

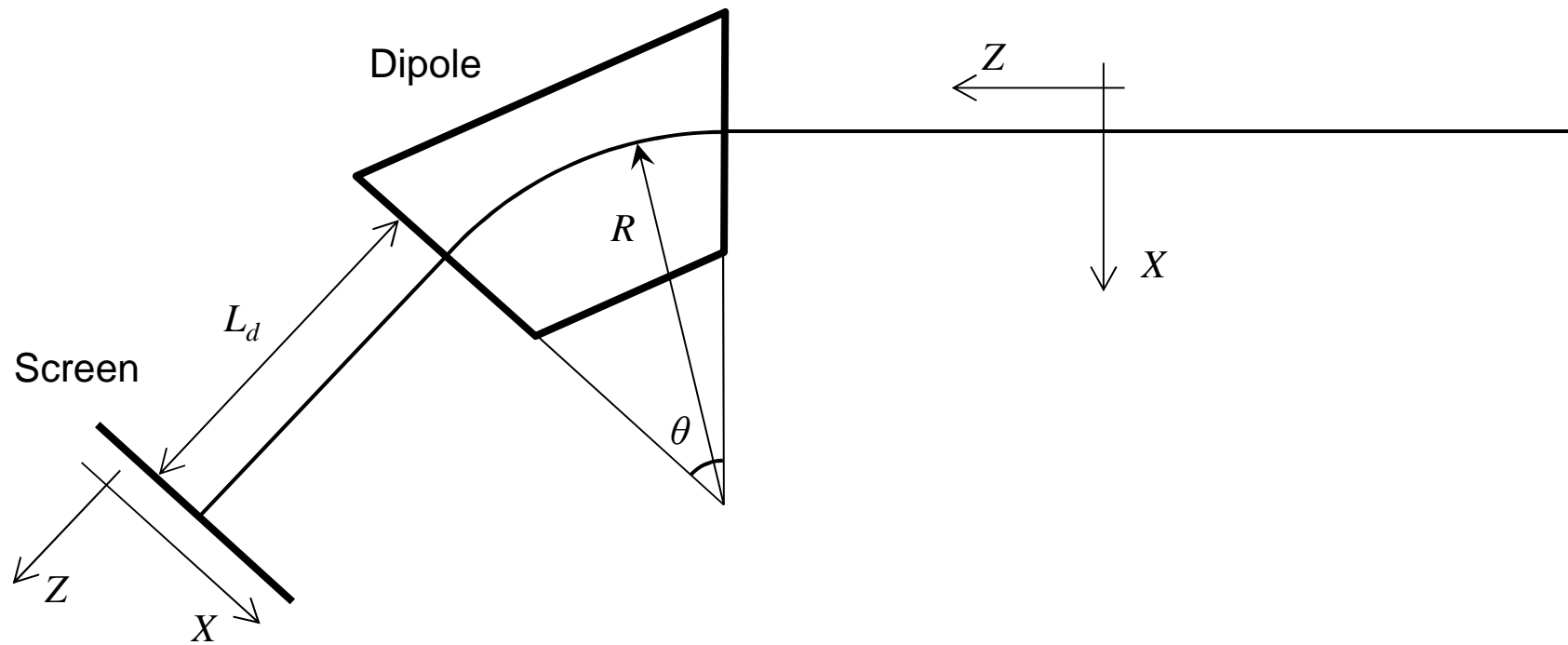
Disp3.D1 resolution limitation

momentum resolution

longitudinal resolution

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PITZ Physics seminar
25th of October, 2012

Definitions



Dipole + drift transport matrix

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ z \\ \delta p \end{pmatrix}_1 = \begin{bmatrix} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & 1 & L_d + R\theta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} x \\ x' \\ y \\ y' \\ z \\ \delta p \end{pmatrix}_0$$

$$x_1 = R_{11}x_0 + R_{12}x'_0 + R_{16}\delta p_0$$

$$z_1 = R_{51}x_0 + R_{52}x'_0 + z_0 + R_{56}\delta p_0$$

$$R_{11} = \cos\theta - \sin\theta \frac{L_d}{R}$$

$$R_{51} = \sin\theta$$

$$R_{12} = L_d \cdot \cos\theta + R \cdot \sin\theta$$

$$R_{52} = R \cdot (1 - \cos\theta)$$

$$R_{16} = L_d \cdot \sin\theta + R \cdot (1 - \cos\theta)$$

$$R_{56} = -R \cdot \theta + R \cdot \sin\theta + \frac{R\theta + L_d}{\gamma^2}$$



