Paper reveiw

Chromatic effects in quadrupole scan emittance measurements

A. Mostacci, M. Bellaveglia, E. Chiadroni, A. Cianchi, M. Ferrario, D. Filippetto, G. Gatti, C. Ronsivalle

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Galina Asova

HELMHOLTZ
GEMEINSCHAFT

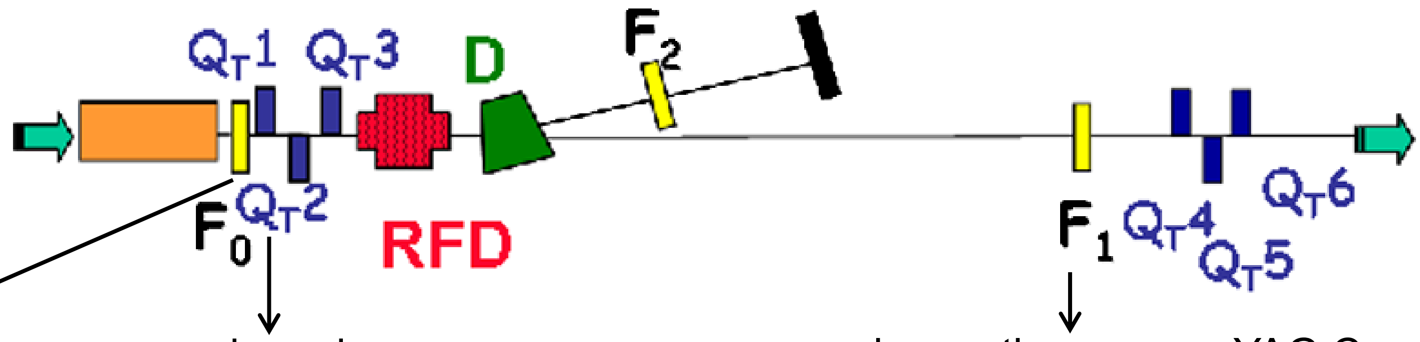
DESY

Overview

- Discusses the emittance degradation due to chromatic effects in measurements deploying quadrupole scans
  - Quad scans are widely used after final acceleration
  - Increased energy spread (velocity bunching VB, wakefield-based acceleration)
- Analytical investigations on the effects using single quad/doublet
- Analytical calculations compared to numerical ones for representative SPARC data
- General assumptions:
  - X and Y planes are not coupled, emittance is geometric
  - Thin-lens approximation

Measurement setup

- 140-170 MeV
- 300 pC
- longitudinal compression via VB



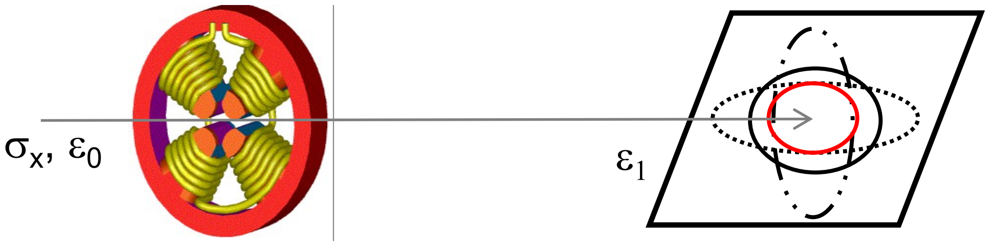
measure spot size

scanned quads Q_T1 or a doublet (Q_T1, Q_T3)

observation screen, YAG:Ce

- When VB → the energy spread $\sigma_\gamma = \sqrt{\langle\delta^2\rangle}$, $\delta = \Delta p/p$ goes to a few percent vs 0.1% without RF compression
  - large σ_{xy} at the entrance of Q_T1 unless additional solenoid focusing is applied

Emittance change in a chromatic single quad line



Quad: k, L_q

$f^{-1} = K(1-\delta)$ for $K = kL_q$

1. **Negligible correlations** between transverse coordinates and energy

Eq. 16

$$\varepsilon_1^2 = \varepsilon_0^2 + K^2\sigma_x^4\sigma_\gamma^2 = \varepsilon_0^2 + \varepsilon_c^2$$

$$\Delta\varepsilon = -\varepsilon_0 + \sqrt{\varepsilon_0^2 + K^2\sigma_x^4\sigma_\gamma^2}$$

