Booster steering procedure

07 December 2012

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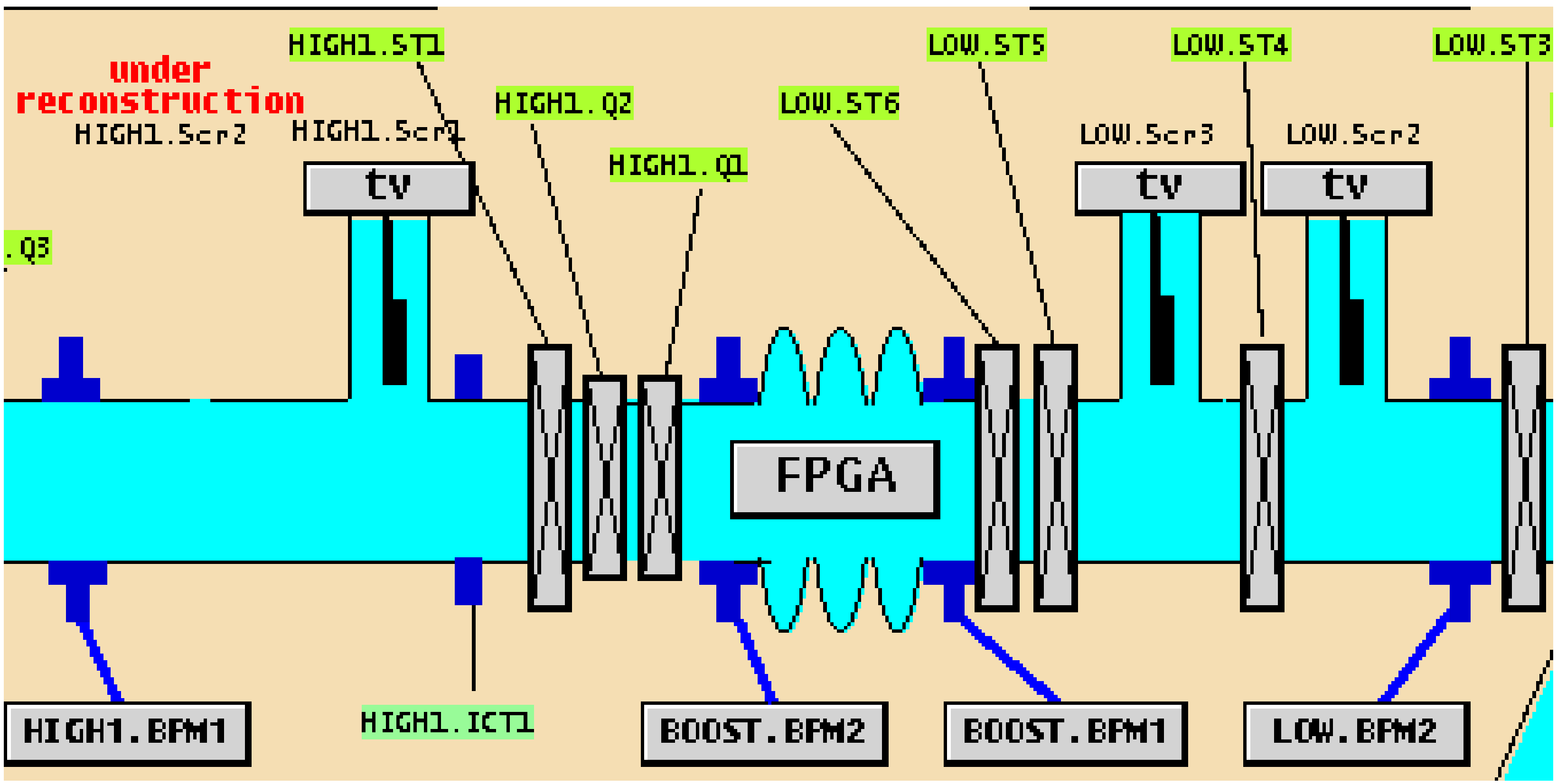
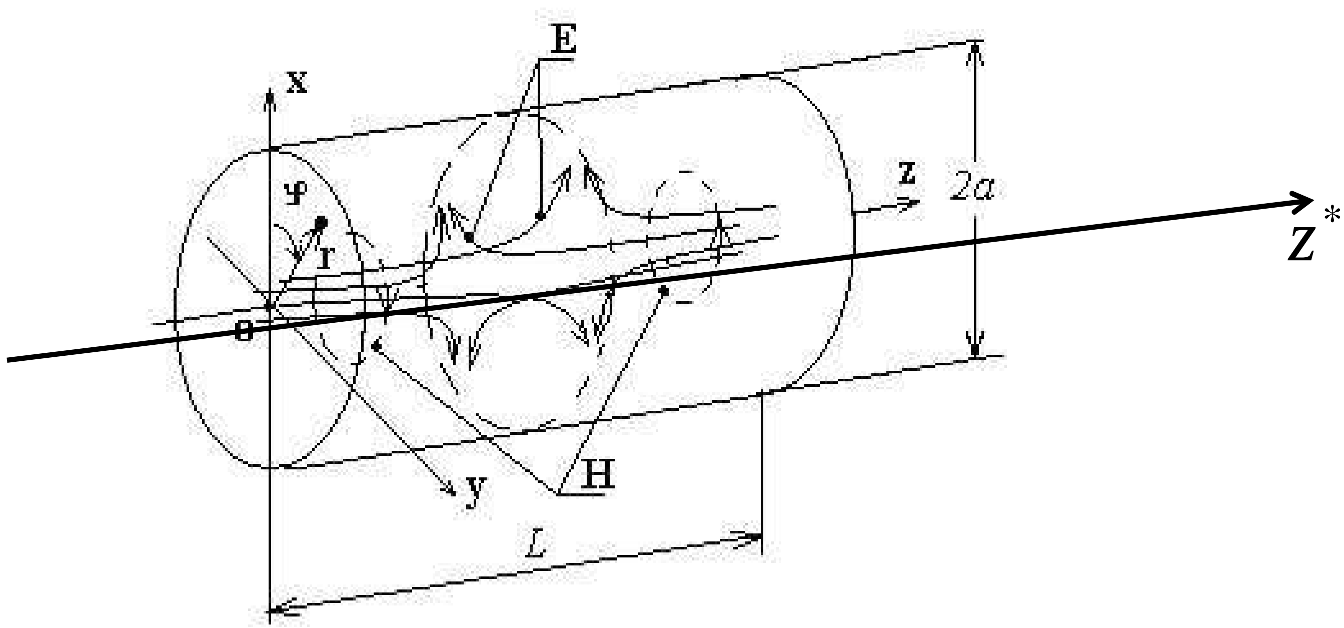
Content

Booster region snapshot from ddd panel



Definition of problem

With some good approximation "steering of booster" means finding central axes of device, where transverse forces are 0.



The variables with asterisks are the variables in coordinate system connected to diagnostic system.

The variables without asterisks are variables in system connected to booster central axes.

