Emittance growth due to multipole transverse magnetic modes in an rf gun*

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OUTLINE

- Introduction
- Theoretical analysis
- Simulation models and RF field manipulation
- Simulation results
- Conclusion

Introduction

- A factor inducing transverse emittance growth in a photocathode rf gun :
  - Time dependent transverse deflection of each electron due to Lorentz force in the cavity
    - Intrinsic kick from the fundamental monopole mode(TM010)
    - Multipole modes due to asymmetric boundary condition of cavity
- Ports break cylindrical symmetry in different cells of the photocathode gun:
  - the primary asymmetry usually lies in the cell with biggest port cavity
  - Nonaxisymmetric structure excites multipole field components
    - Dipole
    - Quadrupole
    - Octopole
    - They kick the beam time dependently

Introduction

> How to produce high-quality beam?

  > The removal of fundamental mode is difficult

  > Try to minimize of the multipole rf field

> Model:

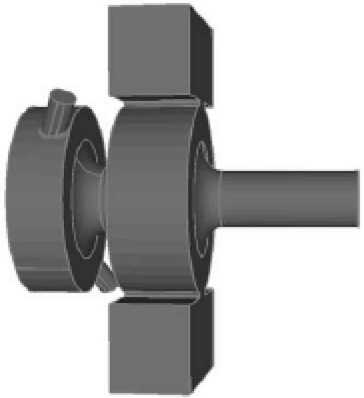
  > Original model: The third generation BNL-type rf gun

    - One-sided coupling scheme causes asymmetry in the electromagnetic field inside the cavity
    - This asymmetric field can be expanded into multipole fields
    - The dipole and quadrupole fields are the main causes of emittance growth

  > LCLS gun : one of the rf guns which many innovative ideas were applied showing a successful result

  > PAL gun : S-band rf gun, four-hole model

    - Two holes added to the LCLS-type gun
    - To reduce the quadrupole mode
    - An additional advantage: Much easier to maintain a high vacuum level



Introduction

- Methodology
  - Artificial map field combines both fundamental monopole mode and one of the multipole fields for beam dynamics simulation, without space charge effect and solenoid field(multipole field effects will be coupled to x and y).
  - Four gun models with zero, one, two, and four holes at the full cell cavity have been studied respectively.
  - Manipulating the field map using Fourier transform technique, a field map generated by controlling the amplitude of each multipole field(TM_{n10} mode)
  - Comparing the emittance evolution with original field map from each gun model with the simulation results using the artificial field map with a single multipole component.

Theoretical analysis

- Simplified consideration:
  - An electron beam passing through a single full cell
  - Velocity of electron : *c*
  - Evaluation of the dominant rf multipole field effect in each model
- Formulas:
  - Panofsky-Wenzel theorem:

$$p_{\perp}=\frac{e}{w}\mathrm{Re}(\int_{-L/2}^{L/2} i\nabla_{\perp}E_z dz)$$

  - Acceleration electric field along the longitudinal axis(sum of all TM_{n10} modes)：

$$E_z(r,\theta,z,t)=e^{i(\omega t+\varphi_o)}\sum_{n=0}^{\infty}E_n\cos(kz)J_n(k_c r)\cos[n(\theta-\theta_0)]$$

  - For each model,the main mode is the monopole mode(TM_{010})：

$$E_z^{010}=e^{i(\omega t+\varphi_o)}E_0\cos(kz)J_0(k_c r)\approx e^{i(\omega t+\varphi_o)}E_0\cos(kz)(1-\frac{k_c}{4}r^2)=e^{i(\omega t+\varphi_o)}E_0\cos(kz)[1-\frac{k_c}{4}(x^2+y^2)]$$

Defination: when t=0, z=0,then t=z/c

$$beamenergy=e\int_{-L/2}^{L/2}\mathrm{Re}(E_z^{010})dz=\frac{1}{2}eE_0L\cos\overline{\varphi}_0$$

Theoretical analysis

$\bar{\varphi}_0$ is the ensemble average of φ_0 along the distribution in the bunch, when $\bar{\varphi}_0$ is zero , the acceleration in the cell is maximized

- Normailized transverse monopole kick for a single electron can be calculated by P-W theorem

$$p_{n,\perp}^{010} = \frac{1}{mc}\mathrm{Re}\left(\frac{e}{\omega}\int_{-L/2}^{L/2} i\nabla_{\perp} E_z^{010} dz\right) = \frac{k_c^2}{2}\alpha L \sin\varphi_0 (x\hat{x} - y\hat{y})$$

$$\alpha = (eE_0)/(2mc^2 k)$$

- The normalized rms emittance growth is calculated by:

$$\varepsilon_{n,x} = \sqrt{\left\langle (p_{n,x} - \langle p_{n,x}\rangle)^2 \right\rangle \left\langle (x - \langle x\rangle)^2 \right\rangle - \left\langle (p_{n,x} - \langle p_{n,x}\rangle)(x - \langle x\rangle) \right\rangle^2}$$

- The emittance due to monopole field :

$$\varepsilon_{n,x}^{010} = \frac{k_c^2}{2}\alpha L \cos\bar{\varphi}_0 \sigma_x^2 \sigma_\varphi \qquad \varepsilon_{n,y}^{010} = \frac{k_c^2}{2}\alpha L \cos\bar{\varphi}_0 \sigma_y^2 \sigma_\varphi$$

rms: σ_x, σ_φ

assumption :<x>,<y> in the full cell are equal to zero (precondition)

Theoretical analysis

> TM110:

- The dipole mode:

$$E_z^{010} = e^{i(\omega t+\varphi_o)} E_0 \cos(kz) J_1(k_c r)\cos(\theta - \frac{\pi}{2}) \approx e^{i(\omega t+\varphi_o)} E_1 \cos(kz)\frac{k_c}{2}\sin\theta = e^{i(\omega t+\varphi_o)} a_1 E_0 \cos(kz) r \sin\theta = e^{i(\omega t+\varphi_o)} a_1 E_0 \cos(kz) y$$

a_2 is a parameter characterizing the relative strength of the dipole field to the monopole field

