Interaction of Ultrarelativistic Electron and Proton Bunches with Dense Plasmas

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Introduction

- **Motivation**
- Small-scale accelerators (several meters in length): plasma wakefield accelerators
- In 1949 A. I. Akhiezer, Ya. B. Fainberg [1] proposed to employ the **nonrelativistic** electron bunches for **generation of high plasma wakefields**
- In 1956 G. I. Budker, V. I. Veksler and Ia. B. Fainberg [2] proposed to **accelerate the charged particles in a plasma** medium using collective plasma fields
- In 1961 A. A. Rukhadze showed that the **relativistic** electron bunches can generate plasma waves with high relativistic phase velocity [3]
- Later on, other acceleration schemes were proposed: Laser-plasma wakefield acceleration, time-shifted sequence of injected into a cold plasma electron bunches
- In the work [4] of Caldwell et al., Nature Phys., 2009 there was considered a possibility of the **proton-driven plasma wakefield acceleration**.

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[1] A. I. Ahiezer, Ya. B. Fainberg, Doklady Akademii Nauk SSSR, **69**, 555 (1949).

[2] G. I. Budker, V. I. Veksler, Ia. B. Fainberg *Proc. CERN Symp. on High Energy Accelerators and Pion Physics*, Vol. 1 (Geneva: CERN, 1956), p. 68, p.80, p. 84.

[3] A. A: Rukhadze, Zhurnal Tekhnicheskoj Fiziki, **31**, Nr.10, 1236 (1961).

[4] Caldwell, A., Lotov K., Pukhov, A. and Simon, F., Nature Physics, **5**, (2009).

Content

Interaction of ultrarelativistic e^--bunches with dense plasma

- **Generation and Amplification of a Plasma Wakefield with the help of Ultrarelativistic e^- bunches**
  - Let the ultrarelativistic monoenergetic e^- bunch ($\gamma = 1/\sqrt{1-u^2/c^2} >> 1$) be injected into the cold plasma at $n_b << n_p$, n_b - e^- bunch, the bunch generates the plane wake plasma wave $\vec{E} = \vec{E}_0 \exp(-i\omega t + i\vec{k}\cdot\vec{r})$;
  - Consider the case when $Z||\vec{u}$, $\vec{u}$ - bunch velocity;

Dispersion relation for a plasma+e^--bunch system in a lab. frame of ref. [3]

$$(k^2c^2 - \omega^2 + \omega_p^2 + \omega_b^2\gamma^{-1})\left(1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega_b^2\gamma^{-3}}{(\omega - k_z u)^2}\right) - \frac{k_\perp^2 u^2}{\omega^2}\frac{\omega_p^2\omega_b^2\gamma^{-1}}{(\omega - k_z u)^2} = 0, \quad (1)$$

where ${\omega_{b,p}}^2 = 4\pi e^2 n_{b,p}/m$, $k_z||\vec{u}$, $k_z \sim 1/R_0$, R_0 - bunch radius, $k_\perp$ - transverse coordinate.

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[3] A.F. Alexandrov, L.S. Bogdankevich, A.A. Rukhadze, *Principles of Plasma Electrodynamics* (Springer, Heidelberg, 1984), pp. 167-170.

Interaction of ultrarelativistic e^--bunches with dense plasma

- The **general solution** $\omega(\vec{k})$ of (1) with $(\mathrm{Im}\,\omega > 0)$, $\gamma^2 >> 1$:

$$\omega = k_z u(1+\delta) = \omega_p(1+\delta)$$

$$\delta = \frac{-1+i\sqrt{3}}{2}\left(\frac{n_b}{2n_p}\frac{1}{\gamma}\right)^{1/3}\left(1-\frac{u^2}{c^2}\frac{\omega_p^2}{\omega_p^2+k_\perp^2 u^2}\right)^{1/3}. \qquad (2)$$

- Equation (2) shows that at the **Cherenkov resonance** $\omega \approx k_z u \approx \omega_p$ and $(\mathrm{Im}\,\omega = \mathrm{Im}\,\delta > 0)$ the oscillations are unstable and $\mathrm{Im}\,\delta$ (increment) is the highest;
- From Eq. (2) follows that at $\mathrm{Re}\,\delta < 0 \Rightarrow u > \omega/k_z$.