Dipole limitations

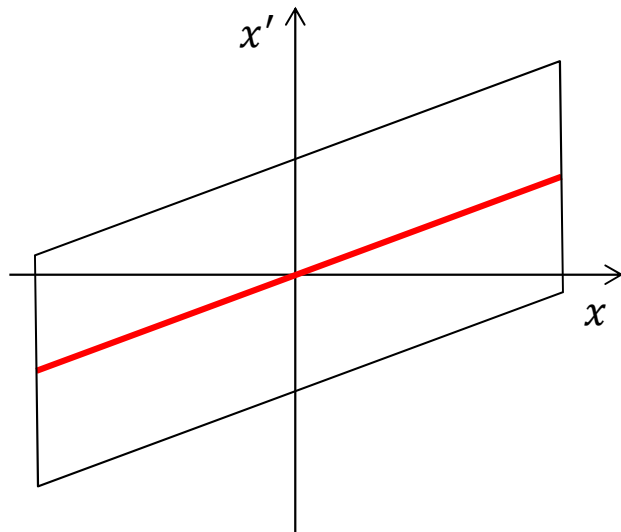
$$x_1 = R_{11}x_0 + R_{12}x'_0 + R_{16}\delta p_0$$

$$R_{11} = -0.516$$

$$R_{12} = 0.867$$

$$R_{16} = 0.905$$

$$|R_{11}x_0 + R_{12}x'_0| < R_{16}\delta p_0$$



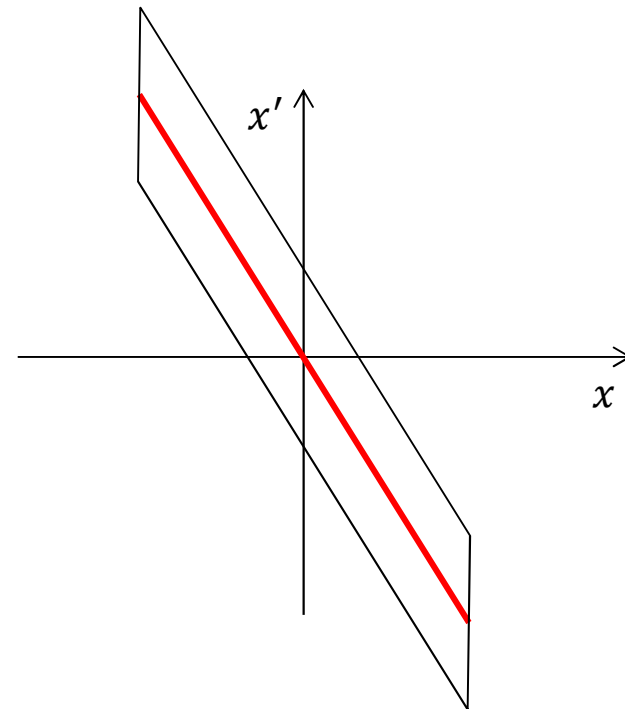
$$z_1 = R_{51}x_0 + R_{52}x'_0 + z_0 + R_{56}\delta p_0$$