So the problem mathematically can be formulated finding offset and orientation between 2 coordinate systems

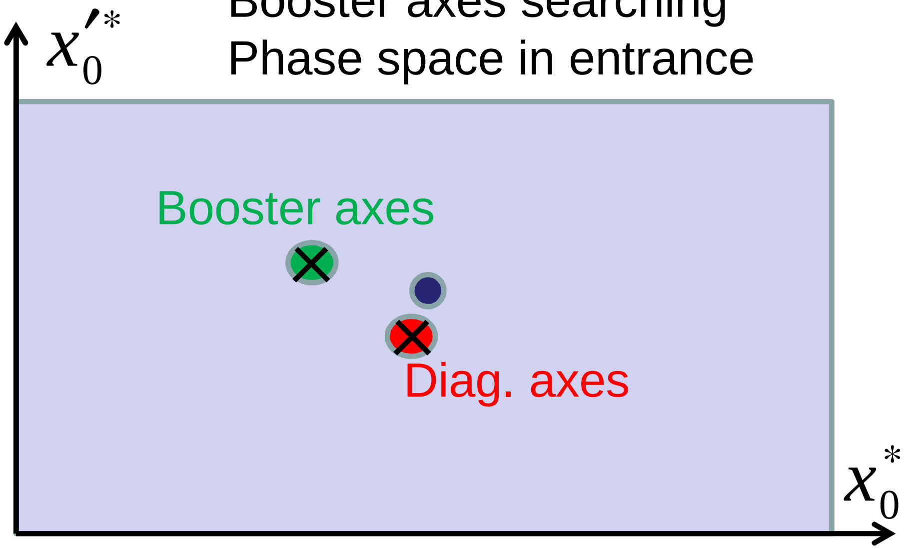
During considering this problem we use transverse radius vector derivative by z instead of transverse impulses

$$\vec{p}_{\perp} \rightarrow \frac{d\vec{r}_{\perp}}{dz} \equiv \vec{\chi}_{\perp}$$

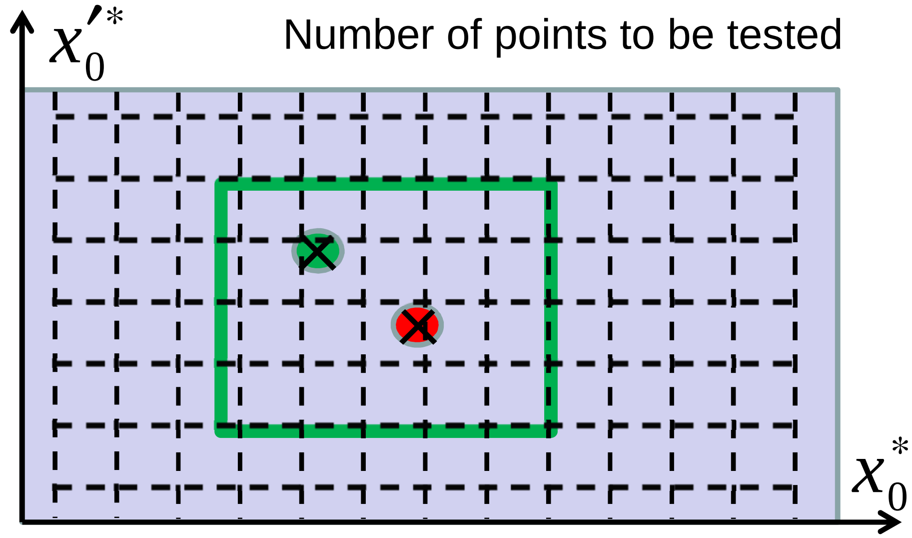
For decartian coordinates $\chi_x = \dfrac{dx}{dz}, \chi_y = \dfrac{dy}{dz}$

For cylindrical coordinates $\chi_r = \dfrac{dr}{dz}, \chi_\varphi = \dfrac{r d\varphi}{dz}$

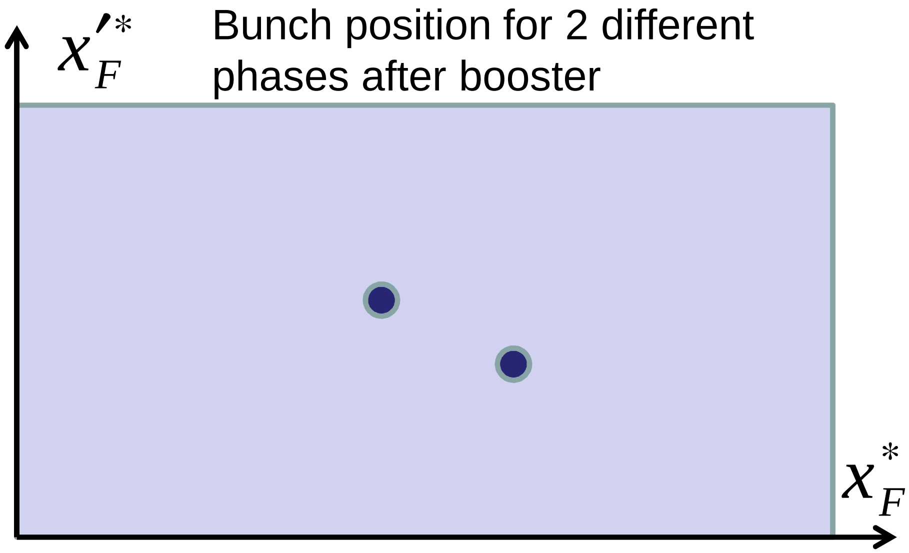
Booster axes searching
Phase space in entrance



Number of points to be tested



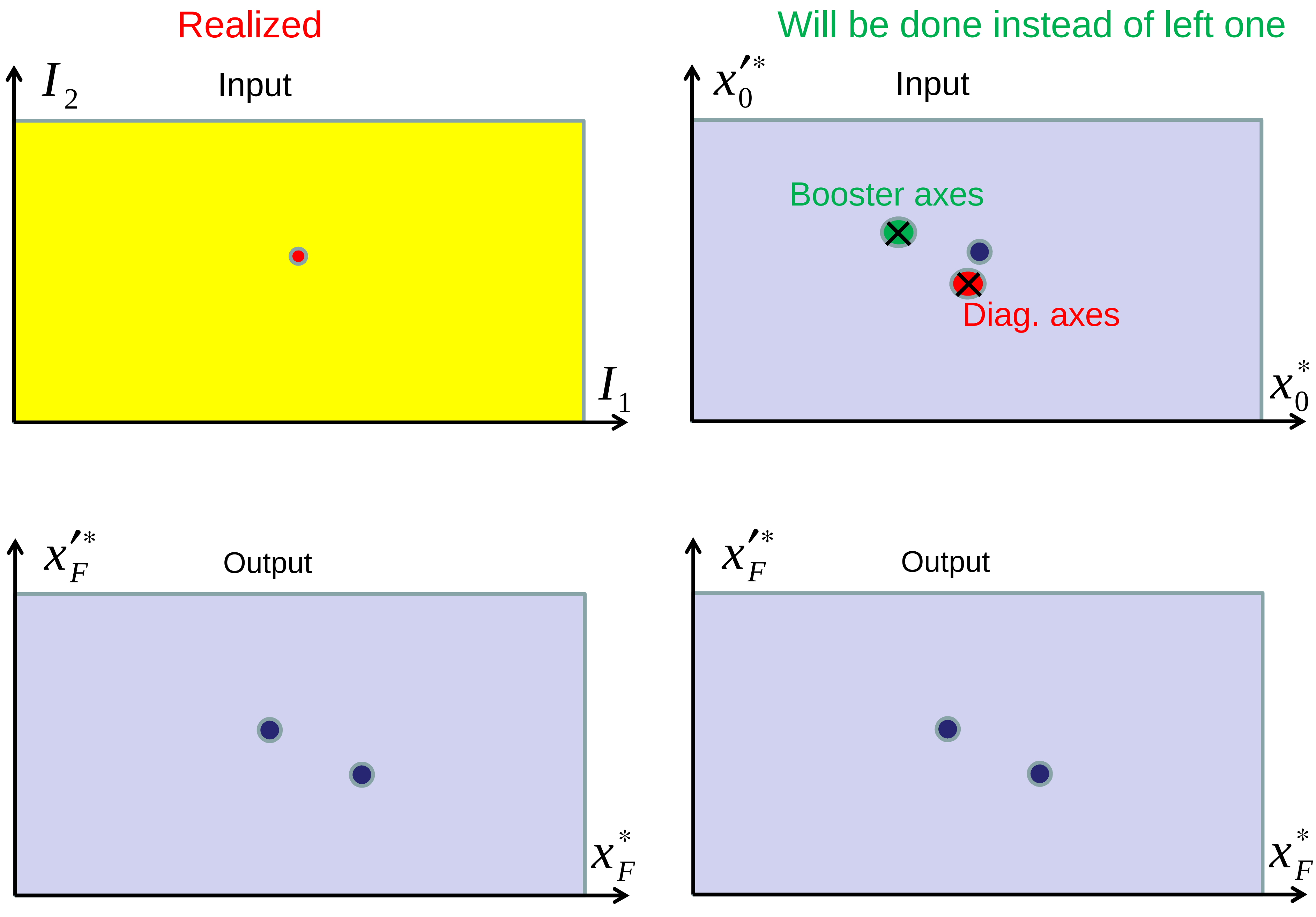
Bunch position for 2 different
phases after booster