> $\varepsilon_0 \ll \varepsilon_c$: $\frac{\Delta\varepsilon}{\varepsilon_0} \sim \frac{\varepsilon_c}{\varepsilon_0}$, i.e. **linear dependance on *k***

for high-brightness beams the geometric emittance ε_0 might be comparable to ε_c

> $\varepsilon_0 \gg \varepsilon_c$: $\frac{\Delta\varepsilon}{\varepsilon_0} \sim \frac{\varepsilon_c^2}{2\varepsilon_0^2}$, i.e. **quadratic dependance**

2. Include **correlations** $x_1 = x_1$, $x_1' = x_0' + K(1-\delta)$

Eq. 20

$$\varepsilon_1^2 = \varepsilon_0^2 + K^2\sigma_x^2\langle(x_0\delta)^2\rangle - K^2\langle x_0^2\delta\rangle^2 + 2K(\langle x_0x_0'\rangle\langle x_0^2\delta\rangle - \langle x_0x_0'\delta\rangle\sigma_x^2)$$

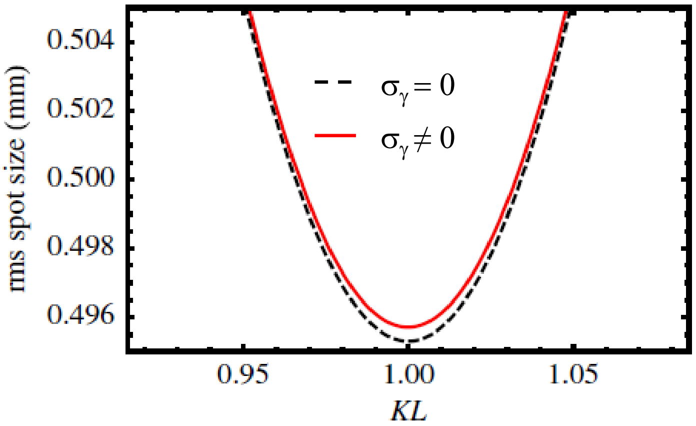
partial compensation of ε_c

DESY

Spot size variations – discard $\sigma_{xx`}$

$\sigma_L^2 = \sigma_{L,\delta=0}^2 + (KL)^2\sigma_x^2\sigma_{x'}^2$, for L – distance from the quad

Before the quad: σ_x = 2 mm $\sigma_{x'}$ = 100 μrad
$\sigma_{xx'}$ = 0 $\sigma_\gamma = 1\%$
L = 5 m

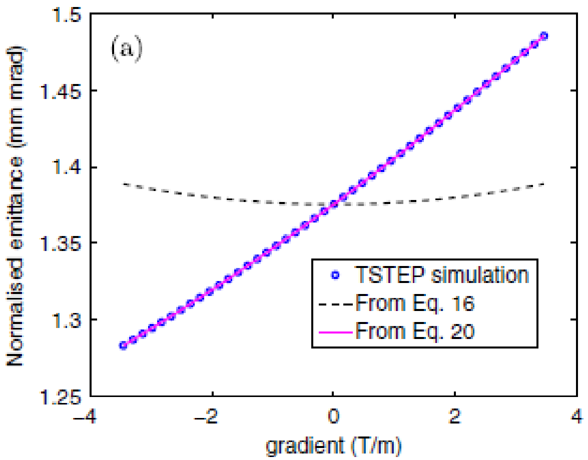


$$\varepsilon_{\mathrm{meas}}^2 = \varepsilon^2 + \frac{\sigma_\gamma^2}{1+\sigma_\gamma^2}\frac{ac}{L^2}$$ **systematics**

as from the scan fit: $a = \sigma_x^2(1+\sigma_\gamma^2),\ c = \sigma_x^2 + L^2\sigma_{x'}^2 + 2L\langle xx'\rangle$

→ **bigger σ_x and/or $\sigma_{x'}$** at the quad entrance, **more the emittance is affected** by energy spread

Single quad calculations



RF comperssion – OFF

p = 148 MeV/c, on crest

$\sigma_{x/y}$ = 0.571 / 0.595 mm

σ_{γ} = 0.169 %

$\varepsilon_{x/y}$ = 4.7e-3/4.8e-3 mm mrad (norm. 1.37/1.4 mm mrad)

$\Delta\varepsilon$ ~ 1% (chromatic term)