- The normalized transverse dipole kick for a single elcetron:

$$p_{n,\perp}^{110} = \frac{1}{mc}\mathrm{Re}(\frac{e}{\omega}\int_{-L/2}^{L/2} i\nabla_\perp E_z^{110} dz) = -a_1 \alpha L \sin\varphi_0 \hat{y}$$

- The normalized rms emittance growth due to dipole kick:

$$\varepsilon_{n,y}^{110} = a_1 \alpha L \cos\overline{\varphi}_0 \sigma_x \sigma_\varphi$$

> TM210:

- The quadrupiole mode:

$$p_{n,\perp}^{210} = \frac{1}{mc}\mathrm{Re}(\frac{e}{\omega}\int_{-L/2}^{L/2} i\nabla_\perp E_z^{210} dz) = 2a_2 \alpha L \sin\varphi_0 (x\hat{x} - y\hat{y})$$

The ensemble average $\langle p_{n,\perp}^{210} \rangle$ of the quadrupole kick is zero, so there is no kick from quadrupole field

Theoretical analysis

- The normalized rms emittance growth due to quadrupole kick :

$$\varepsilon_{n,x}^{210} = 2a_2 \alpha L \cos\overline{\varphi}_0 \sigma_x^2 \sigma_\varphi \qquad \varepsilon_{n,y}^{210} = 2a_2 \alpha L \cos\overline{\varphi}_0 \sigma_y^2 \sigma_\varphi$$

> TM410:

- For the octopole mode ,the accelerating electric field is given by

$$E_z^{410} = e^{i(\omega t+\varphi_o)} E_4 \cos(kz) J_4(k_c r) \cos[4(\theta - \frac{\pi}{2})] \approx e^{i(\omega t+\varphi_o)} E_4 \cos(kz) \frac{1}{4!}\left(\frac{k_c r}{2}\right)^4 \cos 4\theta = e^{i(\omega t+\varphi_o)} a_4 E_0 \cos(kz)(x^4 - 6x^2 y^2 + y^4)$$

a4 is a parameter characterizing the relative strength of the octopole field to the monopole field.

- The normalized transverse octopole kick is

$$p_{n,\perp}^{410} = \frac{1}{mc} \mathrm{Re}\left(\frac{e}{\omega}\int_{-L/2}^{L/2} i\nabla_\perp E_z^{410} dz\right) = -4a_4 \alpha L \sin\varphi_0 [(x^3 - 3xy^2)\hat{x} - (y^3 - 3x^2 y) y]$$

If the multipole field is weak enough so that the correlation between x and y is negligible, and the beam is still symmetric, then $\langle x^2 \rangle = \langle y^2 \rangle \;\; \langle x^4 \rangle = \langle y^4 \rangle$

Theoretical analysis

- The emittcance induced by the octopole field :

$$\varepsilon_{n,x}^{410} = 4\sqrt{6}a_4\alpha L\sigma_x^4\sqrt{\sin^2\overline{\varphi}_0 + \sigma_\varphi^2\cos^2\overline{\varphi}_0} \qquad \varepsilon_{n,x}^{410} = 4\sqrt{6}a_4\alpha L\sigma_y^4\sqrt{\sin^2\overline{\varphi}_0 + \sigma_\varphi^2\cos^2\overline{\varphi}_0}$$

As we can see later in the next slides, the octopole field effect could be neglected\

> Considering only monopole, dipole and quadrupole effect

- Rf field:

$$E_z \approx E_z^{010} + E_z^{110} + E_z^{210} = e^{i(\omega t+\varphi_0)}\cos(kz)E_0[1 + a_1 y - (a_2 + \frac{k_c^2}{4})x^2 - (-a_2 + \frac{k_c^2}{4})y^2]$$

- The totla normalized transverse rf field kick is

$$P_{n,x} = (2a_2 + \frac{k_c^2}{2})x\alpha L\sin\varphi_0\hat{x} \qquad P_{n,y} = -[a_1 + (2a_2 - \frac{k_c^2}{2})y]\alpha L\sin\varphi_0\hat{y}$$

If the multipole field is weak enough so that the correlation between x and y is negligible, and the beam is still symmetric, then

- The total emittance growth due to the rf effect:

$$\varepsilon_{n,x}^{010+110+210} = (\frac{k_c}{2} + 2a_2)\alpha L\cos\overline{\varphi}_0\sigma_x^2\sigma_\varphi = \varepsilon_{n,x}^{010} + \varepsilon_{n,x}^{210}$$

Theoretical analysis

$$\varepsilon_{n,y}^{010+110+210} = \alpha L \cos\overline{\varphi}_0 \sigma_y \sigma_\varphi \sqrt{(\frac{k_c^2}{2} - 2a_2)^2 \sigma_y^2 + a_1^2} = \sqrt{(\varepsilon_{n,y}^{010} - \varepsilon_{n,y}^{210})^2 + (\varepsilon_{n,y}^{110})^2}$$

- If the dipole field is the dominant multipole field(rf gun with one hole),the quadrupole field can be neglected,then

$$\varepsilon_{n,x}^{010+110} = \frac{k_c^2}{2} \alpha L \cos\overline{\varphi}_0 \sigma_y^2 \sigma_\varphi = \varepsilon_{n,x}^{010}$$

$$\varepsilon_{n,y}^{010+110} = \alpha L \cos\overline{\varphi}_0 \sigma_y \sigma_\varphi \sqrt{(\frac{k_c^2}{2} \sigma_y)^2 + a_1^2} = \sqrt{(\varepsilon_{n,y}^{010})^2 + (\varepsilon_{n,y}^{110})^2}$$