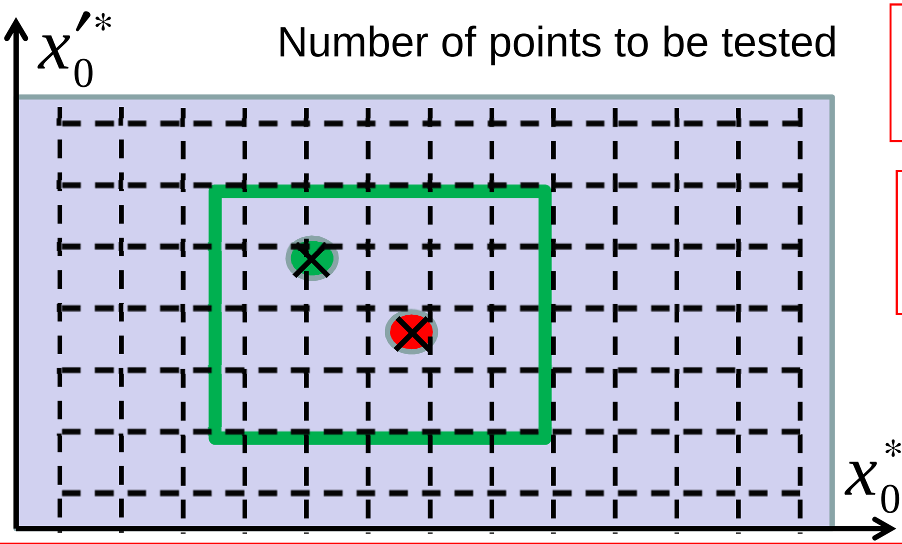


In the worse case number of points in grid by each direction is derived from ratio of acceptance in specified direction of the accuracy in same direction.

$$N_i = \frac{Accept_i}{Accuracy_i}$$

Where i is one of the possible coordinate


Realized
Will be done instead of left one
Input
Input
Booster axes
Diag. axes
Output
Output
I_2
I_1
$x_0'^*$
x_0^*
$x_F'^*$
x_F^*
$x_F'^*$
x_F^*



$$N_x \approx N_{\chi_x} \approx N_y \approx N_{\chi_y} \approx 100$$

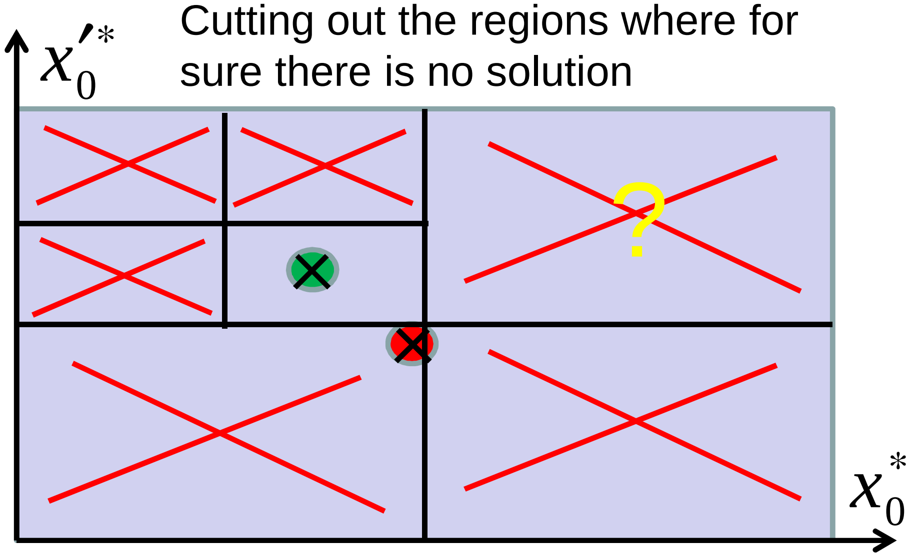
$$N_4 = N_x N_{\chi_x} N_y N_{\chi_y} \approx 10^8$$

$$N_2 = N_x N_{\chi_x} = N_y N_{\chi_y} \approx 10^4$$

Each iteration (one point in above grid) needs booster phase scan. If phase scan will consist of 2 phases (2 phases enough?) then 6 DOOCS call (4 for steerers and 2 for booster phase) must be fulfilled during each iteration. If system frequency is 10Hz, and we assume that in average 200ms is needed for reaching steerers currents in power supplies to set values, then 400ms is needed for trying one point in the grid.

This means that in the case of coupled (4-d) motion up to one year can be needed to find booster axes why in the case of not coupled (2-d) motion maximum one hour is needed to fulfill steering procedure.

Cutting out not needed regions



If we can find good criteria for decision whether there is solution in given region, then we can have in the best case following