Eq. 16: ε_x as $\langle xx' \rangle = 0$

Eq. 20: ε_x as $\langle xx' \rangle \neq 0$

TSTEP - tracking

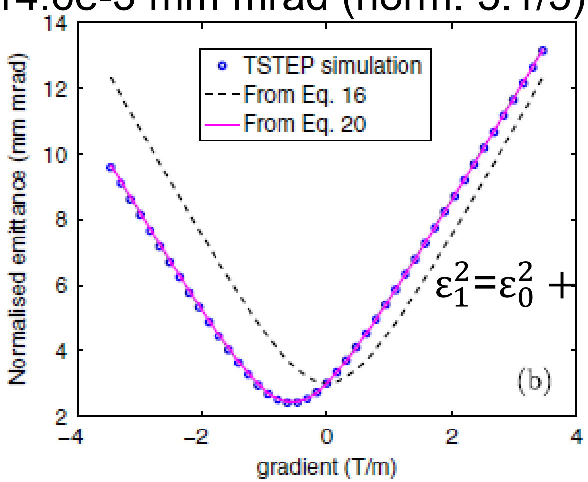
RF comperssion – ON, p = 105 MeV/c

$\sigma_{x/y}$ = 1.74/1.79 mm

σ_{γ} = 1 %

$\varepsilon_{x/y}$ = 14.8e-3/14.6e-3 mm mrad (norm. 3.1/3)

$\Delta\varepsilon$ ~ 30%



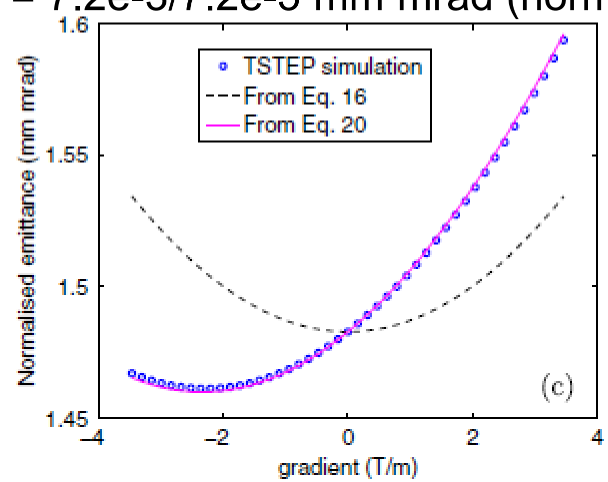
$$\varepsilon_1^2 = \varepsilon_0^2 + K^2 \sigma_x^4 \sigma_{x'}^2$$

$\sigma_{x/y}$ = 0.34/0.34 mm

σ_{γ} = 1 %

$\varepsilon_{x/y}$ = 7.2e-3/7.2e-3 mm mrad (norm. 1.48/1.48)

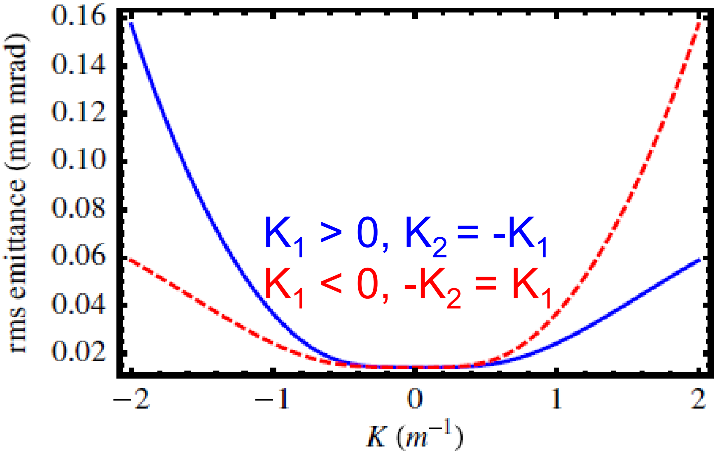
$\Delta\varepsilon$ ~ 1%



Chromatic effect in a doublet

$$\varepsilon_1^2 = \varepsilon_0^2 + F_1\sigma_\gamma^2\sigma_x^4 + F_2\sigma_\gamma^2\sigma_{x'}^4 + F_3\sigma_\gamma^2\sigma_{xx'}^2 + F_4\sigma_\gamma^2\sigma_x^2\sigma_{x'}^2 + F_5\sigma_\gamma^2\sigma_x^2\sigma_{xx'} + F_6\sigma_\gamma^2\sigma_{x'}^2\sigma_{xx'},$$
$$F_i(K_1, K_2, L_{12}), K_2 = -K_1, K_i = K_i(1-\delta)$$