There is no emittance growth in the x axis for the dipole mode case

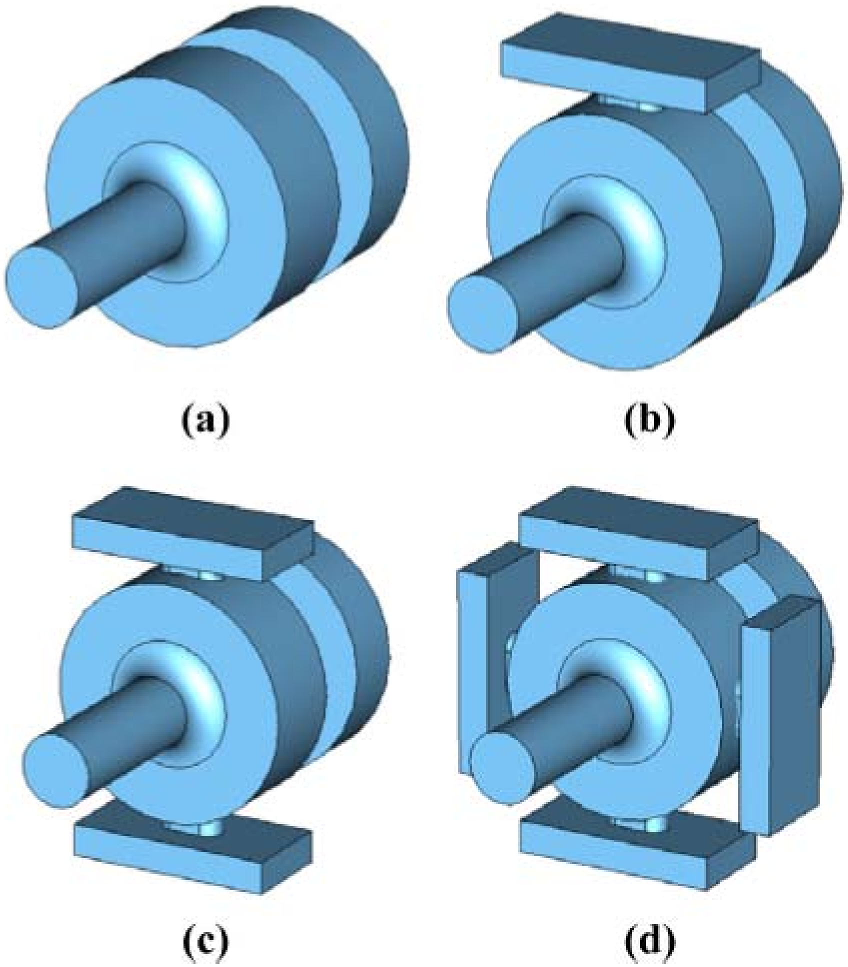
- If the quadrupole field is dominant multipole field, the dipole can be neglected then

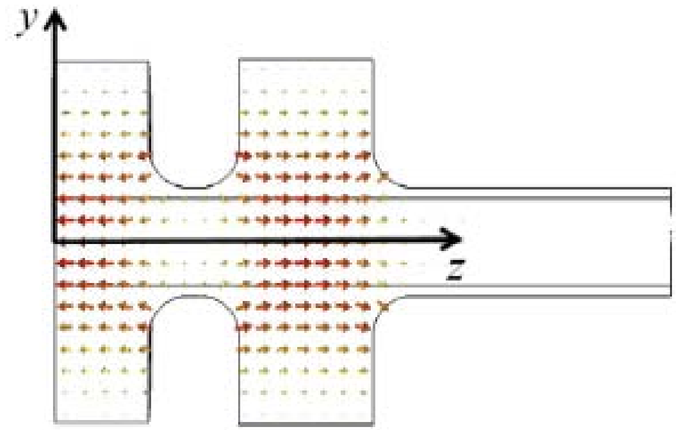
$$\varepsilon_{n,x}^{010+210} = (\frac{k_c^2}{2} + 2a_2) \alpha L \cos\overline{\varphi}_0 \sigma_x^2 \sigma_\varphi = \varepsilon_{n,x}^{010} + \varepsilon_{n,x}^{210} \qquad \varepsilon_{n,y}^{010+210} = (\frac{k_c^2}{2} - 2a_2) \alpha L \cos\overline{\varphi}_0 \sigma_y^2 \sigma_\varphi = \varepsilon_{n,y}^{010} - \varepsilon_{n,y}^{210}$$

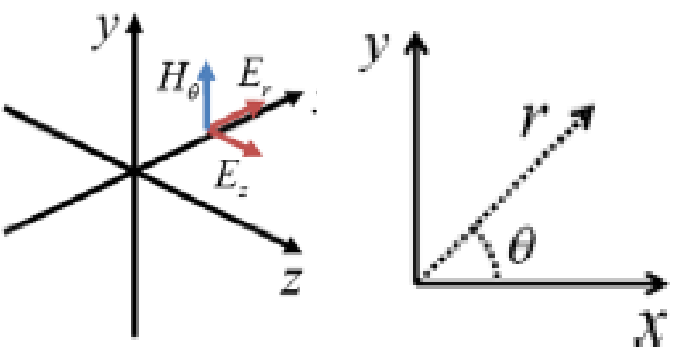
that is why in the gun model with two holes.emittance seems to be different in x and y direction emittance in the x direction is bigger than the monopole mode and emittace in the y direction is smaller than that in the one-hole model

Simulation models and rf field manipulation

> Simulation models:

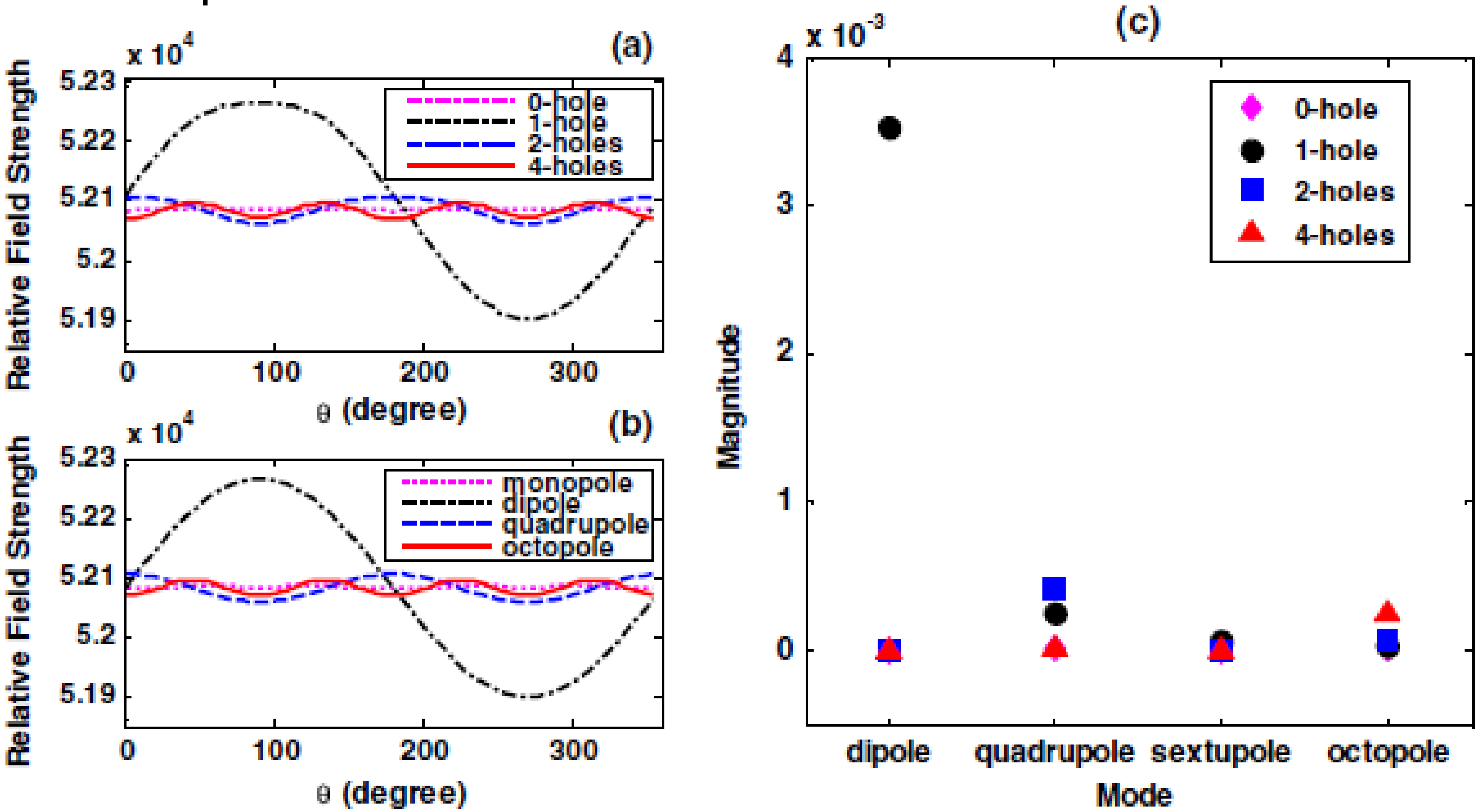






Simulation models and rf field manipulation

> Rf field manipulation

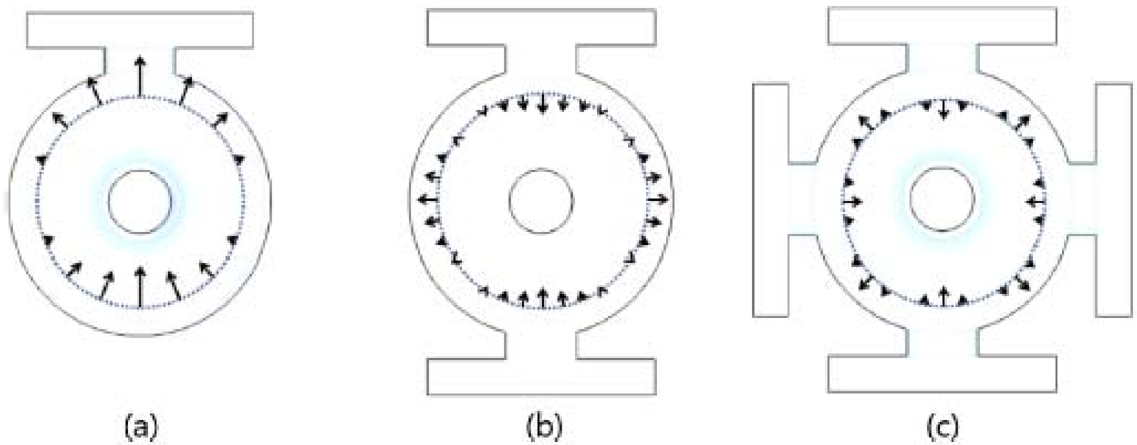


(a) Rf field Er distribution along the direction for the models at z =61 mm (middle of the full cell), r =9.5 mm.

(b) Distribution of the artificial rf field, which is the combination of the monopole and the dominant component of each model.

(c) Fourier coefficient of Er of 1.6 cell models with a different number of holes.

Simulation models and rf field manipulation



Schematic drawing of each multipole mode in the rf gun.
(a) Dipole mode. (b) Quadrupole mode. (c) Octopole mode.