A . In a case of **dense plasma** when

$$\omega_p^2 >> k_\perp^2 u^2 \sim u^2/R_0^2 \qquad (3)$$

$$\delta = \frac{-1+i\sqrt{3}}{2}\left(\frac{n_b}{2n_p}\right)^{1/3}\frac{1}{\gamma}. \qquad (4)$$

B . In a case of **rare plasma** when the inverse to (3) inequality is satisfied

$$\delta = \frac{-1+i\sqrt{3}}{2}\left(\frac{n_b}{2n_p}\frac{1}{\gamma}\right)^{1/3}. \qquad (5)$$

Interaction of ultrarelativistic e^--bunches with dense plasma

- For the development of instability and **plasma wake growth** the following constraint for a plasma length must be satisfied:

$$L > \frac{u}{\delta\omega_p} \tag{6}$$

being at the same time limited due to the **bunch divergence** to:

$$L < \frac{u\sqrt{\gamma}}{\omega_b} = \frac{u\sqrt{\gamma}}{(n_b/n_p)^{1/2}\omega_p},$$

here δ ($\mathrm{Im}\,\delta$).

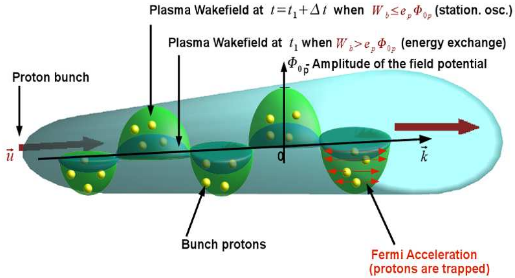
- In accordance with Lorentz transformations the **speed of bunch electrons** in the wake frame of reference will be

$$u_1 = -\frac{u\delta\gamma^2}{1-\frac{2u^2}{c^2}\delta\gamma^2} = -\frac{u\delta\gamma^2}{1-2\delta(\gamma^2-1)}, \tag{7}$$

which remains relativistic!, here the real part of δ ($\mathrm{Re}\,\delta$) is considered.
In the **relativistic limit** when $|\delta|\gamma^2 << 1 \Rightarrow u_1 \approx -u\delta\gamma^2 << u$, whereas in the **ultrarelativistic limit** $|\delta|\gamma^2 >> 1 \Rightarrow u_1 \approx u/2$.

Interaction of ultrarelativistic e^--bunches with dense plasma

- **Saturation amplitude of the plasma wake potential**



The relative potential energy[4]

$$\frac{e\Phi_0}{mc^2} = \frac{1}{\gamma}\left\{ \frac{1}{\sqrt{1-\frac{u^2\delta^2\gamma^4}{c^2(1-2\delta(\gamma^2-1))^2}}} - 1 \right\}, \tag{8}$$

here $\Phi_0\gamma$ is measured in the wake frame of reference.

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[4] A. A. Rukhadze, S. P. Sadykova, Phys. Rev. ST Accel. Beams **15**, 041302 (2012); arXiv:1210.0610.

Interaction of ultrarelativistic e^--bunches with dense plasma

- **Saturation amplitude of the electric wakefield**

The relative energy density of the electric wakefield[4]

$$\frac{E_0^2}{8\pi n_p m c^2 \gamma} \simeq \frac{c^2}{u^2}\frac{1}{2\gamma^3}\left\{\frac{1}{\sqrt{1-\frac{u^2\delta^2\gamma^4}{c^2(1-2\delta(\gamma^2-1))^2}}}-1\right\}^2 . \qquad (9)$$

- In the **relativistic limit** when $|\delta|\gamma^2 << 1 \Rightarrow$ the result obtained by *Kovtun, R. I. and Rukhadze, A. A., JETP **58**, 1970* for dense plasma Eq. (4)
- In the **ultrarelativistic limit** when $|\delta|\gamma^2 >> 1 \Rightarrow$

$$\frac{e\Phi_0}{mc^2} \approx \frac{0.154}{\gamma}, \quad \frac{E_0^2}{8\pi n_p m c^2\gamma} \simeq \frac{0.012}{\gamma^3} \qquad (10)$$

here Φ_0 is measured in the laboratory frame of reference.