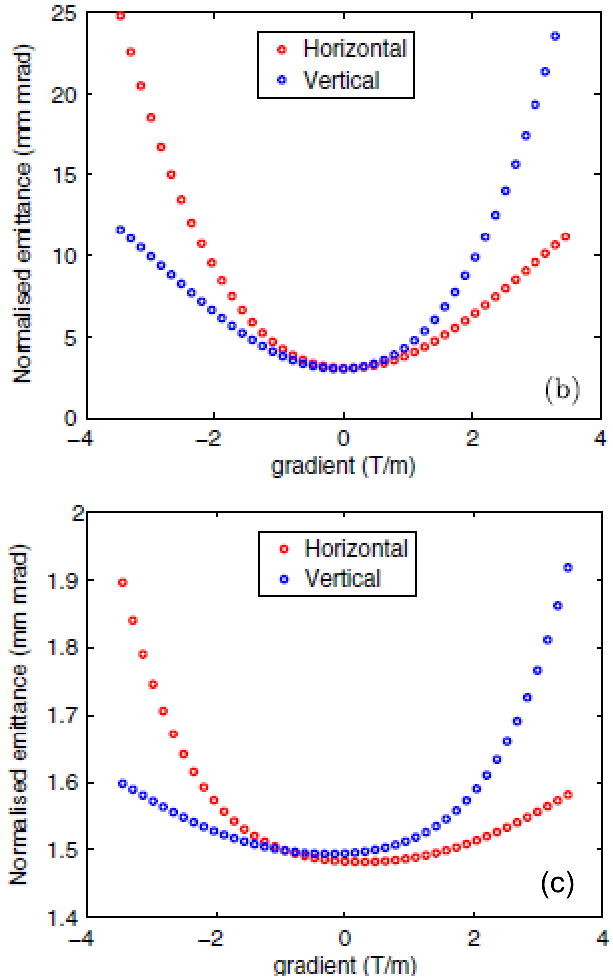
If K_1 focusing in X (> 0) → smaller chromatic contribution $\varepsilon_{1,K_1>0}^2 - \varepsilon_{1,K_1<0}^2 \sim -8K_1^5 L_{12}^3 \sigma_x^4 \sigma_\gamma^2$



Same machine/beam as (b), previous slide

But $\Delta\varepsilon_{doublet} > \Delta\varepsilon_{singleQuad}$

> asymmetric effect due to smaller focusing range



smaller initial spot

Effect of sampling the quadrupole gradient

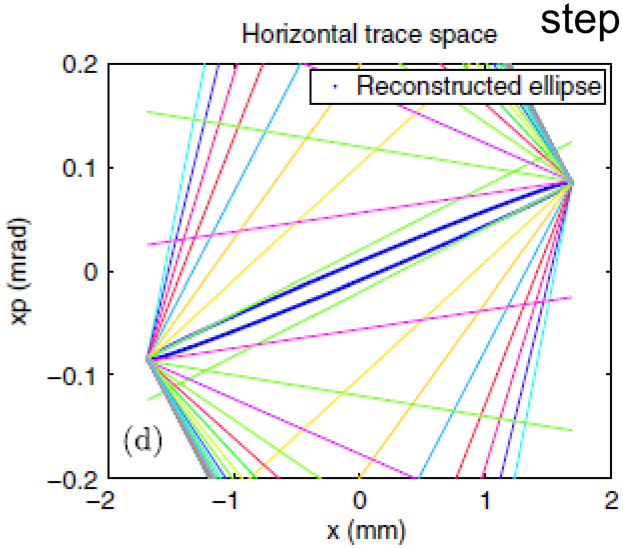
300 pC, monochromatic beam, case (b)

TABLE V. Virtual emittance measurement (TSTEP simulation) for different quadrupole scans of the same beam (parameters of Case 2 in Table I, but with $\sigma_\gamma = 0$) compared to the expected value ε_{ref}.