$$N_1 = N_x \approx N_{\chi_x} \approx N_y \approx N_{\chi_y} \approx 100$$

$$N_4 = 4 \cdot \log_2 (N_1) = 54$$

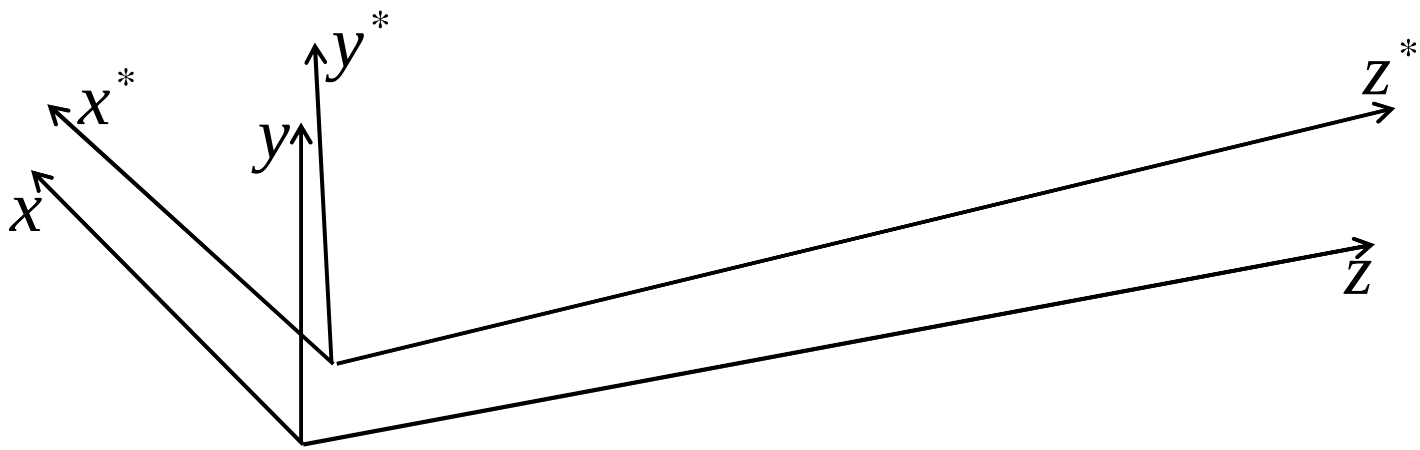
$$N_2 = 2 \cdot \log_2 (N_1) = 27$$

For example one of criterion can be this one: If

$$\left|\Delta x_F^*\right| > \left|\frac{\partial(\Delta x_F^*)}{\partial x_0^*}\right|_{\max} \cdot \Delta x_0^* + \left|\frac{\partial(\Delta x_F^*)}{\partial x_0'^*}\right|_{\max} \cdot \Delta x_0'^* + \left|\frac{\partial(\Delta x_F^*)}{\partial y_0^*}\right|_{\max} \cdot \Delta y_0^* + \left|\frac{\partial(\Delta x_F^*)}{\partial y_0'^*}\right|_{\max} \cdot \Delta y_0'^*$$

then solution is not possible in this region

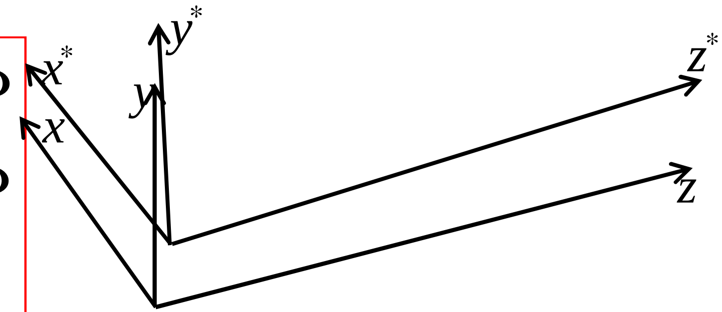
Finding approximate place for searching



$$x = x_d + x^* \cdot \cos\Theta\cos\Phi + y^* \cdot \sin\Phi - z^* \cdot \sin\Theta\cos\Phi$$
$$y = y_d - x^* \cdot \cos\Theta\sin\Phi + y^* \cdot \cos\Phi + z^* \cdot \sin\Theta\sin\Phi \qquad (1)$$
$$z = z_d + x^* \cdot \sin\Theta + y^* \cdot 0 + z^* \cdot \cos\Theta$$

$$x^* = (x - x_d)\cdot\cos\Theta\cos\Phi - (y - y_d)\cdot\cos\Theta\sin\Phi - (z - z_d)\cdot\sin\Theta$$
$$y^* = (x - x_d)\cdot\sin\Phi + (y - y_d)\cdot\cos\Phi + (z - z_d)\cdot 0 \qquad (2)$$
$$z^* = -(x - x_d)\cdot\sin\Theta\cos\Phi + (y - y_d)\cdot\sin\Theta\sin\Phi + (z - z_d)\cdot\cos\Theta$$

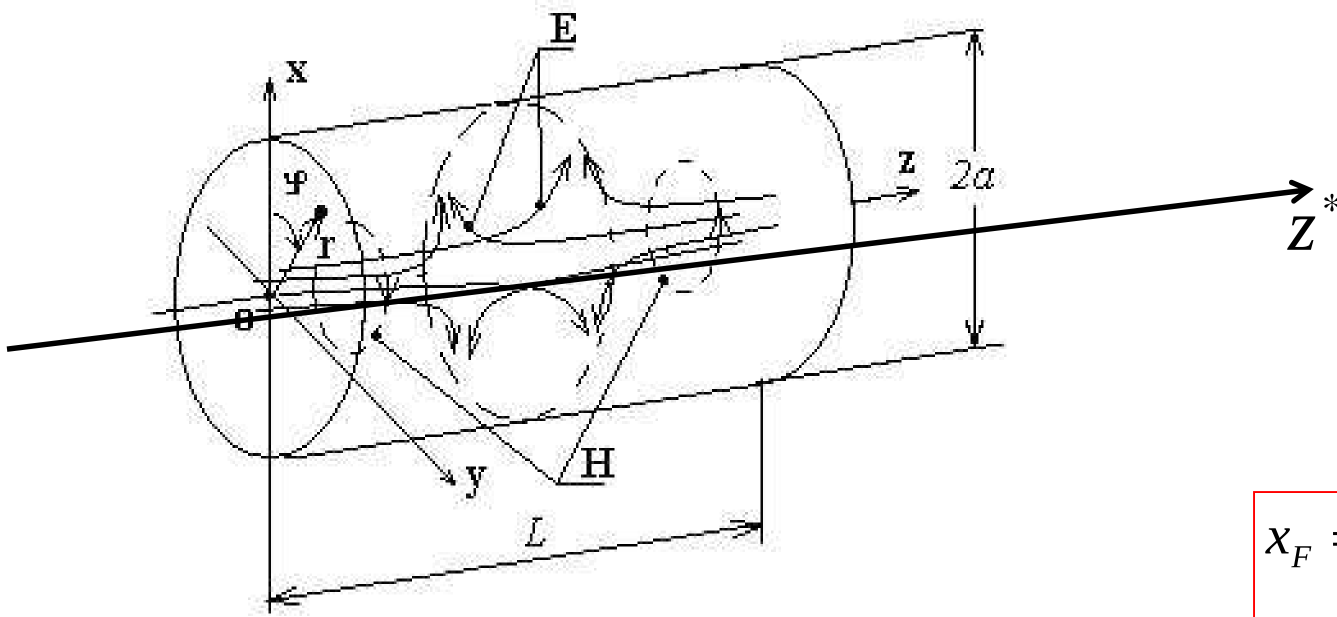
$$x = x_d + x^* \cdot \cos\Theta\cos\Phi + y^* \cdot \sin\Phi - z^* \cdot \sin\Theta\cos\Phi$$
$$y = y_d - x^* \cdot \cos\Theta\sin\Phi + y^* \cdot \cos\Phi + z^* \cdot \sin\Theta\sin\Phi$$
$$z = z_d + x^* \cdot \sin\Theta + y^* \cdot 0 + z^* \cdot \cos\Theta$$



$$x^* = (x - x_d)\cdot\cos\Theta\cos\Phi - (y - y_d)\cdot\cos\Theta\sin\Phi - (z - z_d)\cdot\sin\Theta$$
$$y^* = (x - x_d)\cdot\sin\Phi + (y - y_d)\cdot\cos\Phi + (z - z_d)\cdot 0$$
$$z^* = -(x - x_d)\cdot\sin\Theta\cos\Phi + (y - y_d)\cdot\sin\Theta\sin\Phi + (z - z_d)\cdot\cos\Theta$$

$$\chi_x = \frac{\chi_x^* \cdot \cos\Theta\cos\Phi - \chi_y^* \cdot \cos\Theta\sin\Phi - \sin\Theta\cos\Phi}{\chi_x^* \cdot \sin\Theta + \cos\Theta}$$
$$\chi_y = \frac{-\chi_x^* \cdot \cos\Theta\sin\Phi + \chi_y^* \cdot \cos\Phi + \sin\Theta\sin\Phi}{\chi_x^* \cdot \sin\Theta + \cos\Theta} \qquad (3)$$

$$\chi_x^* = \frac{\chi_x \cdot \cos\Theta\cos\Phi - \chi_y \cdot \cos\Theta\sin\Phi - \sin\Theta}{-\chi_x \cdot \sin\Theta\cos\Phi + \chi_y \cdot \sin\Theta\sin\Phi + \cos\Theta}$$
$$\chi_y^* = \frac{\chi_x \cdot \sin\Phi + \chi_y \cdot \cos\Phi}{-\chi_x \cdot \sin\Theta\cos\Phi + \chi_y \cdot \sin\Theta\sin\Phi + \cos\Theta} \qquad (4)$$