- Dipole mode gives the electron beam a kick in y dirction, no effect in x direction
- Quadrupole mode applies a kick inward in y direction and outward in the x direction
- Octopole mode applies a kick outward in the x and y directions and inward in a direction that is rotated 45 degree with respect to the x and y axes

Simulation results

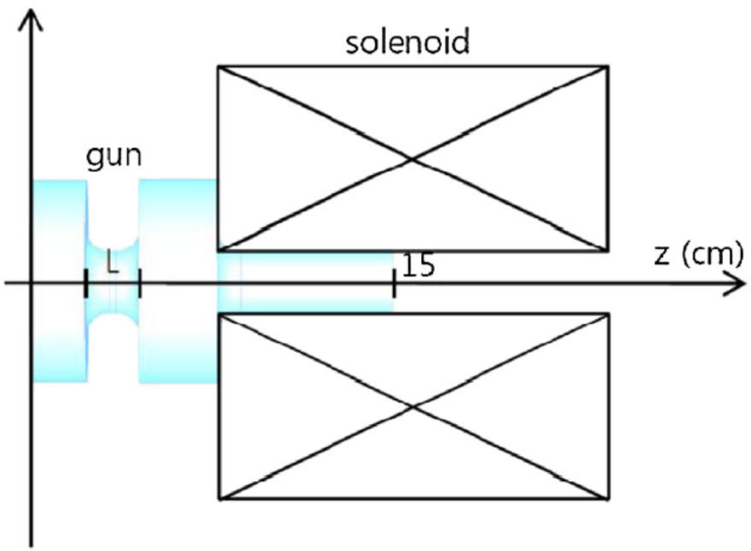
> Methods used in Parmela simulation

> Rf gun model with 3D rf field maps

- Gun models with zero hole, one hole, two holes, four holes
- Manipulated artifical field map: monople field plus one of the multipole fields, respectively

> Three kinds of beam charges

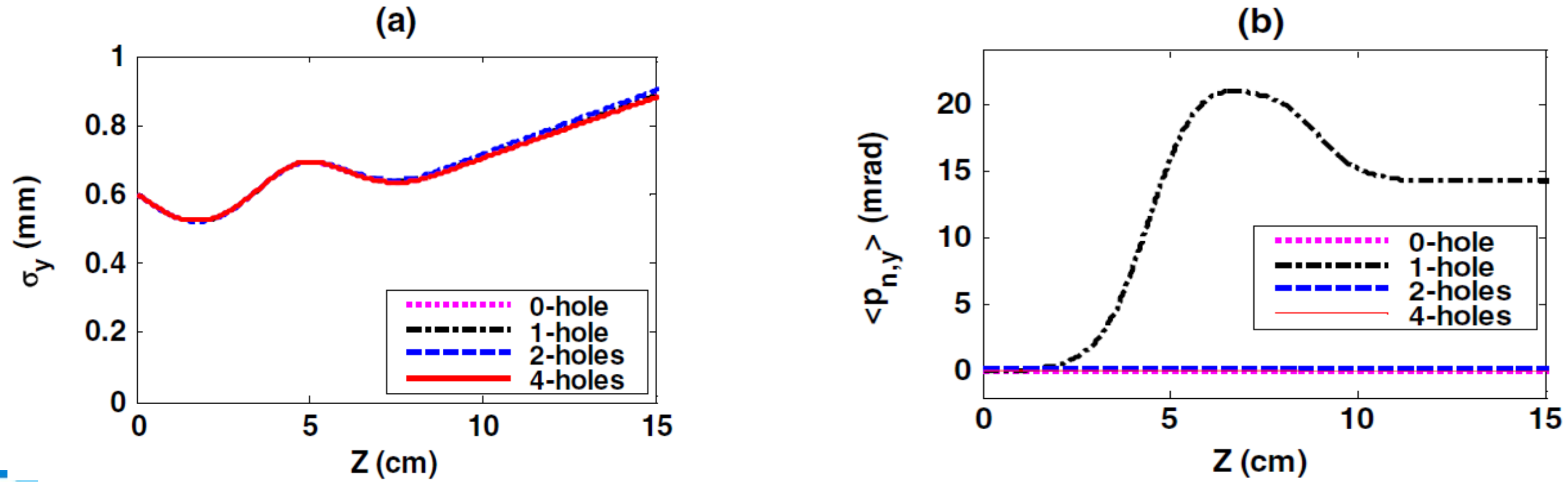
- First, the beam charge of 0 nC without solenoid field to understand the effect of rf field
- Then other beams with charges of 0.1 nC and 1 nC studied with solenoid field



Number of particles	100 000
Total charge	0/0.1/1 nC
UV laser pulse length (FWHM)	10 ps
UV laser radius	1.2 mm
Initial kinetic energy at the cathode	1 eV
Peak E_0 at cathode	120 MV/m
Length of the rf gun	15 cm
Initial laser-rf phase	32 degree

Simulation results: 0 nC charge without solenoid field

- Evolution of the rms beam size
  - Similar behaviors shown in the figure
- Evolution of the average transverse momentum of electron beam
  - $\langle p_{n,y} \rangle$ in the one-hole model is different from other
    - Obvious effect of transverse kick due to dipole field
    - The normalized momentum is 14.2 mrad at the exit of the cavity, while it can be calculated by using Eq.9, 7.25 mrad
    - Differences between simulation model and theorem model



Simulation results: 0 nC charge without solenoid field

- 1.6 cell in the simulation, a single full cell in the theory consideration

  Panofsky-Wenzel can not be applied to the half , velocity is much smaller than c
- 4 mrad kick from the half cell, so 10 mrad kick in the full cell in the simulation (good agreement)

> Two-hole model and four-hole model

- No average kicks calculated by Eq.12 and Eq.15 ,
- No transverse momentum in the simulation results
- No dipole kick at all for the rf guns with two and four holes