	$(\varepsilon - \varepsilon_{ref})/\varepsilon_{ref}$ (horizontal plane)	$(\varepsilon - \varepsilon_{ref})/\varepsilon_{ref}$ (vertical plane)
Single quad	$-1.6\%/-3\%$[a]	$-1.6\%/-3\%$[a]
Two quads (+ , −)	$-1.9\%/-4\%$[a]	$+1.98\%/+66\%$[a]
Two quads (− , +)	$+1.4\%/+57\%$[a]	$-1.7\%/-5\%$[a]

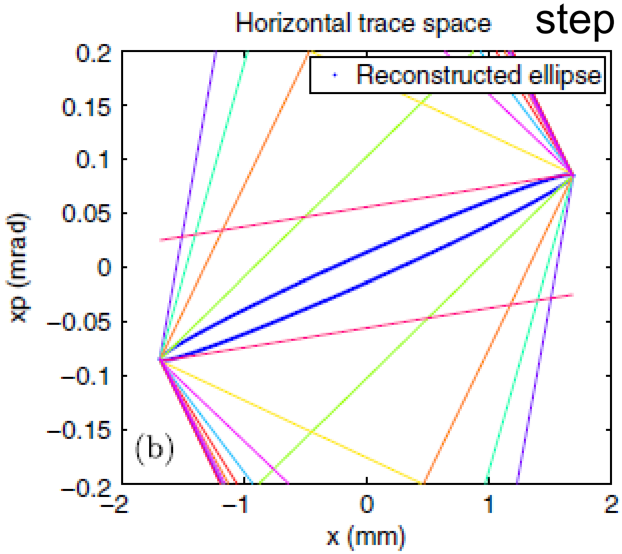
[a]The results are obtained with a gradient step of 0.16 T/m, against the step of 0.08 T/m for all the others scans (300 pC beam).

step 0.08 T/m



ε_n = 3.1 μm
α = -9.5
β = 187 m

step 0.16 T/m



ε_n = 4.8 μm
α = -6.1
β = 121 m

Measurements, 200 pC

1. small incoming beam spot $\sigma_{x/y}$ = 0.32/0.36 mm, negligible σ_γ = 0.1 %, no VB

	Single quad	Two quads (+, −)	Two quads (−, +)
$\varepsilon_{n,x}$ (mm mrad)	1.607 (0.075)	1.852 (0.083)	2.44 (0.20)
$\varepsilon_{n,y}$ (mm mrad)	1.41 (0.26)	2.54 (0.46)	1.77 (0.22)

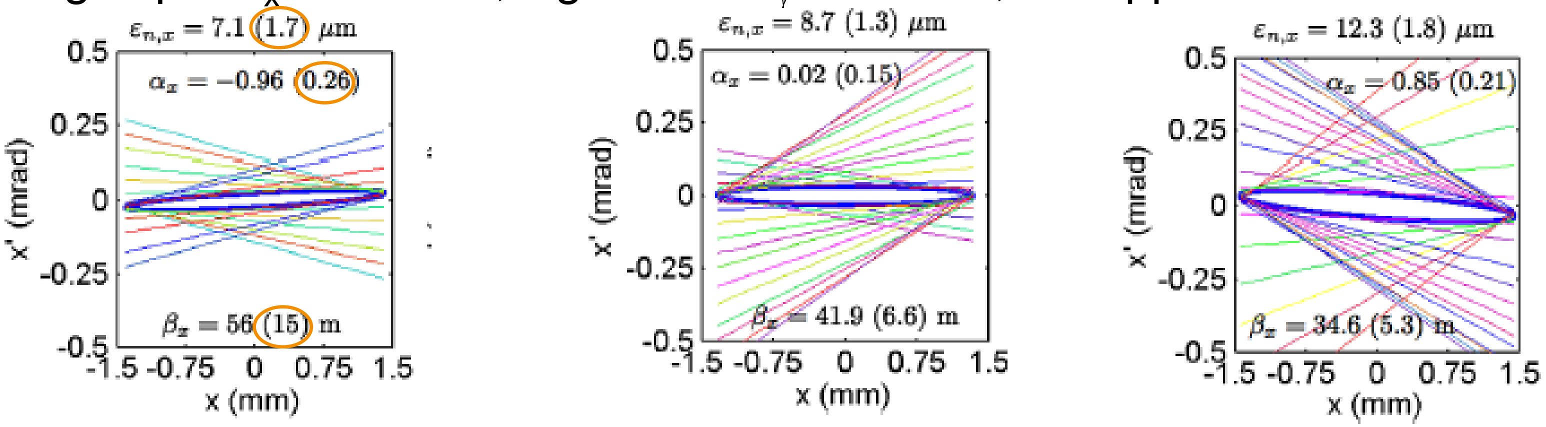
(uncertainty)