In the booster coordinate system, it is possible to obtain final coordinates dependence on initial coordinates (by means of simulations, or by some analytical approximations)(for fixed energy)

$$x_F = x^d(x_0, \chi_{x0}, y_0, \chi_{y0}, \psi)$$
$$\chi_{xF} = \chi_x^d(x_0, \chi_{x0}, y_0, \chi_{y0}, \psi)$$
$$y_F = y^d(x_0, \chi_{x0}, y_0, \chi_{y0}, \psi)$$
$$\chi_{yF} = \chi_y^d(x_0, \chi_{x0}, y_0, \chi_{y0}, \psi)$$

We will direct bunch via z axes connected to our measurement system, then from equations (1)-(4) we can obtain

Writing this for 2 different fixed phases we will obtain

$$x_0 = x_d$$
$$y_0 = y_d$$
$$\chi_{x0} = -\frac{\sin\Theta\cos\Phi}{\cos\Theta}$$
$$\chi_{y0} = \frac{\sin\Theta\sin\Phi}{\cos\Theta}$$

$$\Delta x_F = \Delta x^d(x_0, \chi_{x0}, y_0, \chi_{y0})$$
$$\Delta\chi_{xF} = \Delta\chi_x^d(x_0, \chi_{x0}, y_0, \chi_{y0})$$
$$\Delta\chi_{xF} = \Delta\chi_x^d(x_0, \chi_{x0}, y_0, \chi_{y0})$$
$$\Delta y_F = \Delta y^d(x_0, \chi_{x0}, y_0, \chi_{y0})$$
$$\Delta\chi_{yF} = \Delta\chi_y^d(x_0, \chi_{x0}, y_0, \chi_{y0})$$

=>

$$x_0 = x^{\Delta i}(\Delta x_F, \Delta\chi_{xF}, \Delta y_F, \Delta\chi_{yF})$$
$$\chi_{x0} = \chi_x^{\Delta i}(\Delta x_F, \Delta\chi_{xF}, \Delta y_F, \Delta\chi_{yF})$$
$$y_0 = y^{\Delta i}(\Delta x_F, \Delta\chi_{xF}, \Delta y_F, \Delta\chi_{yF})$$
$$\chi_{y0} = \chi_y^{\Delta i}(\Delta x_F, \Delta\chi_{xF}, \Delta y_F, \Delta\chi_{yF})$$

$$x_d = x^{\Delta i}(\Delta x_F, \Delta\chi_{xF}, \Delta y_F, \Delta\chi_F)$$
$$\chi_{xd} = \chi_x^{\Delta i}(\Delta x_F, \Delta\chi_{xF}, \Delta y_F, \Delta\chi_F)$$
$$y_d = y^{\Delta i}(\Delta x_F, \Delta\chi_{xF}, \Delta y_F, \Delta\chi_F)$$
$$\chi_{yd} = \chi_y^{\Delta i}(\Delta x_F, \Delta\chi_{xF}, \Delta y_F, \Delta\chi_F)$$

Where

$$\chi_{xd} = -\frac{\sin\Theta\cos\Phi}{\cos\Theta}$$
$$\chi_{yd} = \frac{\sin\Theta\sin\Phi}{\cos\Theta}$$

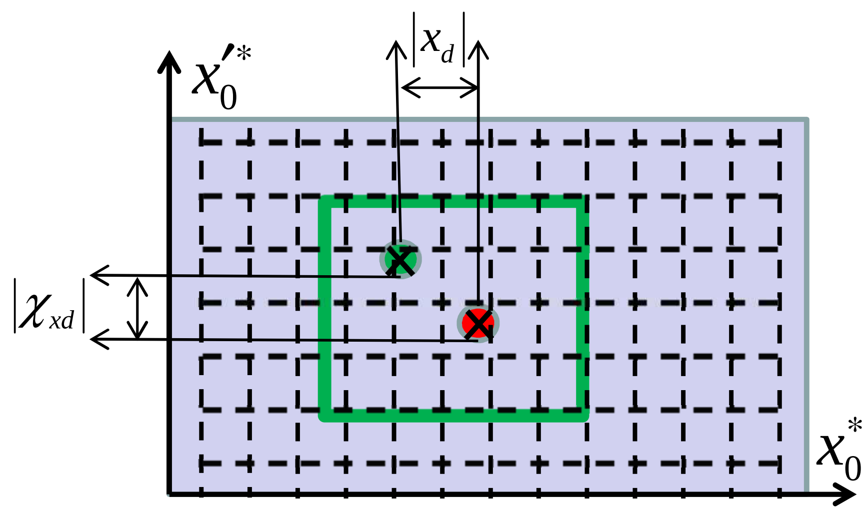
$$\Delta x_F = \Delta x_F^* \cdot \cos\Theta\cos\Phi + \Delta y_F^* \cdot \sin\Phi \approx \Delta x_F^*$$
$$\Delta y_F = -\Delta x_F^* \cdot \cos\Theta\sin\Phi + \Delta y_F^* \cdot \cos\Phi \approx \Delta y_F^*$$

So we will use pertrurbretion theory

$$x_{d0} = x^{\Delta i}(\Delta x_F^*, \Delta\chi_{xF}^*, \Delta y_F^*, \Delta\chi_{yF}^*)$$
$$\chi_{xd0} = \chi_x^{\Delta i}(\Delta x_F^*, \Delta\chi_{xF}^*, \Delta y_F^*, \Delta\chi_{yF}^*)$$
$$y_{d0} = y^{\Delta i}(\Delta x_F^*, \Delta\chi_{xF}^*, \Delta y_F^*, \Delta\chi_{yF}^*)$$
$$\chi_{yd0} = \chi_y^{\Delta i}(\Delta x_F^*, \Delta\chi_{xF}^*, \Delta y_F^*, \Delta\chi_{yF}^*)$$

