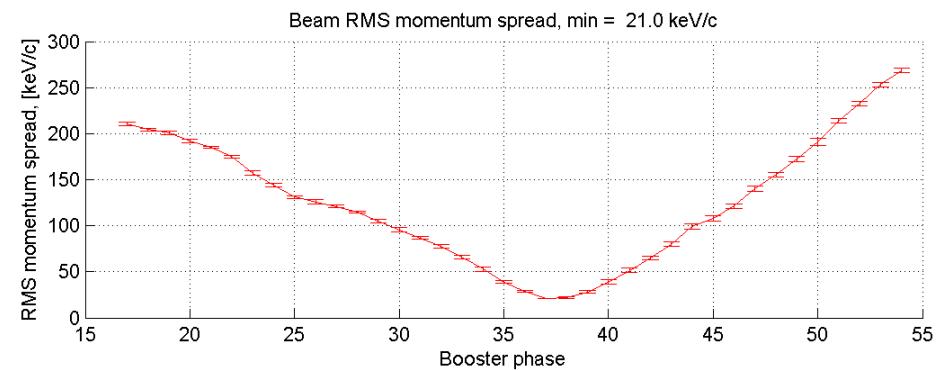
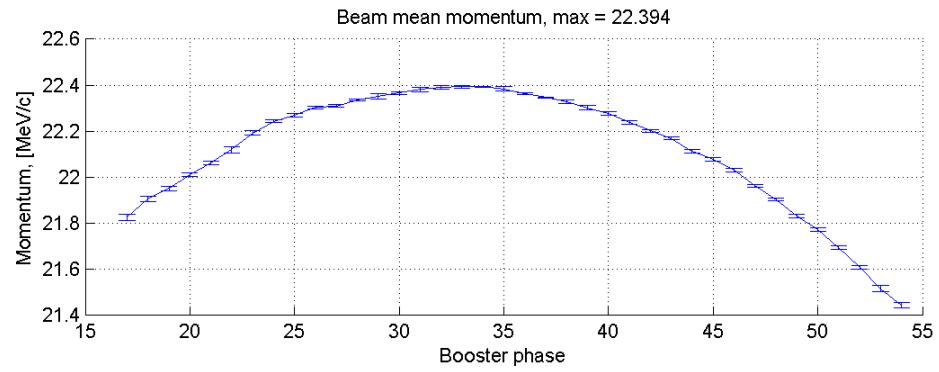
# Experimental results, 2013.02.14 A 21:03

Disp3.D1 = 8.9 A (rbk = 8.97271 A)  
I<sub>main</sub> = 377A  
High2.Q1 = 1.7 A  
NoP = 1  
LT = 100%

Gun:  
6.8 MW in the gun

Booster:  
3.2 MW in booster

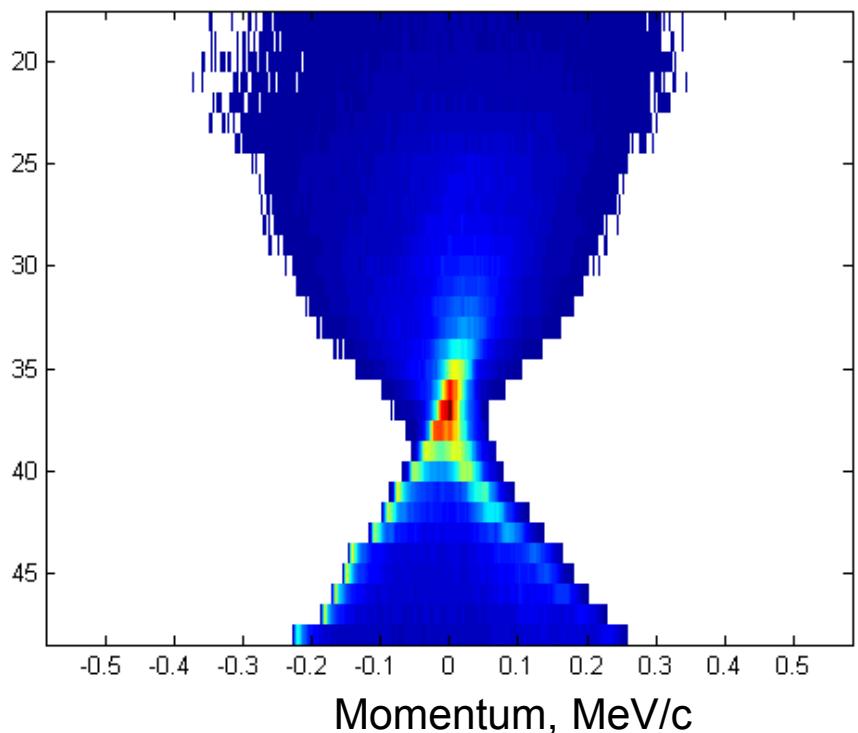
HEDA2 scan, 1 nC, flat top, 17.5 ps FWHM,  
gun on-crest.