$$R_{51} = 0.866$$

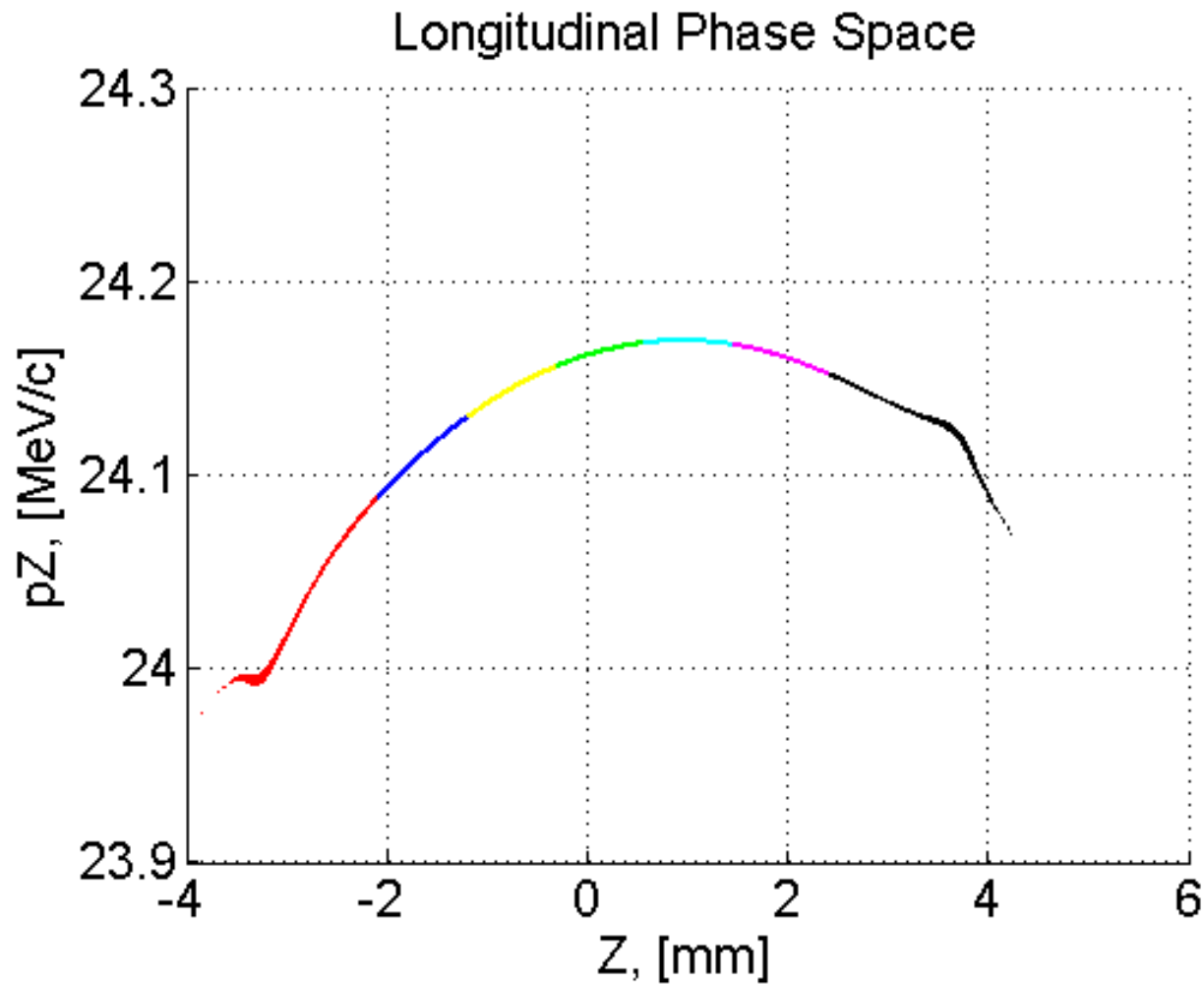
$$R_{52} = 0.298$$

$$R_{56} = -0.107$$

$$|R_{51}x_0 + R_{52}x'_0 + R_{56}\delta p_0| < z_0$$



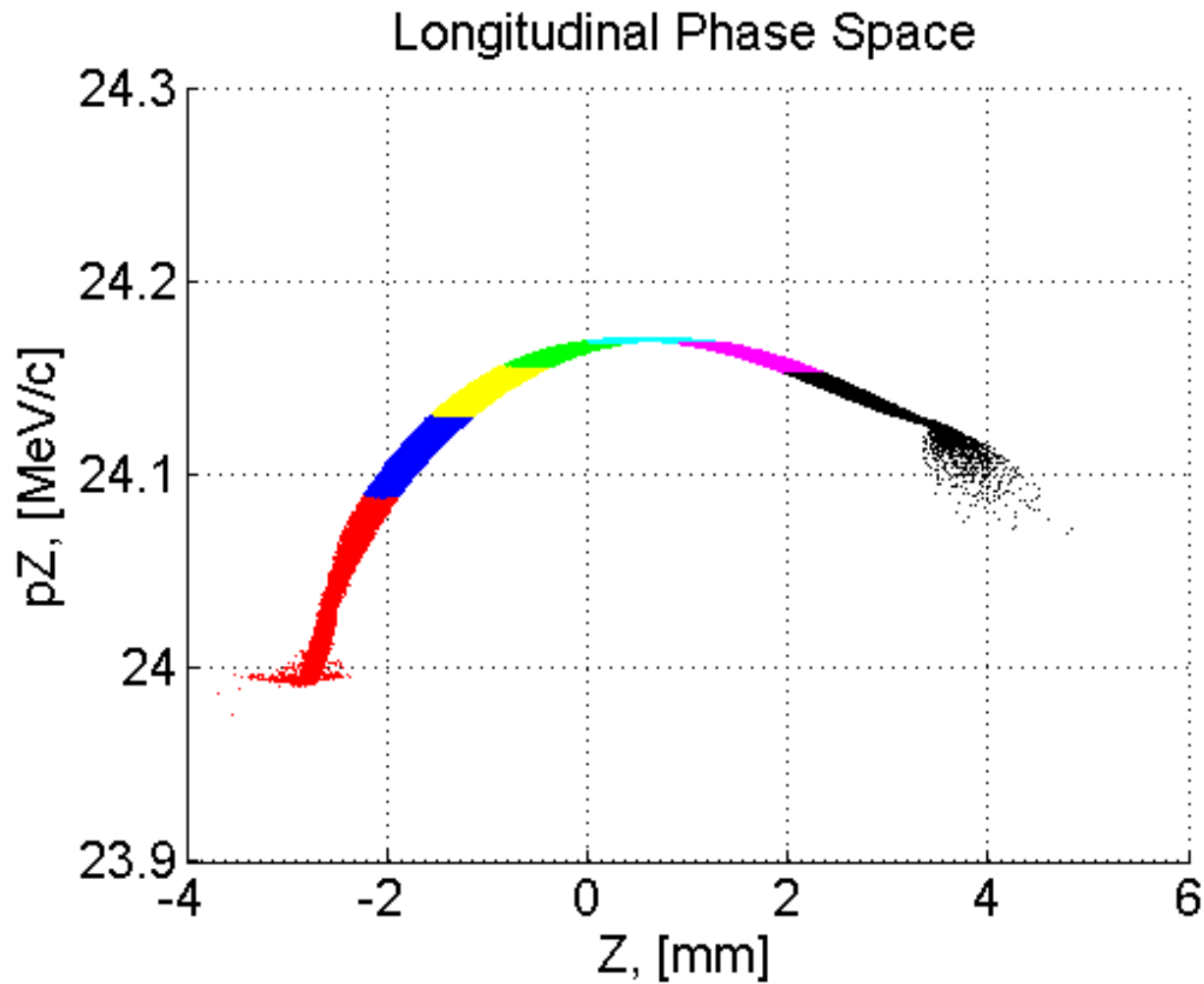
Longitudinal phase space, 17 m



Example of longitudinal phase space for 100pC bunch charge.