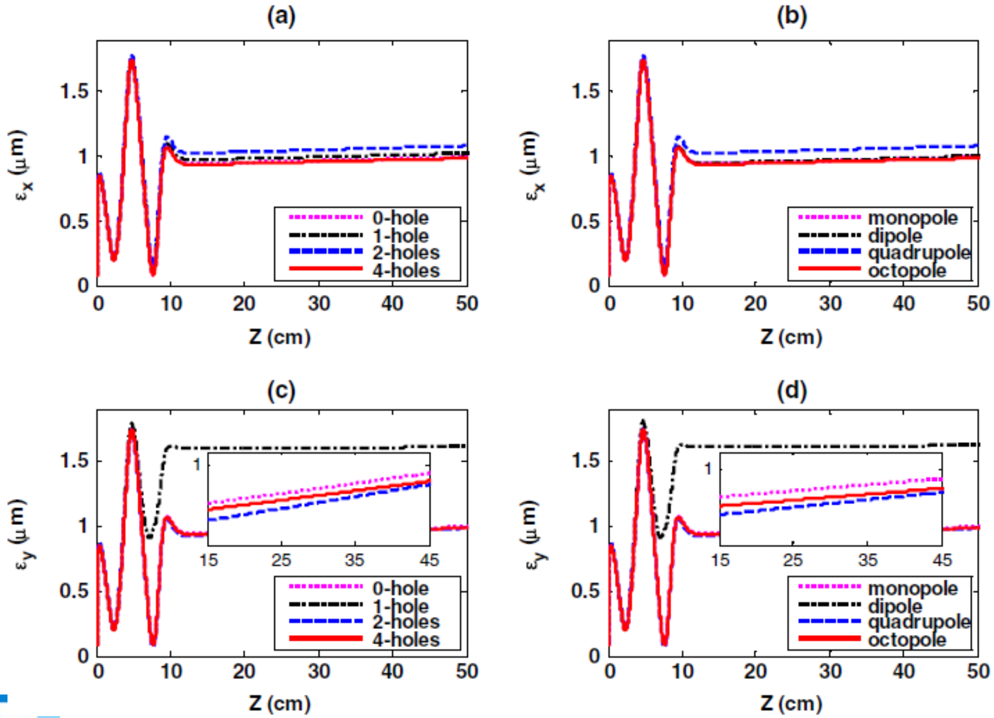
> Evolution of the transverse emittance

> The emittance due to the monopole field is 0.948 μm, while the theoretical emittance is $\varepsilon_{n,x}^{010} \approx \varepsilon_{n,y}^{010} = \frac{k_c^2}{2} \alpha L \cos\overline{\varphi_0} \sigma_x^2 \sigma_\varphi = 2.624$

- The model in theory analysis is a single full cell
- The emittance from monopole calculated by a fomula from a reference papar is 0.052 μm
- The differences are not fully understood yet.

Simulation results: 0 nC charge without solenoid field

- The emittance in the x direction was almost unaffected by all modes except two-hole model
  - Emittance in this paper means projected emittance
  - Slice emittances show similar behaviors
  - The quadrupole mode is the dominant mode in the two-holes models
  - Only quradrupole mode cases shows higher emittance than the other cases

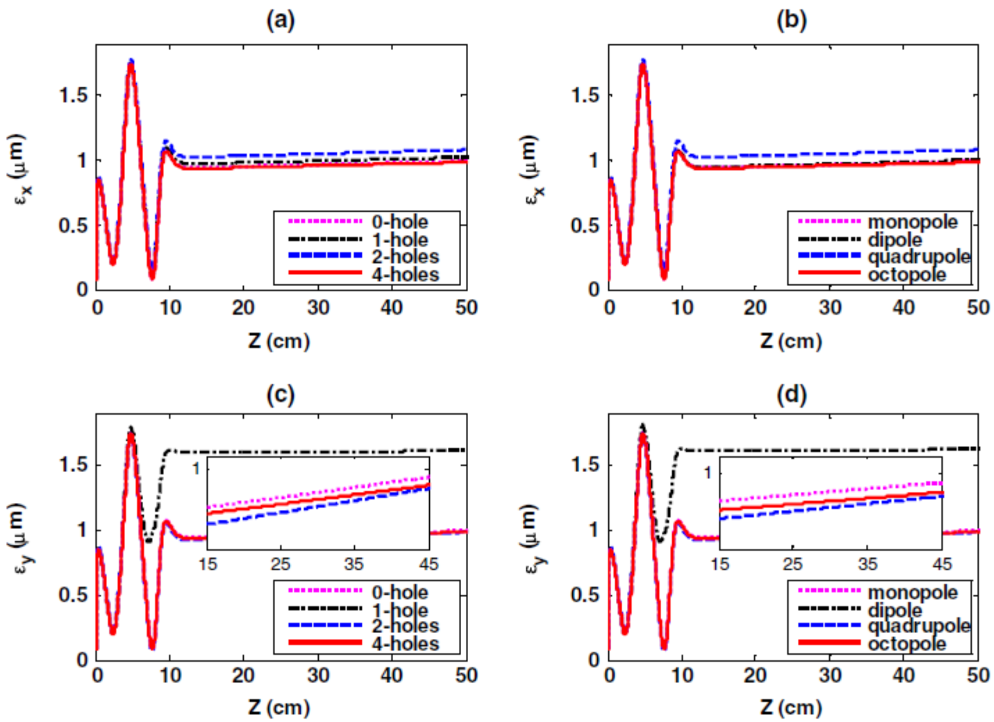


(a) Emittance evolutions in the horizontal (x) direction for each model.
(b) Emittance evolutions from the dominant multipole field from each model in the horizontal (x) direction.
(c) Emittance evolutions in the vertical (y) direction for each model.
(d) Emittance evolutions from the dominant multipole field from each model in the vertical (y) direction.