# Momentum projections, initial data

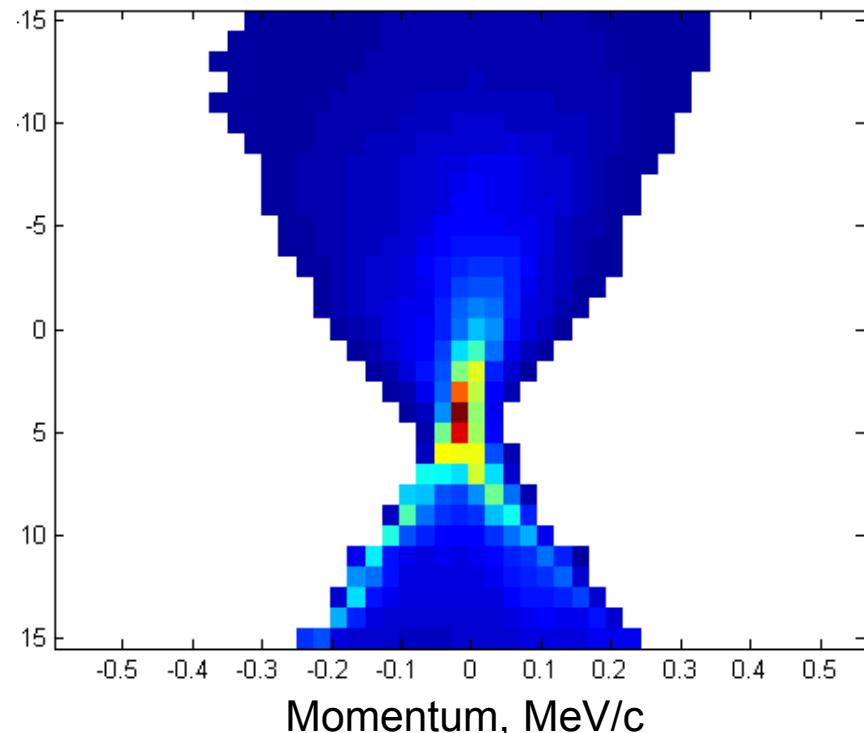
Booster  
phase

31 bins



$z = [-6.0 \quad 8.0]$

57 bins



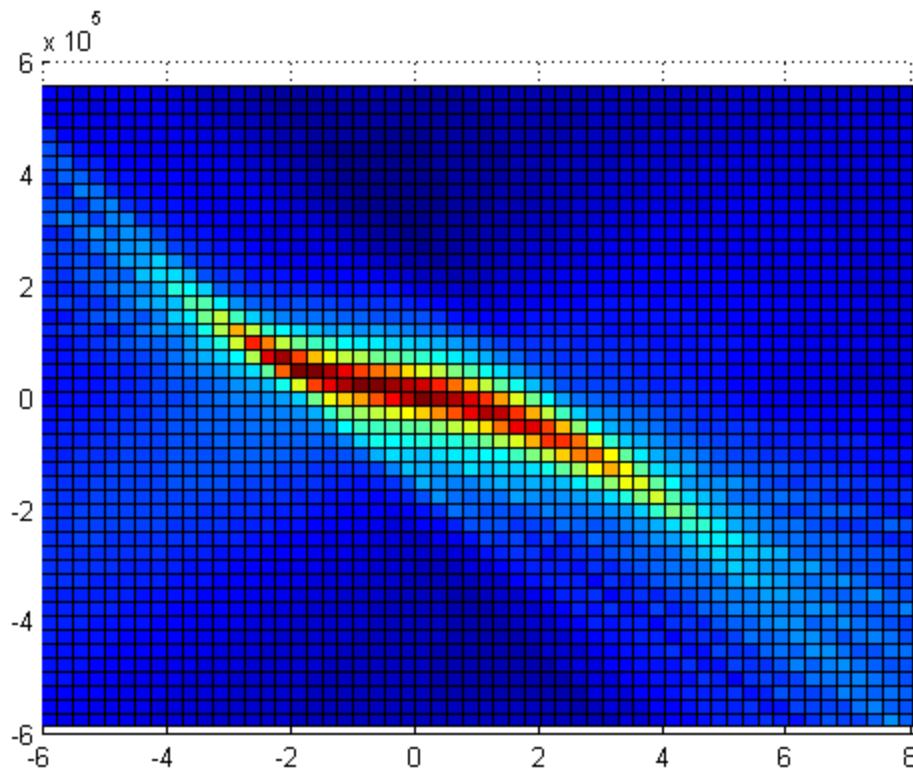