Longitudinal phase space, 18.5 m, after the dipole



Example of longitudinal phase space for 100pC bunch charge.



Dipole limitations with the streak camera

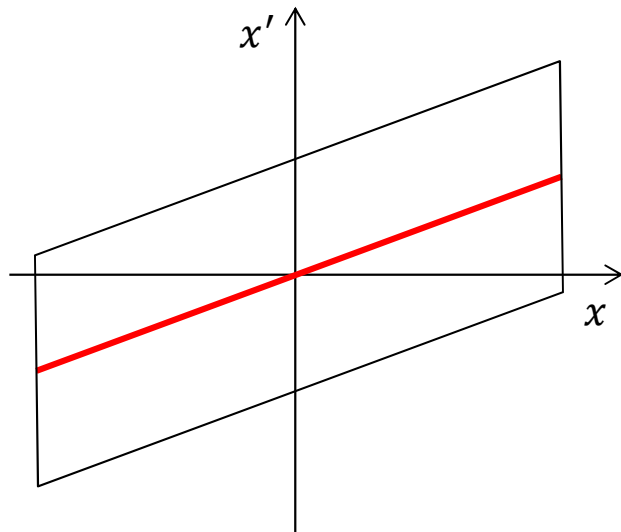
$$x_1 = R_{11}x_0 + R_{12}x'_0 + R_{16}\delta p_0$$