Systematically higher ε_{xy} in a doublet scan

ε_{xy} systematically higher for a defocusing first quad

second order effects also for low σ_γ and small σ_{xy}

2. Large spot σ_x = 1.4 mm, significant σ_γ = 0.86 %, VB applied – increased uncertainty



single quad (+, -) (-, +)

Conclusions

- Analytical evaluation of the chromatic effects on the measured emittance

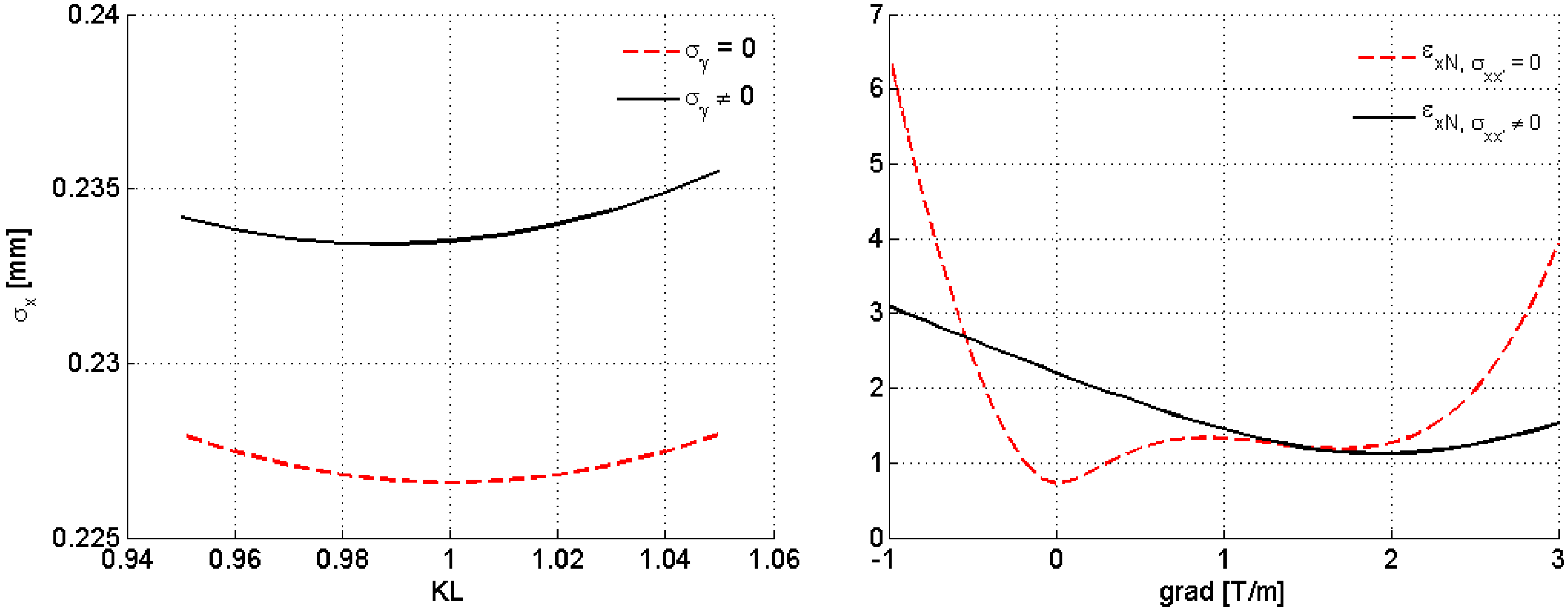
- 2-quad scan:
  - practical advantages (X and Y simultaneously, avoid looses due to aperture)
  - introduces additional errors for beams with high energy spread and large spot size

1 nC, 25 MeV/c

$\varepsilon_{xy, N}$ = 0.73 mm mrad on EMSY1 (ε_x = 0.015 mm mrad)