47 bins

# Reconstructed phase space, 1 iteration

Momentum, eV/c

47 bins

25 keV/c



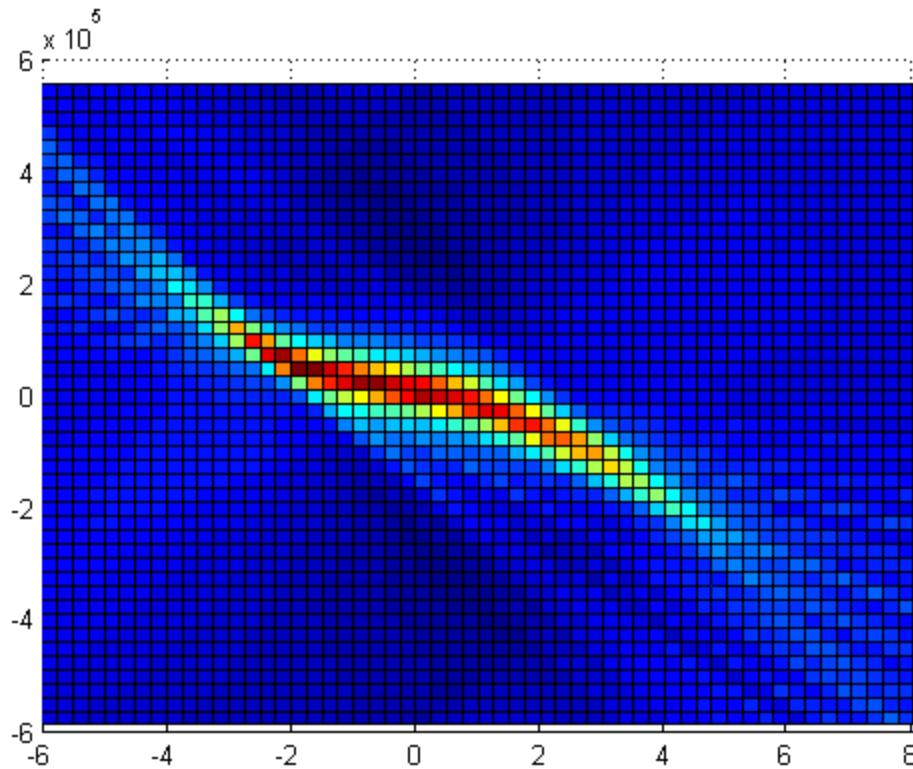
$z, \text{ mm}$  57 bins  $0.25 \text{ mm}$