Interaction of ultrarelativistic e^--bunches with dense plasma

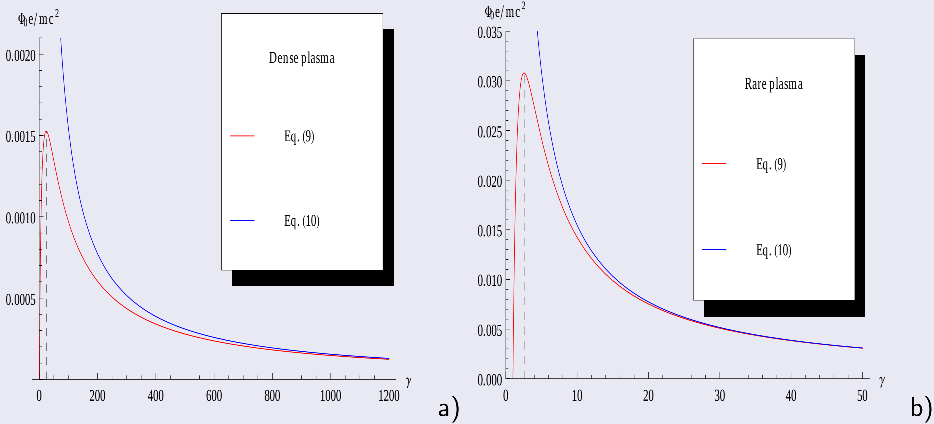

Figure: a) The **relative stationary saturation amplitude of the plasma wake potential** generated by the **electron bunch** Eq. (8) for a) **dense** (4) at $n_b = 2 \cdot 10^{12} cm^{-3}$, $n_p = 10^{16} cm^{-3}$, $\Phi_{0\,max} \simeq 770$ V, $E_{0\,max} \simeq 14.5$ M V/m at $\gamma_0 = 24$ and b) **rare** (5) plasmas at $n_b = 2 \cdot 10^{12} cm^{-3}$, $n_p = 6 \cdot 10^{12} cm^{-3}$, $\Phi_{0\,max} \simeq 15.5$ k V, $E_{0\,max} \simeq 7.2$ M V/m at $\gamma_0 = 2.6$. In a) and b) the relative stationary saturation amplitude in the ultrarelativistic limit (10) is presented for comparison.

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Interaction of ultrarelativistic p^+-bunches with dense plasma

- **Generation and Amplification of a Plasma Wakefield with the help of relativistic p^+ bunches** [4]
  - Let the ultrarelativistic monoenergetic $\boldsymbol{p^+}$ bunch ($\gamma = 1/\sqrt{1-u^2/c^2} >> 1$) be injected into the cold plasma at $\boldsymbol{n_{bi}} << \boldsymbol{n_p}$, n_{bi}- p^+-bunch, the bunch generates the plane wake plasma wave $\vec{E} = \vec{E}_0 \exp(-i\omega t + i\vec{k}\cdot\vec{r})$;
  - Consider the case when $Z||\vec{u}$, $\vec{u}$ - the bunch velocity;

Dispersion relation for a plasma+p^+-bunch system in a lab. frame of ref. [4]

$$(k^2c^2 - \omega^2 + \omega_p^2 + \omega_{bi}^2\gamma^{-1})\left(1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega_{bi}^2\gamma^{-3}}{(\omega - k_z u)^2}\right) - \frac{k_\perp^2 u^2}{\omega^2}\frac{\omega_p^2\omega_{bi}^2\gamma^{-1}}{(\omega - k_z u)^2} = 0, \quad (11)$$

where ${\omega_{bi,p}}^2 = 4\pi e^2 n_{bi,p}/\{M, m\}$, $k_z||\vec{u}$, $k_z \sim 1/R_0$, R_0 - the bunch radius, $k_\perp$ - transverse coordinate.