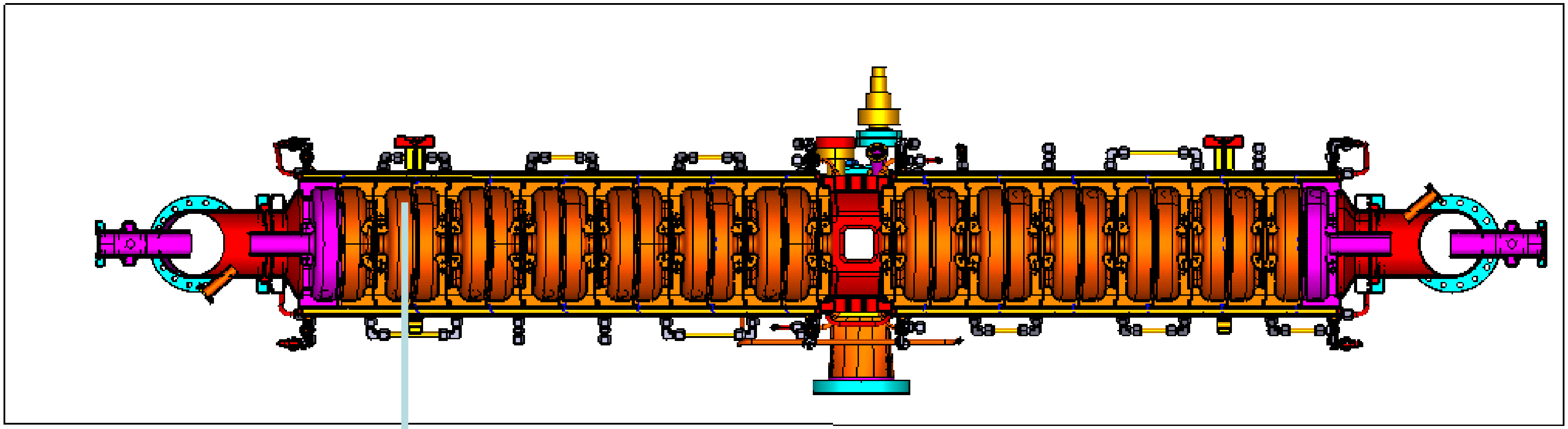
$$\Delta x_{F1} = \Delta x_F^* \cdot \cos\Theta_0\cos\Phi_0 + \Delta y_F^* \cdot \sin\Phi_0$$
$$\Delta y_{F1} = -\Delta x_F^* \cdot \cos\Theta_0\sin\Phi_0 + \Delta y_F^* \cdot \cos\Phi_0$$

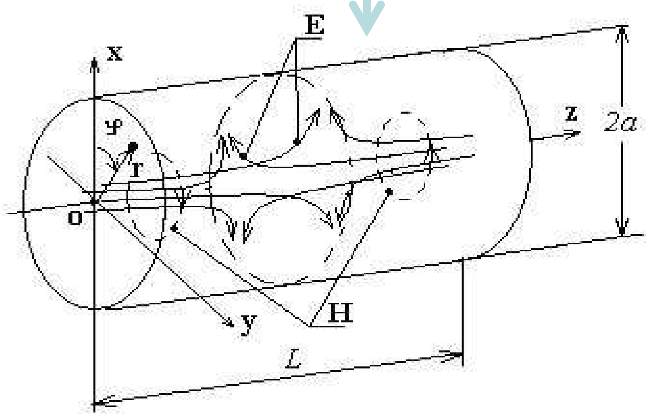
$$x_{d1} = x^{\Delta i}(\Delta x_{F1}, \Delta\chi_{xF1}, \Delta y_{F1}, \Delta\chi_{yF1}, \Delta p_{zF1}, \Delta\psi)$$
$$\chi_{xd1} = \chi_x^{\Delta i}(\Delta x_{F1}, \Delta\chi_{xF1}, \Delta y_{F1}, \Delta\chi_{yF1}, \Delta p_{zF1}, \Delta\psi)$$
$$y_{d1} = y^{\Delta i}(\Delta x_{F1}, \Delta\chi_{xF1}, \Delta y_{F1}, \Delta\chi_{yF1}, \Delta p_{zF1}, \Delta\psi)$$
$$\chi_{yd1} = \chi_y^{\Delta i}(\Delta x_{F1}, \Delta\chi_{xF1}, \Delta y_{F1}, \Delta\chi_{yF1}, \Delta p_{zF1}, \Delta\psi)$$

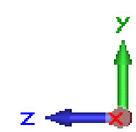


EM fields and equation of motions

With some approximation each cell can be discussed us cylindrical resonator





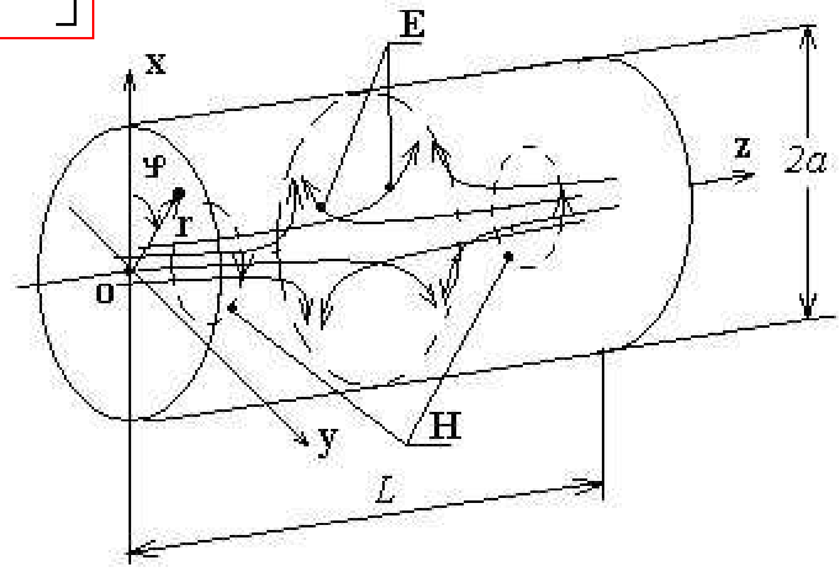


$$E_z(r,\varphi,z,t)=\sum_{m,n,p}\left\{E_{mnp}J_m\left(\frac{\chi_{mn}}{a}r\right)\cos(m\varphi)\cos\left(\frac{p\pi}{L}z\right)\cos\left(\omega_{mnp}t+\psi_{mnpE}\right)\right\}$$

$$H_z(r,\varphi,z,t)=\sum_{m,n,p}\left\{H_{mnp}J_m\left(\frac{\chi_{mn}}{a}r\right)\cos(m\varphi)\sin\left(\frac{p\pi}{L}z\right)\cos\left(\omega_{mnp}t+\psi_{mnpH}\right)\right\}$$

$$\frac{\partial^2\vec{E}_\perp}{\partial z^2}-\varepsilon\mu\frac{\partial^2\vec{E}_\perp}{\partial t^2}=-\vec{\nabla}_\perp\frac{\partial E_z}{\partial z}+\mu\cdot\left[\vec{\nabla}\frac{\partial H_z}{\partial t}\times\hat{e}_z\right]$$

$$\frac{\partial^2\vec{H}_\perp}{\partial z^2}-\varepsilon\mu\frac{\partial^2\vec{H}_\perp}{\partial t^2}=-\vec{\nabla}_\perp\frac{\partial H_z}{\partial z}-\varepsilon\cdot\left[\vec{\nabla}\frac{\partial E_z}{\partial t}\times\hat{e}_z\right]$$



$$\lambda_{mnp} = \frac{2}{\sqrt{\left(\frac{\chi_{mn}}{\pi a}\right)^2 + \left(\frac{p}{L}\right)^2}}$$

$$f_{mnp} = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{\chi_{mn}}{\pi a}\right)^2 + \left(\frac{p}{L}\right)^2}$$

$$\omega_{mnp} = \frac{1}{\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{\chi_{mn}}{a}\right)^2 + \left(\frac{\pi p}{L}\right)^2}$$