σ_x = 0.5 mm

σ_γ = 0.4%



$\Delta\sigma_x \sim 4\% \rightarrow \varepsilon_{meas,x}$ = 0.09 mm mrad

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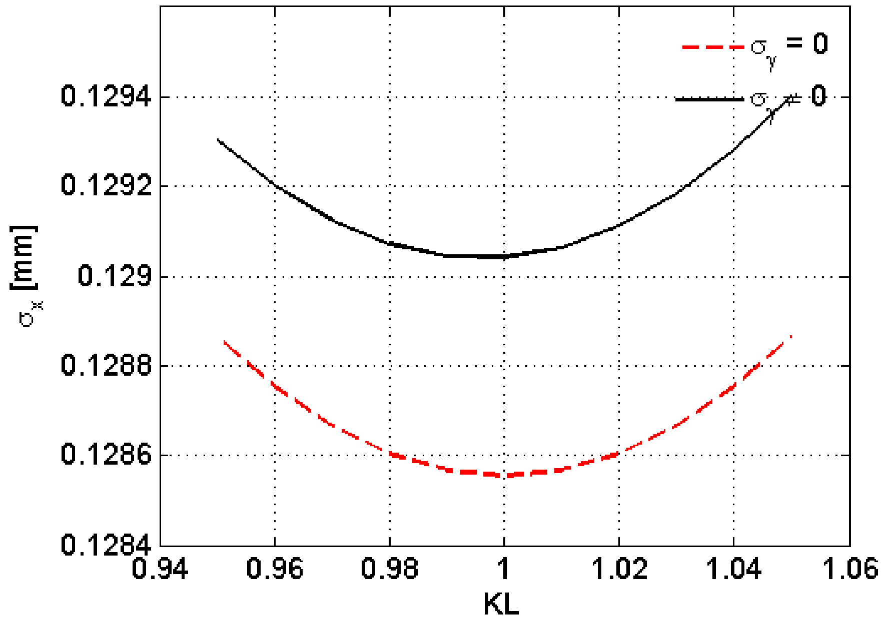
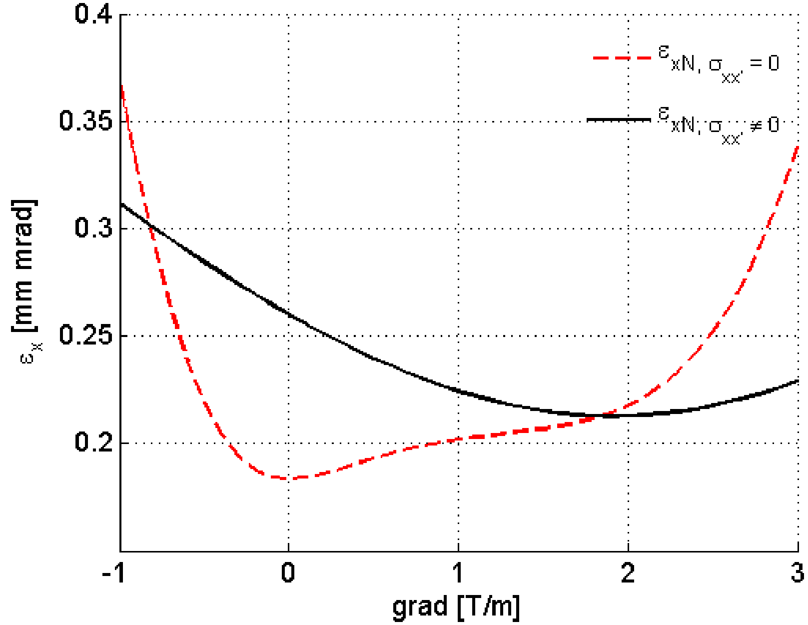
100 pC, 25 MeV/c

$\varepsilon_{xy,\,N}$ = 0.19 mm mrad on EMSY1 (ε_x = 0.004 mm mrad)
σ_x = 0.18 mm
σ_γ = 0.14%





$\Delta\sigma_x \sim 3\% \rightarrow \varepsilon_{meas,\,x}$ = 0.06 mm mrad

Spot size variations

1. Negligible correlations b/n energy and transverse coordinates

$$\sigma_L^2 = \sigma_{L,\delta=0}^2 + (KL)^2 \sigma_x^2 \sigma_\gamma^2$$, for L – distance from the quad

2. Including correlations

$$\sigma_L^2 = \sigma_{L,\delta=0}^2 + (KL)^2 \langle \delta^2 x_0^2 \rangle + 2KL(1 - KL)\langle \delta x_0^2 \rangle + 2KL^2 \langle \delta x_0 x_0' \rangle$$