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[4] A. A. Rukhadze and S. P. Sadykova, Phys. Rev. ST Accel. Beams **15**, 041302 (2012).

Interaction of ultrarelativistic p^+-bunches with dense plasma

- The **general solution** $\omega(\vec{k})$ of (11) with $(\operatorname{Im}\omega > 0)$, $\gamma^2 >> 1$:

$$\omega = k_z u(1 + \delta_1) = \omega_p(1 + \delta_1)$$
$$\delta_1 = \frac{-1 + i\sqrt{3}}{2}\left(\frac{n_{bi}}{2n_p}\frac{m}{M}\frac{1}{\gamma}\right)^{1/3}\left(1 - \frac{u^2}{c^2}\frac{\omega_p^2}{\omega_p^2 + k_\perp^2 u^2}\right)^{1/3}. \qquad (12)$$

- Equation (2) shows that at the **Cherenkov resonance** $\omega \approx k_z u \approx \omega_p$ and $(\operatorname{Im}\omega = \operatorname{Im}\delta_1 > 0)$ the oscillations are unstable and $\operatorname{Im}\delta_1$ (increment) is the highest;
- From Eq. (2) follows that at $\operatorname{Re}\delta_1 < 0 \Rightarrow u > \omega/k_z$.

A . In a case of **dense plasma** when $\omega_p^2 >> k_\perp^2 u^2 \sim u^2/R_0^2$ (Eq. (3)):

- Eq. (4) with $\delta \Rightarrow \delta_1$ and $n_b \Rightarrow n_{bi}\frac{m}{M}$

B . In a case of **rare plasma** when the inverse to (3) inequality is satisfied :

- Eq. (5) with $\delta \Rightarrow \delta_1$ and $n_b \Rightarrow n_{bi}\frac{m}{M}$

- The constraint for a plasma length for the **plasma wake growth**:

$$L > \frac{u}{\delta_1\omega_p} \qquad (13)$$

Interaction of ultrarelativistic p^+-bunches with dense plasma

- **Saturation amplitude of the plasma wake potential**

The relative potential energy[4]

$$\frac{e\Phi_0}{Mc^2} = \frac{1}{\gamma}\left\{\frac{1}{\sqrt{1-\frac{u^2\delta_1^2\gamma^4}{c^2(1-2\delta_1(\gamma^2-1))^2}}}-1\right\}. \tag{14}$$

- **Saturation amplitude of the wake electric wakefield**

The relative energy density of the electric wakefield[4]

$$\frac{E_0^2}{8\pi n_p Mc^2\gamma} \simeq \frac{c^2}{u^2}\frac{1}{2\gamma^3}\left\{\frac{1}{\sqrt{1-\frac{u^2\delta_1^2\gamma^4}{c^2(1-2\delta_1(\gamma^2-1))^2}}}-1\right\}^2. \tag{15}$$

- In the **ultrarelativistic limit** when $|\delta_1|\gamma^2 >> 1 \Rightarrow$

$$\frac{e\Phi_0}{Mc^2} \approx \frac{0.154}{\gamma}, \quad \frac{E_0^2}{8\pi n_p Mc^2\gamma} \simeq \frac{0.012}{\gamma^3}. \tag{16}$$

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[4] A. A. Rukhadze and S. P. Sadykova, Phys. Rev. ST Accel. Beams **15**, 041302 (2012).