# Reconstructed phase space, 5 iteration

Momentum, eV/c

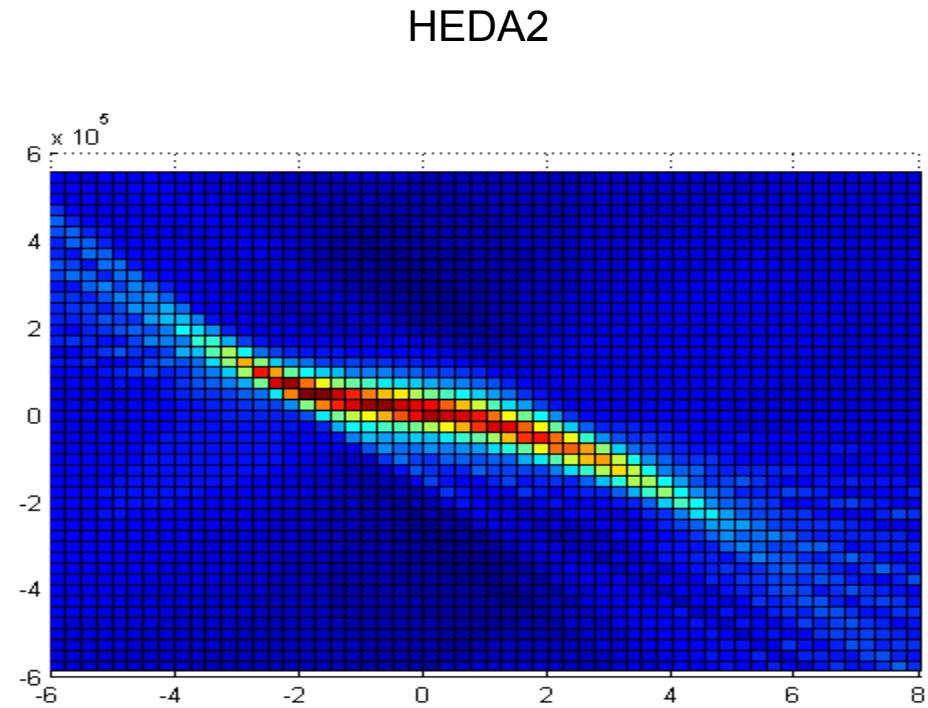
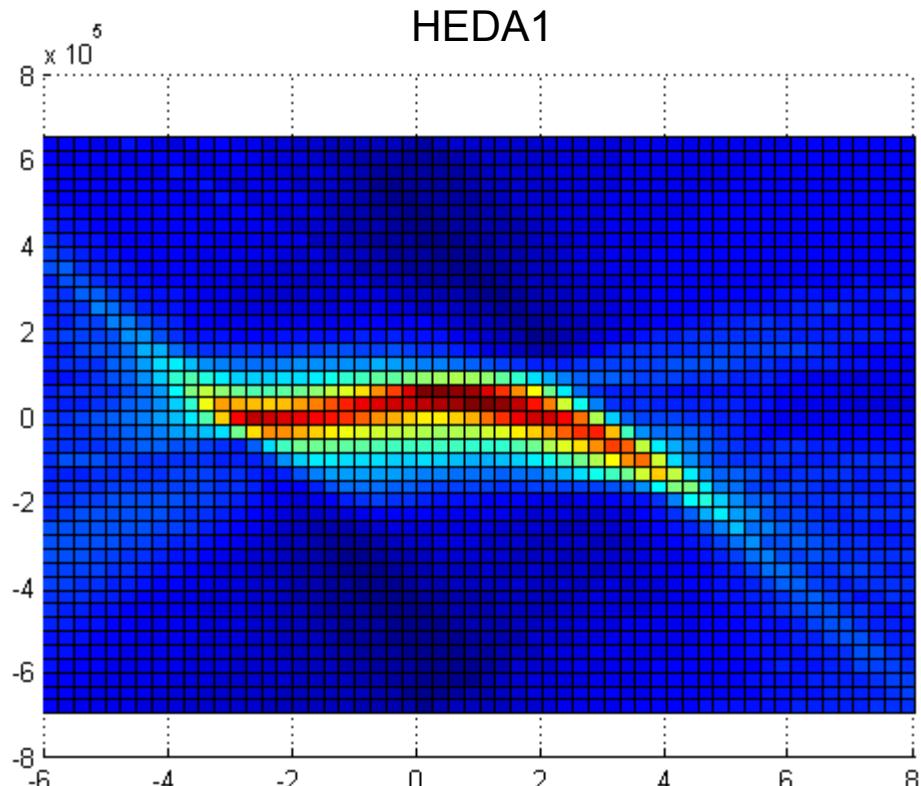
47 bins