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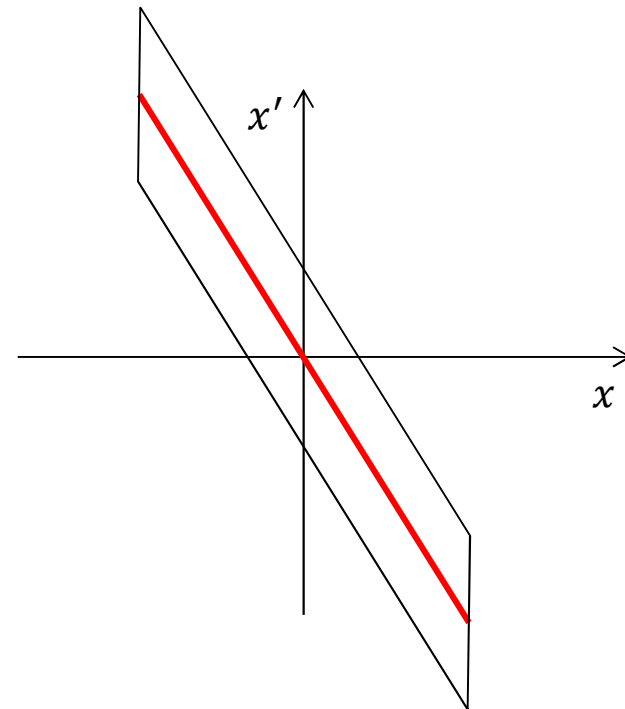
$$z_1 = R_{51}x_0 + R_{52}x'_0 + z_0 + R_{56}\delta p_0$$