Simulation results: 0 nC charge without solenoid field

> The emittance in the y direction

- Emittances in the one-hole model and dipole mode are significantly increased
- Emittances in the two-hole model and is lower than the emittance in the zero-hole model, and same behavior observed in the quadrupole field
- The dipole(quadrupole) mode is the dominant mode in the one-hole(two-hole) model



Simulation results: 0 nC charge without solenoid field

- Dipole mode contribution to emittance growth
  - Emittances growth in the y direction at the gun exit is calculated as 1.257 μm
  - Dipole case means monopole mode plus dipole mode
  - Simulated emittance in the dipole case: $\varepsilon_{n,y}^{010+110} = 1.601(\mu m)$

$$\varepsilon_{n,y}^{110} = \sqrt{\left(\varepsilon_{n,y}^{010+110}\right)^2 - \left(\varepsilon_{n,y}^{010}\right)} = 1.290(\mu m)$$

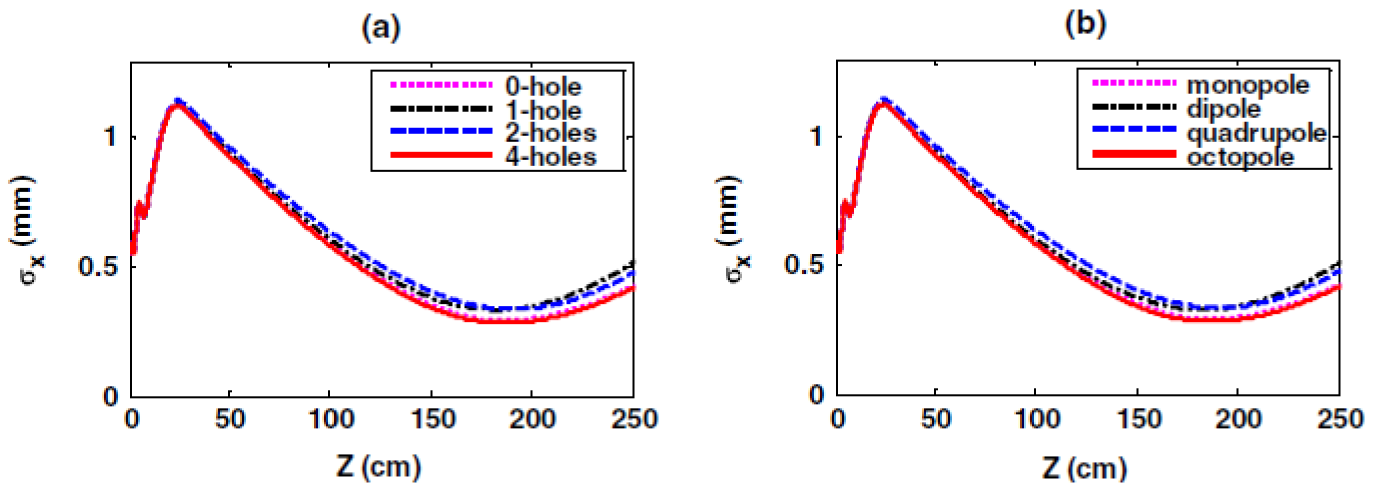
  - They match well
- Quadrupole mode contribution to emittance growth
  - $\varepsilon_{n,x}^{010+210} = 1.027(\mu m)$ $\varepsilon_{n,y}^{010+210} = 0.924(\mu m)$
  - Quadrupole case means monopole mode plus quadrupole mode
  - $\varepsilon_{n,x}^{210} = \varepsilon_{n,x}^{010+210} - \varepsilon_{n,x}^{010} = 1.027 - 0.947 = 0.080(\mu m)$

$$\varepsilon_{n,y}^{210} = -\varepsilon_{n,y}^{010+210} + \varepsilon_{n,y}^{010} = -0.924 + 0.948 = 0.024(\mu m)$$

  - 0.08 μm emittance growth in x direction and 0.024 μm decrease due to quadrupole mode
  - The simulation results are close to the predicted value 0.0536(x) and 0.0542(y.decrease)

Simulation results: 0.1 nC charge with solenoid field

- The results of rms beam size:
  - The rms beam size were not significantly affected by field asymmetric



X rms beam size of the rf gun with solenoid field for the beam charge 0.1 nC

(a) with the four rf gun models and (b) with four artificial field maps.

Simulation results: 0.1 nC charge with solenoid field

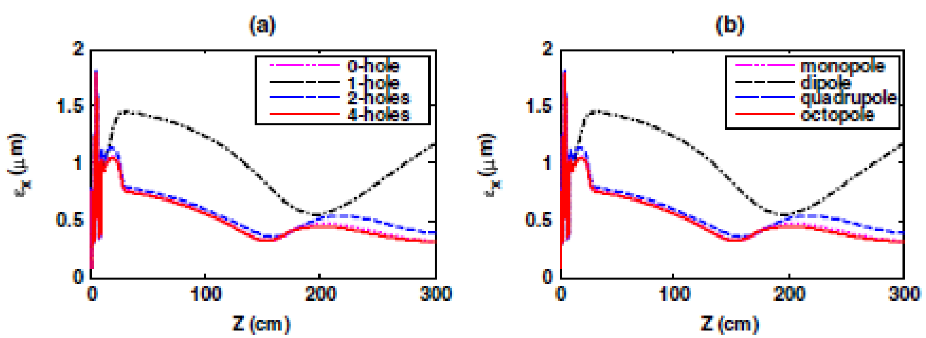
> Emittance evolution

> Emittance evolution in the x direction in each model

- The emittance of one-hole model increases rapidly
- The emittance evolution in other models show double minimums
- At all longitudinal positions, the emittance of one-hole model is the highest; two-hole model shows the second highest value; The zero-hole model and four-hole model do not show much difference, and their emittance value are the lowest