Interaction of ultrarelativistic p^+-bunches with dense plasma

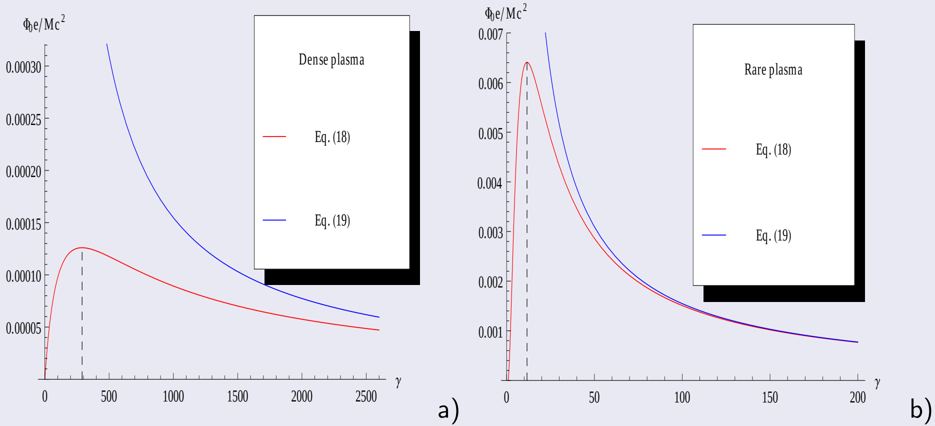

Figure: a) The **relative stationary saturation amplitude of the plasma wake potential** generated by the **proton bunch** Eq. (14) for a) dense (4) at $n_b = 2 \cdot 10^{12} cm^{-3}$, $n_p = 10^{16} cm^{-3}$, $\Phi_{0\,max} \simeq 0.1$ M V, $E_{0\,max} \simeq 50$ M V/m at $\gamma_0 = 289$ and b) rare (5) plasmas at $n_b = 2 \cdot 10^{12} cm^{-3}$, $n_p = 6 \cdot 10^{12} cm^{-3}$, $\Phi_{0\,max} \simeq 5.74$ M V, $E_{0\,max} \simeq 63$ M V/m at $\gamma_0 = 11.6$. In a) and b) the relative stationary saturation amplitude in the ultrarelativistic limit (16) is presented for comparison.

Interaction of ultrarelativistic e^-- and p^+-bunches with dense plasma

The **maximum** magnitudes of the wakefield and wake potential for **dense** plasma

$$\frac{e\Phi_{0\,max}}{\{M,m\}c^2} = -0.06576\alpha, \quad \frac{E_{0\,max}}{\sqrt{\{M,m\}c^2}} = -0.06576\alpha\sqrt{4\pi n_p} \tag{17}$$

The **maximum** magnitudes of the wakefield and wake potential for **rare** plasma

$$\frac{e\Phi_{0\,max}}{\{M,m\}c^2} = -0.0618\alpha^{3/5}, \quad \frac{E_{0\,max}}{\sqrt{\{M,m\}c^2}} = -0.0618\alpha^{3/5}\sqrt{4\pi n_p}, \tag{18}$$

where $\alpha = \frac{-1}{2}\left(\frac{n_{bi}}{2n_p}\frac{m}{M}\right)^{1/3}$ for **proton** bunch and $\alpha = \frac{-1}{2}\left(\frac{n_b}{2n_p}\right)^{1/3}$ for **electron** bunch.

Content I

Conclusion

- In the present work for the first time the analytical problem of interaction of **ultrarelativistic electron and proton bunches with plasmas** has been solved. These bunches remain relativistic in the frame of reference of generated by these bunches wakes compared to those considered earlier which were nonrelativistic.
- The wake amplitude growth produced by the bunches gets saturated with an increase of bunch energy at a not quite high level. The **saturation amplitude of the electric wakefield** possesses a **maximum** in dependence on the relativity factor γ and should be tuned in accordance with the constraints for **Cherenkov resonance** in either dense or rare plasmas, plasma and bunch densities. This amplitude can be increased by increasing the plasma and bunch densities. $E_{0max} \simeq 14.5$ M V/m produced by the **electron bunch** at $\gamma_0 = 24$, $n_p = 10^{16} cm^{-3}$ (dense), whereas that produced by the **proton bunch** is $E_{0max} \simeq 63$ M V/m at $\gamma_0 = 11.6$, $n_p = 6 \cdot 10^{12} cm^{-3}$ (rare). These **longitudinal magnitudes are much less** than the transverse one[4] forming the longitudinal Poynting vector which could be of the same order as that of the **contemporary quite powerful pulse lasers** (10^{15} W/cm^2).