$$R_{51} = 0.866$$

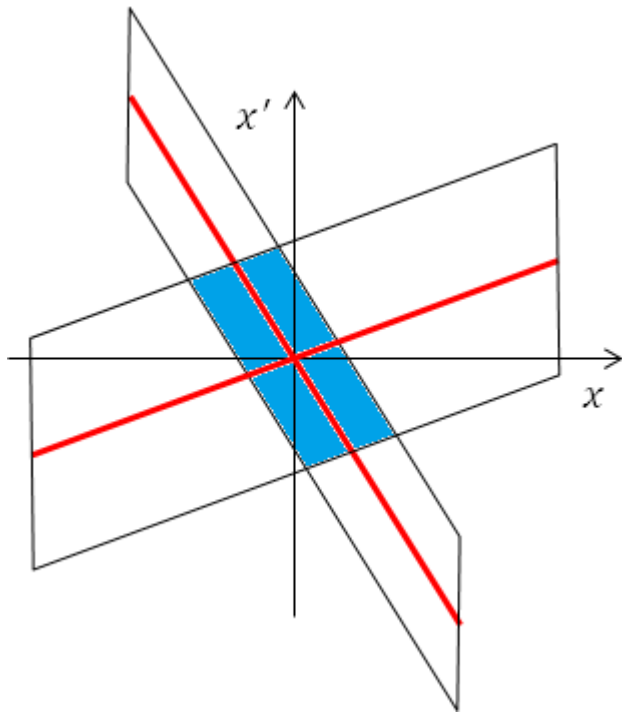
$$R_{52} = 0.298$$

$$R_{56} = -0.107$$

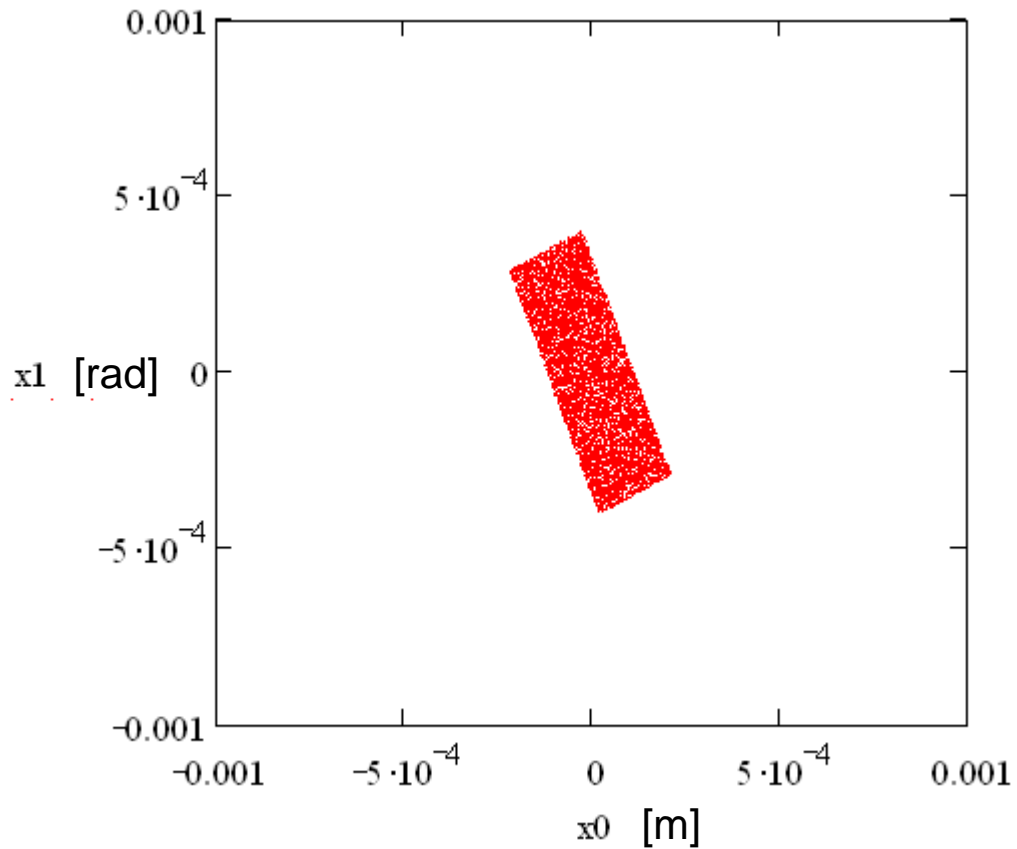
$$|R_{51}x_0 + R_{52}x'_0 + R_{56}\delta p_0| < z_0$$



Overlap



Numbers I



$$\Delta\delta p_0 = \frac{10 \text{ keV}}{25 \text{ MeV}} = 4 \cdot 10^{-4}$$

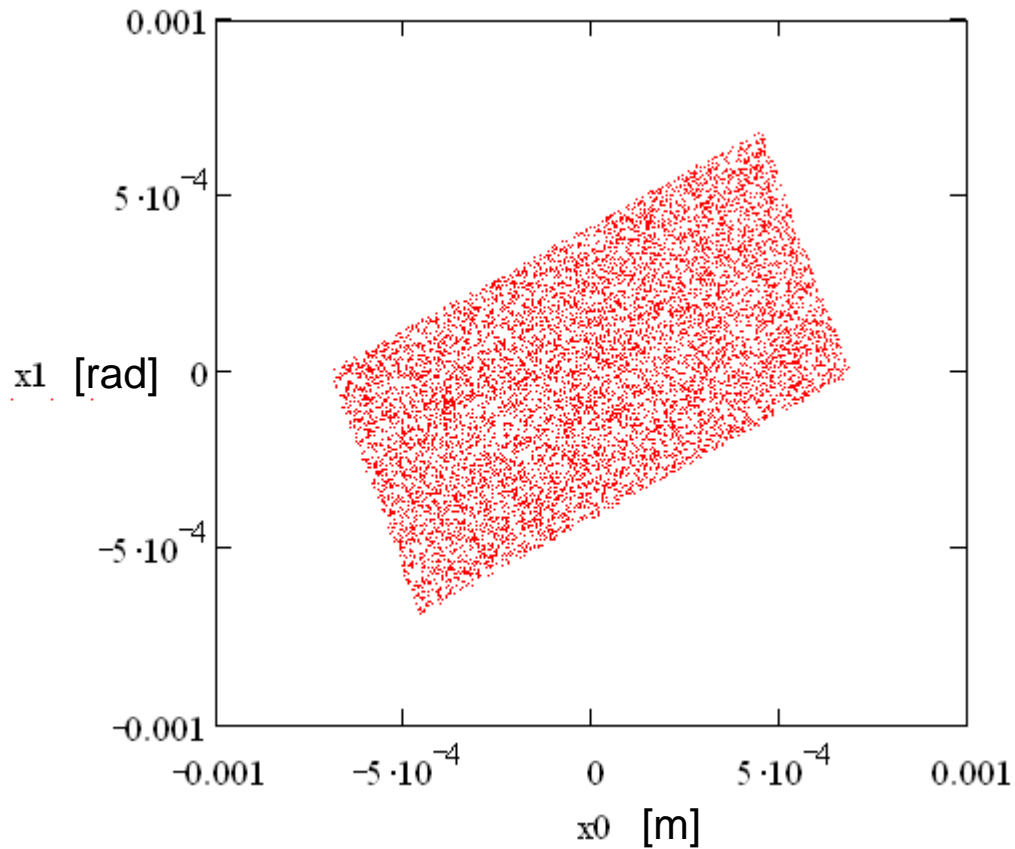
$$\Delta z = 0.1 \text{ mm (0.33 ps)}$$

$$\varepsilon := \sqrt{\text{stdev}(x0)^2 \cdot \text{stdev}(x1)^2 - \text{mean}(x01)^2}$$