25 keV/c

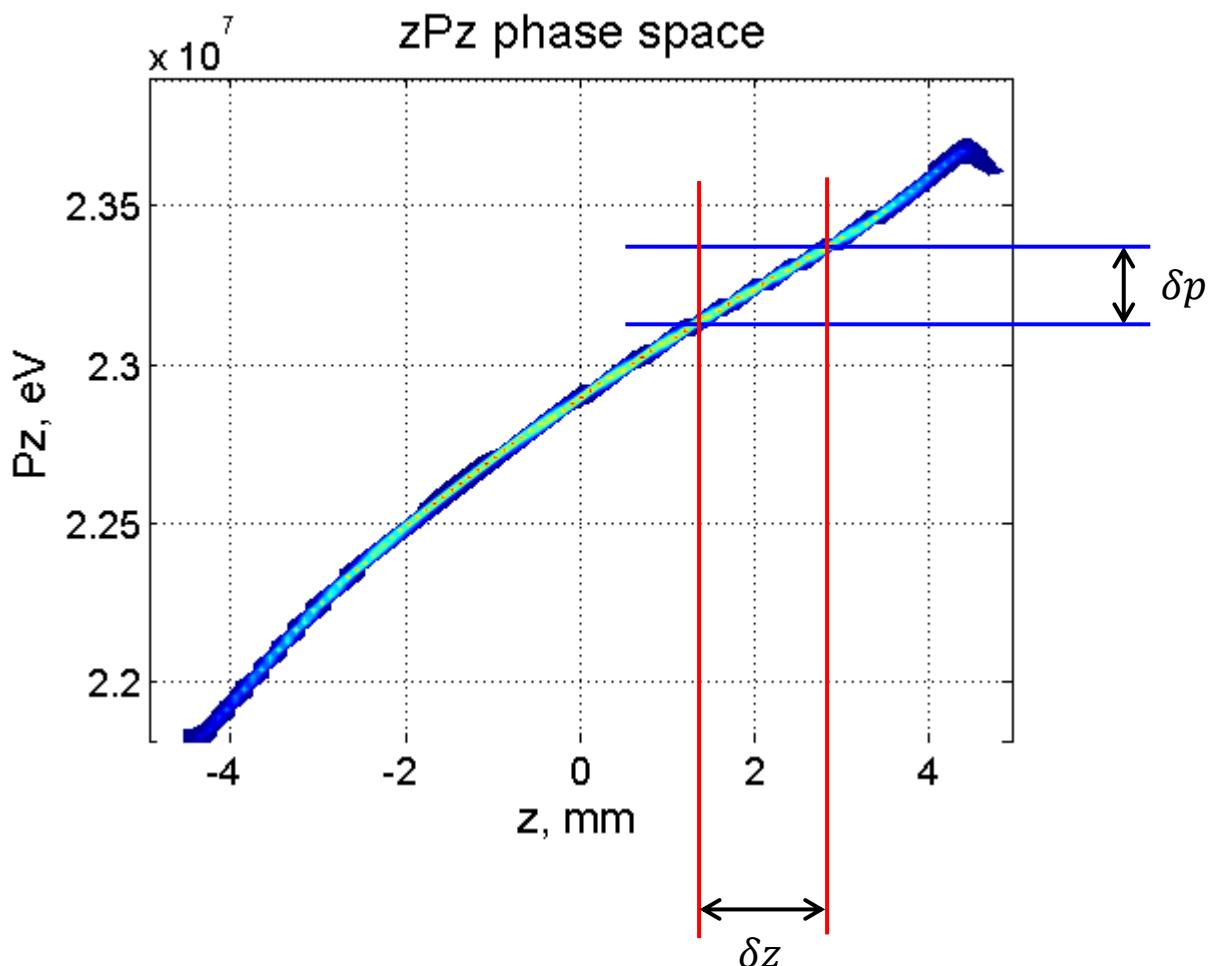


$z, \text{ mm}$  57 bins  $0.25 \text{ mm}$

# Reconstructed phase space, HEDA1 and HEDA2



# Longitudinal resolution



$$\delta z = \frac{\delta p}{k}$$

# Longitudinal resolution (see part I)

$$\frac{dp}{dt} = -18 \cdot 2\pi f \cdot \sin(\varphi) = -147 \frac{keV/c}{ps} \cdot \sin(\varphi)$$

$$k = 20 \frac{keV/c}{ps}$$

(for 8° phase offset) → 0.05 ps resolution??? (1 keV energy spread)  
→ 1.5 ps resolution??? (30 keV/c resolution)

???

$$\delta z = \frac{\delta p}{k} \rightarrow f_{res} = \frac{k_{max}}{2\pi\delta p} \rightarrow \delta z = \frac{2\pi\delta p}{k}$$