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[4] S. P. Sadykova, A. A. Rukhadze et al., arXiv:1210.0610.

Conclusion

- Nonlinear plasma Langmuir wave is unstable and it breaks down when

$$e\Phi_0 > mc^2\sqrt{\gamma}_w,$$

where $\gamma_w = 1/\sqrt{1 - u_{ph}^2/c^2}$ and u_{ph} is the wave phase velocity. In our case $u_{ph} = u$ and $\gamma_w = \gamma$. The resonance Cherenkov instabilities arise when the **plasma wake amplitudes are much less than the breakdown threshholds**.

- For a better and detailed research we are planing to run simulations of the considered phenomena using the **"KARAT"** code [5]

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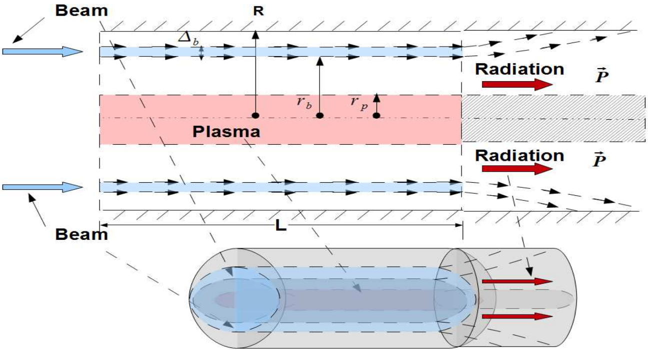
[5] Collaboration with Prof. Rukhadze,A.A. and Tarakanov, V.P., authors of the code, (**Prokhorov General Physics Institute of RAS**); V. P. Tarakanov, *User's Manual for Code KARAT* (USA, VA: Berkeley Research Associates Inc., 1992).

Content

Amplification of a surface electromagnetic wave by running over plasma surface ultrarelativistic electron bunch as a new scheme for generation of Terahertz radiation

- **The surface wave is a wave of E-type with the nonzero field components E_x, E_z, B_y** [3]



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[3] S. P.Sadykova, A. A. Rukhadze, et al. archive: arXiv:1210.0610.

Amplification of a surface electromagnetic wave by running over plasma surface ultrarelativistic electron bunch as a new scheme for generation of Terahertz radiation

- **Estimated parameters**

- $f_0 \simeq 0.5 \cdot 10^{12}$ Hz, $\omega_0 = 3 \cdot 10^{12}$ s^{-1} Since $\omega_0 = \omega_p/\gamma$ then the plasma and bunch parameters can be chosen respectively.
- Electron energy of 50 MeV ($\gamma = 100$) and current density of 500 A/cm^{-2} ($n_b = 10^{11}$ cm^{-3}, net current $I = 3$ A).
- The plasma frequency should be of order $\omega_p = \gamma\omega_0 = 3 \cdot 10^{14}$ s^{-1} and $n_p \simeq 3 \cdot 10^{19}$ cm^{-3}, atmospheric pressure.
- Correspondingly, the time increment will be $\omega_0\delta'' \simeq 2 \cdot 10^8$ s^{-1} $<< \omega_0$ at $a = 0.1$ and the amplification coefficient $\delta'' k_z \simeq 1/L \simeq 7.4 \cdot 10^{-3}$ cm^{-1} leading to the system length of 1.34 m where the plasma radius is $r_p = 0.1$ cm and beam radius is $r_b = 0.2$ cm; $c/\omega_p = 10^{-4} << r_p$, i.e. the plasma surface can be considered as a flat one; the condition $\gamma^2\delta' \simeq 3 > 1$ is satisfied.
- The SEW radiation Poynting vector $|P| = c/4\pi(E_x^2) \simeq 6 \cdot 10^{11}$ W/cm^{-2} and the fields - $|E_x| \simeq |B_y| \simeq 10^7$ V/cm=10^9 V/m.

Acknowledgements

Thank you very much for your Attention !