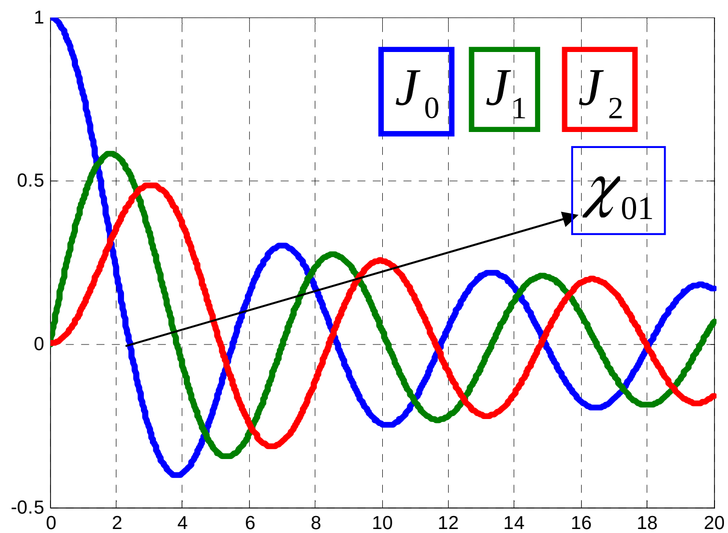


Table for Bessel functions zeros (χ_{mn})

n \ m	0	1	2	3
1	2.405	3.832	5.135	6.379
2	5.520	7.016	8.417	9.760
3	8.654	10.173	11.620	13.015
4	11.792	13.324	14.796	16.223

In our case working mode is E_{001} and we will consider the case of vacuum, this means $v_{light} = c$.

$$a = 0.088m$$

$$f_{010} = \frac{c\chi_{01}}{2\pi a} \approx 1.3 \cdot 10^9\ Hz$$

Equation of motion in cylindrical coordinate system

1. $$\frac{dr}{dz} = \chi_r$$

2. $$\frac{d\chi_r}{dz} = -\frac{1}{p_z}\left\{eB_\varphi + \left(\frac{dp_z}{dz}\right)\chi_r\right\}$$

3. $$r\frac{d\varphi}{dz} = \chi_\varphi$$

4. $$\frac{d\chi_\varphi}{dz} = -\frac{1}{\gamma_z}\left(\frac{d\gamma_z}{dz}\right)\chi_\varphi$$

5. $$\frac{d\tau}{dz} = \frac{\sqrt{1+\gamma_z^2\left(1+\chi_r^2+\chi_\varphi^2\right)}}{\gamma_z}$$

6. $$\frac{d\gamma_z}{dz} = \frac{eE_z\sqrt{1+\gamma_z^2\left(1+\chi_r^2+\chi_\varphi^2\right)}}{m_0c^2\gamma_z} + \frac{eB_\varphi\chi_r}{m_0c}$$

$$\tau = c\cdot t$$

$$\gamma_z = \frac{p_z}{m_0c}$$

$$\chi_r = \frac{p_r}{p_z}$$

$$\chi_\varphi = \frac{p_\varphi}{p_z}$$

$$k_{01} = \frac{\chi_{01}}{a} = 27.33m^{-1}$$

$$E_z = E_0J_0\left(k_{01}r\right)\cos\left(k_{01}\tau+\psi\right)$$

$$B_\varphi = -\frac{E_0}{c}J_1\left(k_{01}r\right)\sin\left(k_{01}\tau+\psi\right)$$

Some analytical approximate results

In the booster the motion is relativistic. For initial energy 7MeV β=0.9974

1. $\tau = z$

2. $\gamma_z = \gamma_{z0} + \dfrac{aE_0}{\chi_{01} m_0 c^2}\left\{\sin\left(k_{01} z + \psi_0\right) - \sin\psi_0\right\}$

3. $\dfrac{dr}{dz} = \chi_r$

4. $r\dfrac{d\varphi}{dz} = \chi_\varphi$

5. $\dfrac{d\chi_r}{dz} = -\dfrac{e}{p_z c}\left\{cB_\varphi(r,z) + E_z(z)\chi_r\right\}$

6. $\dfrac{d\chi_\varphi}{dz} = -\dfrac{eE_z(z)}{cp_z(z)}\chi_\varphi, \Rightarrow$

$$z = L_{cell}, k_{01}L_{cell} = \frac{2\pi}{3}$$

We will consider the case, when r<<a . Then we can expand to Tailor series Bessel functions and cut out not linear terms

$$J_\nu(x) = \left(\frac{x}{2}\right)^\nu \sum_{k=0}^{\infty}\left(-\frac{x^2}{4}\right)^k \frac{1}{k!\,\Gamma(\nu+k+1)}$$

$$E_z = E_0\cos\left(k_0 z + \psi\right)$$

$$B_\varphi = \frac{E_0\omega_0 r}{2c^2}\sin\left(k_0 z + \psi\right)$$

$$\Rightarrow \chi_\varphi(z) = \chi_{\varphi 0}\cdot\exp\left(-\frac{e}{m_0 c^2}\int_0^z \frac{E_z(\tau)d\tau}{p_z(\tau)}\right) = \frac{\chi_{\varphi 0}\gamma_{z0}}{\gamma_z}$$

This means that because there is no forces in φ direction. Motion in this direction will dump (like adiabatic damping)

Some results

$$\gamma_z = \gamma_{z0} + R \cdot E_0 \cdot \cos\left(\frac{k_{01}L}{2} + \psi\right), \qquad R = \frac{2N_{cell}ae}{\chi_{01}m_0c^2}\sin\left(\frac{k_{01}L_{cell}}{2}\right) \approx 1.734 \cdot 10^{-6}\,\frac{m}{V}$$

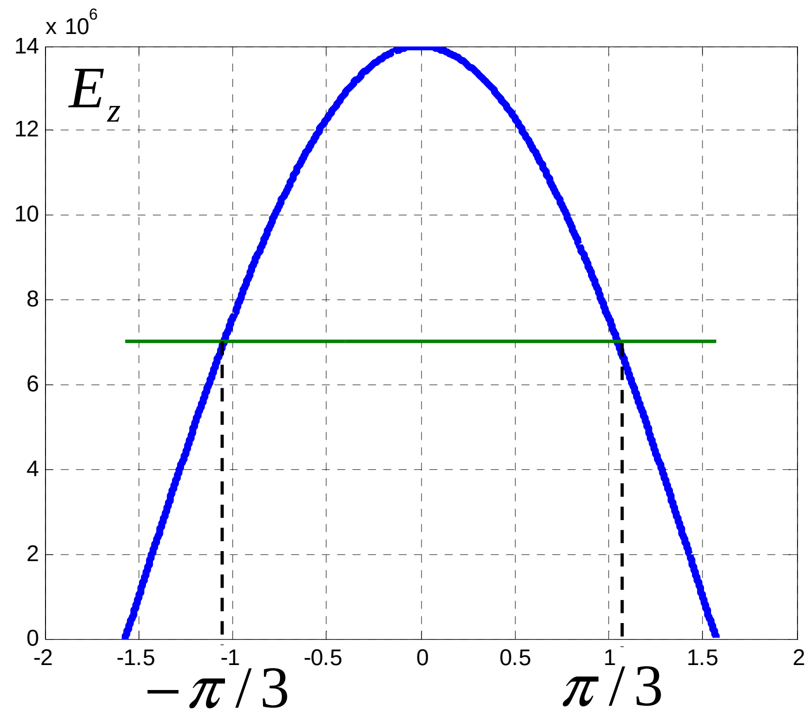
Maximum energy gain will be for phase $\psi = -\frac{k_{01}L}{2} = -\frac{\pi}{3}$ This means