$$\varepsilon \cdot 45 \cdot 10^6 = 0.60542$$



Numbers II



$$\Delta\delta p_0 = \frac{10 \text{ keV}}{25 \text{ MeV}} = 4 \cdot 10^{-4}$$

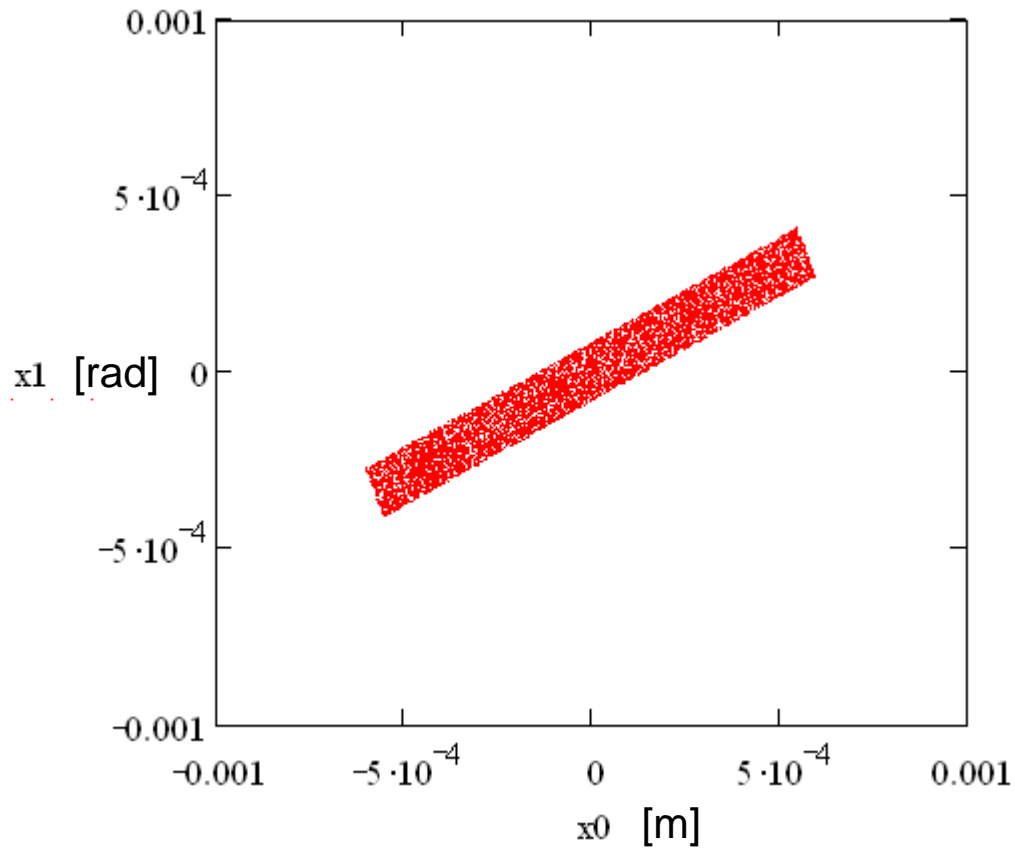
$$\Delta z = 0.6 \text{ mm (2 ps)}$$

$$\varepsilon := \sqrt{\text{stdev}(x_0)^2 \cdot \text{stdev}(x_1)^2 - \text{mean}(x_0 x_1)^2}$$

$$\varepsilon \cdot 45 \cdot 10^6 = 3.586274$$



Numbers III



$$\Delta\delta p_0 = \frac{2 \text{ keV}}{25 \text{ MeV}} = 0.8 \cdot 10^{-4}$$

$$\Delta z = 0.6 \text{ mm (2 ps)}$$

$$\varepsilon := \sqrt{\text{stdev}(x_0)^2 \cdot \text{stdev}(x_1)^2 - \text{mean}(x_0 x_1)^2}$$

$$\varepsilon \cdot 45 \cdot 10^6 = 0.728411$$



TDS transport matrix

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ z \\ \delta p \end{pmatrix}_1 = \begin{bmatrix} 1 & L_c & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L_c & \frac{KL_c}{2} & 0 \\ 0 & 0 & 0 & 1 & K & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & K & \frac{KL_c}{2} & \frac{K^2 L_c}{6} & 1 \end{bmatrix} \cdot \begin{pmatrix} x \\ x' \\ y \\ y' \\ z \\ \delta p \end{pmatrix}_0$$

$$K = \frac{eV_0 k}{pc},$$

$$k = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}.$$

$$y_1 = y_0 + L_c \cdot y_0' + \frac{KL_c}{2} \cdot z_0,$$

$$y_1' = y_0' + K \cdot z_0,$$

$$\delta p_1 = \delta p_0 + K \cdot y_0 + \frac{KL_c}{2} \cdot y_0' + \frac{K^2 L_c}{6} \cdot z_0,$$

Vertical kick

Induced slice energy spread



Dipole transport matrix

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ z \\ \delta p \end{pmatrix}_2 = \begin{bmatrix} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & 1 & L_d + R\theta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} x \\ x' \\ y \\ y' \\ z \\ \delta p \end{pmatrix}_1$$

$$y_2 = y_1 + (L_d + R\theta) \cdot y_1' = y_0 + L_c \cdot y_0' + \frac{KL_c}{2} \cdot z_0 + (L_d + R\theta) \cdot (y_0' + K \cdot z_0)$$

$$y_2 = y_0 + y_0' \cdot (L_d + R\theta + L_c) + z_0 K \cdot (L_d + R\theta + \frac{L_c}{2})$$



Dipole limitation

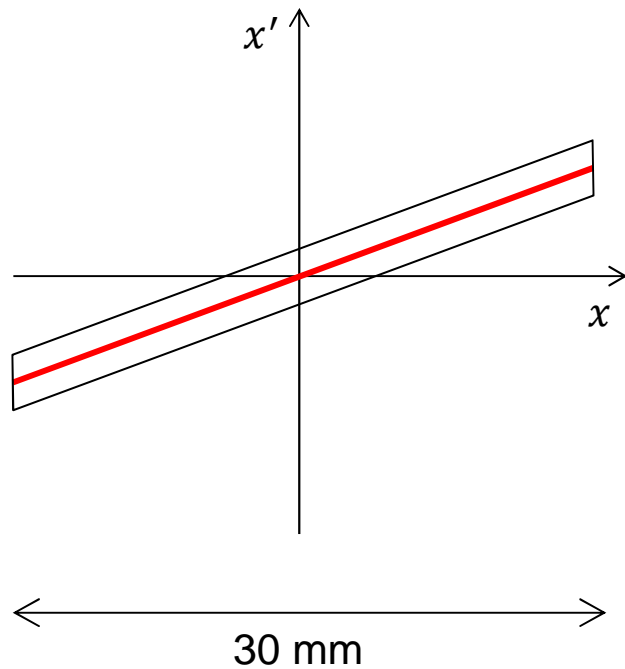
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$$R_{16} = 0.905$$

$$|R_{11}x_0 + R_{12}x'_0| < R_{16}\delta p_0$$



$$x'_{max} \approx \frac{\pi \cdot 0.6 \cdot \frac{10^{-6}}{45}}{2 \cdot 3 \cdot 10^{-2}} = 0.7 \cdot 10^{-6}$$

$$\delta p_0 \approx 0.7 \cdot 10^{-6} \quad 16 \text{ eV}$$



Disp3.Scr1 camera resolution

YAG screen calibration:

> lens f200 -> 37 um/pixel

> lens f80 -> 126 um/pixel

1 pixel -> 37 um -> $\frac{\Delta p}{p} = 4 \cdot 10^{-5}$ -> 1 keV



Conclusion

For momentum measurements:

- Momentum resolution limit: **16 eV**, for **0.6 mm*mrad** emittance
- Video camera limitation: **1 keV**

For longitudinal phase space measurements:

- With the streak camera limitation: **2 keV** and **2 ps**
- With TDS (video camera limitation): **1 keV** and **0.3* ps**