To obtain maximum energy gain for given maximum field in the entrance of booster the field seen by center of bunch must be the half of the amplitude.

$$E_z(entrance) = E_0\cos\left(-\frac{\pi}{3}\right) = \frac{E_0}{2}$$

For maximum energy gain in the exit of booster the field seen by center of bunch must be again the half of the amplitude, but with opposite phase

$$E_z(exit) = E_0\cos\left(\frac{\pi}{3}\right) = \frac{E_0}{2}$$



Equation of motion in Decartian coordinate system

1. $$\frac{dx}{dz} = \chi_x$$

2. $$\frac{d\chi_x}{dz} = -\frac{1}{\gamma_z}\left\{\frac{eB_y}{m_0 c} + \chi_x \frac{d\gamma_z}{dz}\right\}$$

3. $$\frac{dy}{dz} = \chi_y$$

4. $$\frac{d\chi_y}{dz} = \frac{1}{\gamma_z}\left\{\frac{eB_x}{m_0 c} - \chi_y \frac{d\gamma_z}{dz}\right\}$$

5. $$\frac{d\tau}{dz} = \frac{\sqrt{1+\gamma_z^2\left(1+\chi_r^2+\chi_\varphi^2\right)}}{\gamma_z}$$

6. $$\frac{d\gamma_z}{dz} = \frac{eE_z\sqrt{1+\gamma_z^2\left(1+\chi_r^2+\chi_\varphi^2\right)}}{m_0 c^2 \gamma_z} + \frac{e}{m_0 c}\left[B_y\chi_x - B_x\chi_y\right]$$

$$\tau = c \cdot t$$

$$\gamma_z = \frac{p_z}{m_0 c}$$

$$\chi_x = \frac{p_x}{p_z}$$

$$\chi_y = \frac{p_y}{p_z}$$

$$k_{01} = \frac{\chi_{01}}{a} = 27.33 m^{-1}$$

$$\frac{\partial B_x}{\partial x} << \frac{\partial B_x}{\partial y}$$

$$\frac{\partial B_y}{\partial y} << \frac{\partial B_y}{\partial x}$$

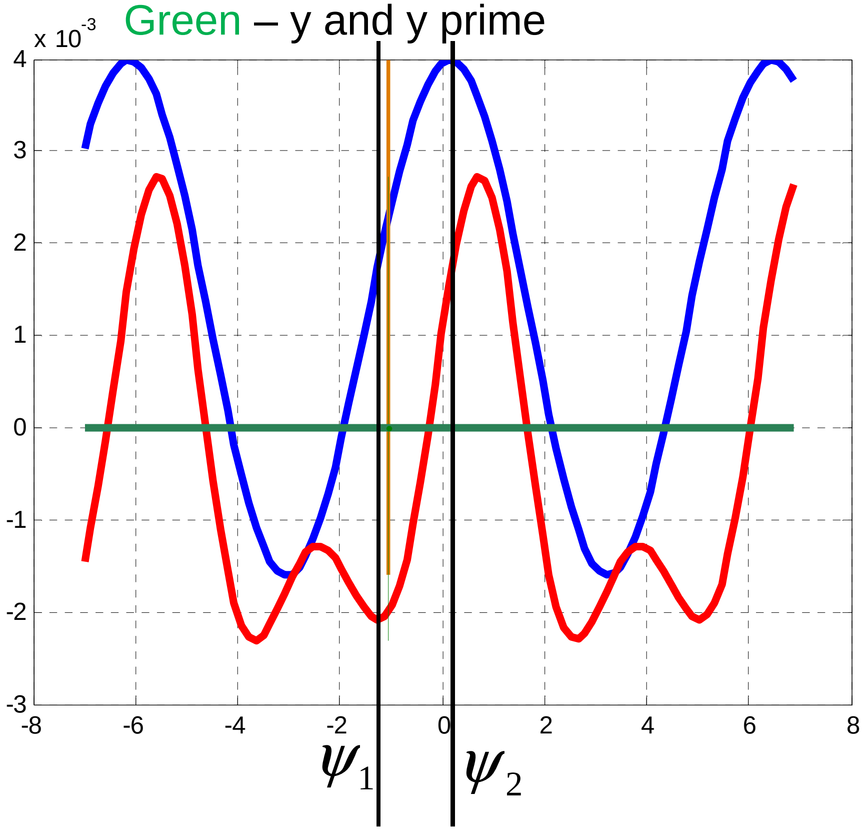
This means that motions are almost not correlated. And it is possible by some approximation (may be bad) to do steering independently.

$$E_z = E_0 J_0\left(k_{01}\sqrt{x^2+y^2}\right)\cos\left(k_{01}\tau+\psi\right)$$

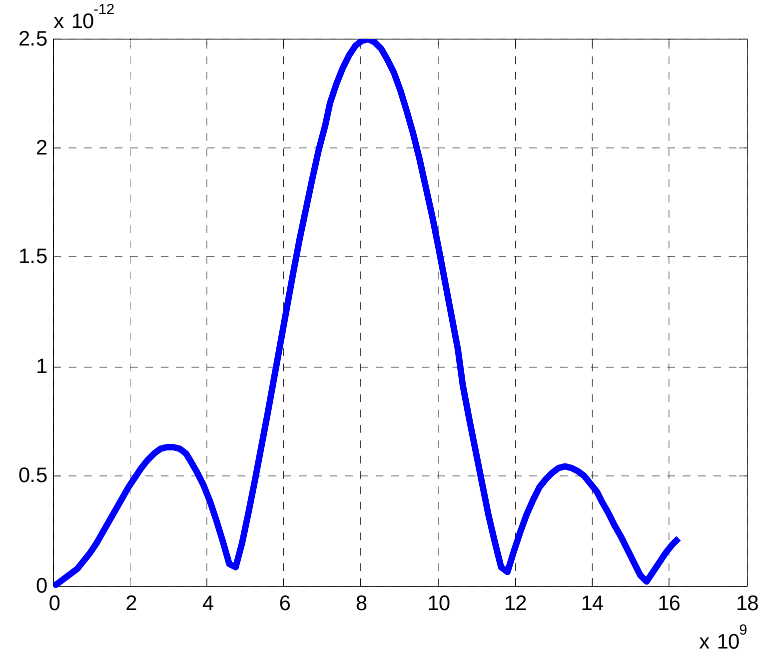
$$B_x = \frac{E_0 y}{c\sqrt{x^2+y^2}} J_1\left(k_{01}\sqrt{x^2+y^2}\right)\sin\left(k_{01}\tau+\psi\right), B_y = -\frac{E_0 x}{c\sqrt{x^2+y^2}} J_1\left(k_{01}\sqrt{x^2+y^2}\right)\sin\left(k_{01}\tau+\psi\right)$$

Final deflection dependences on booster phase

Blue is x dependence on phase
Red is x prime dependence on phase
Green – y and y prime



Fourier transfer of blue curve



One can see from left curves:

a) if y direction is already steered, then changes in x direction will not disturb y steering.
b) 2 phases are enough to distinguish if steering is reached or not.

Conclusion

1. Blindly searching algorithm is realized. MATLAB interface is almost ready.

2. Finding small region for blindly searching can be done ?

3. Genetic algorithm (by A. Bacci) has been realized for searching solutions.

4. At last one more algorithm will be realized ? (for example cutting not needed regions).

5. Equation of motion shows that motion is quasi not correlated, so application will have option to reach steering independently: First for x direction, then for y direction (not parallel). And then 4-d search can be performed in small region (or not, it will be clear after tests).