> Emittance evolution due to each multipole mode in the x direction

- Similar behavior



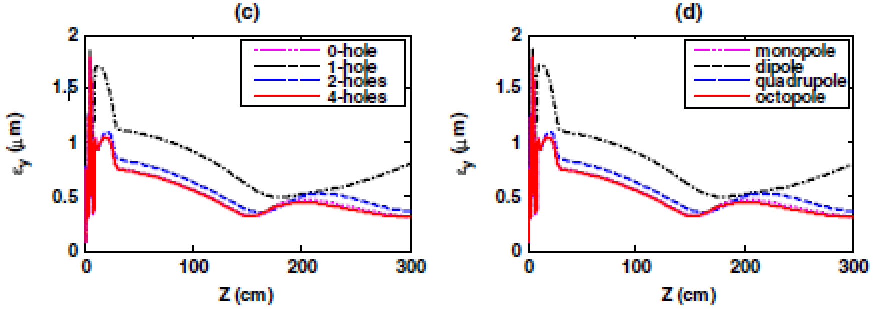
（a）Emittance evolutions in the horizontal (x) direction for each model.

(b) Emittance evolutions from the dominant multipole field from each model in the horizontal (x) direction

Simulation results: 0.1 nC charge with solenoid field

> Emittance evolution in the y direction

- The overall behaviors in the vertical direction are same with that in the horizontal direction
- A comparison of figure shows that the dominant modes are dipole, quadrupole and otopole modes in the one-hole, two-hole and four-hole models, respectively



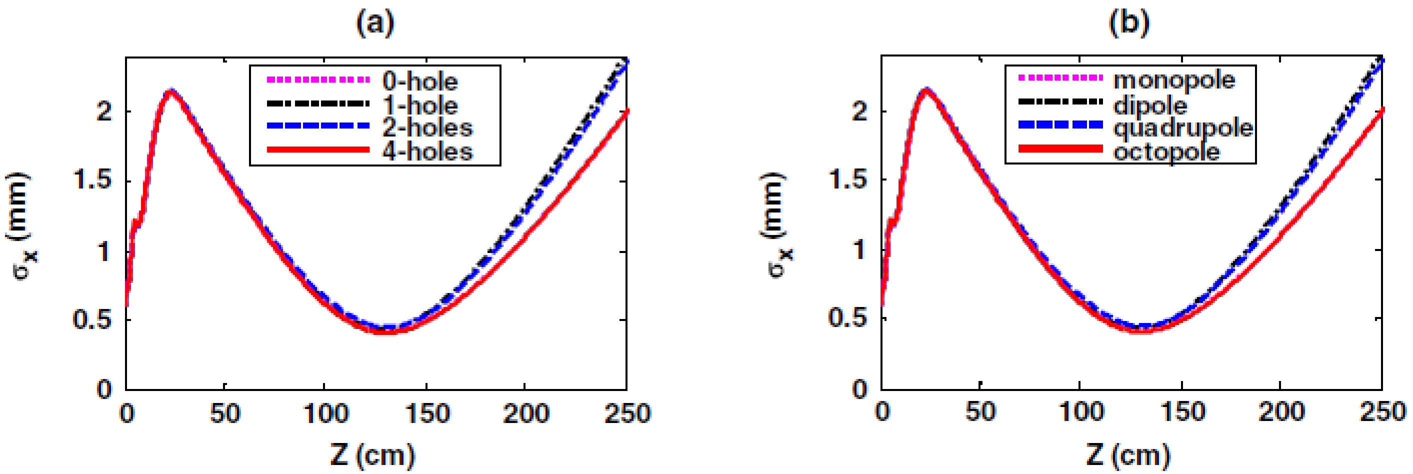
（a）Emittance evolutions in the vertical(y) direction for each model.

(b) Emittance evolutions from the dominant multipole field from each model in the vertical (y) direction

Simulation results: 1 nC charge with solenoid field

> The rms beam size evolution:

> The results of the rms beam size in the x direction showed insignificant diffferences with all kinds of models as the 0.1 nC case.



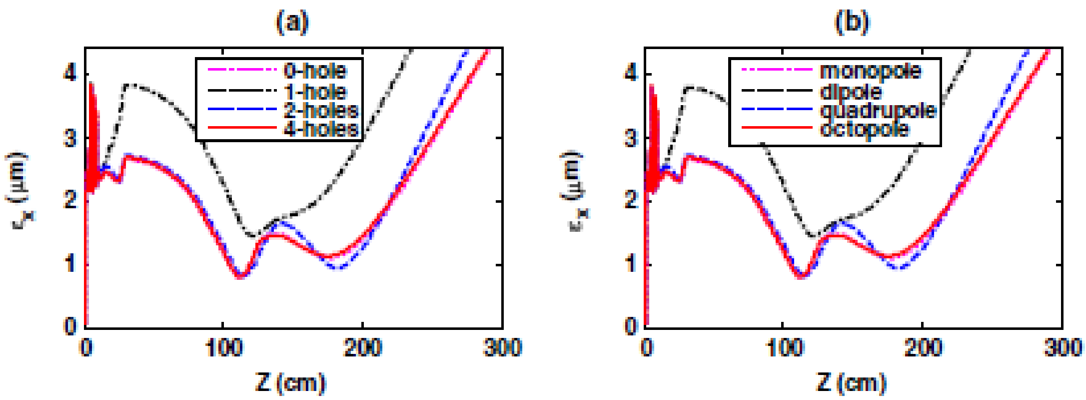
(a) with the four rf gun models and (b) with four artificial field maps.

Simulation results: 1 nC charge with solenoid field

> The emittance evolution:

  > Emittance evolution in the x direction

  - Double minimums in the emittance evolution are clearly seen
  - Emittance values of the one-hole model and the dipole model show the highest values



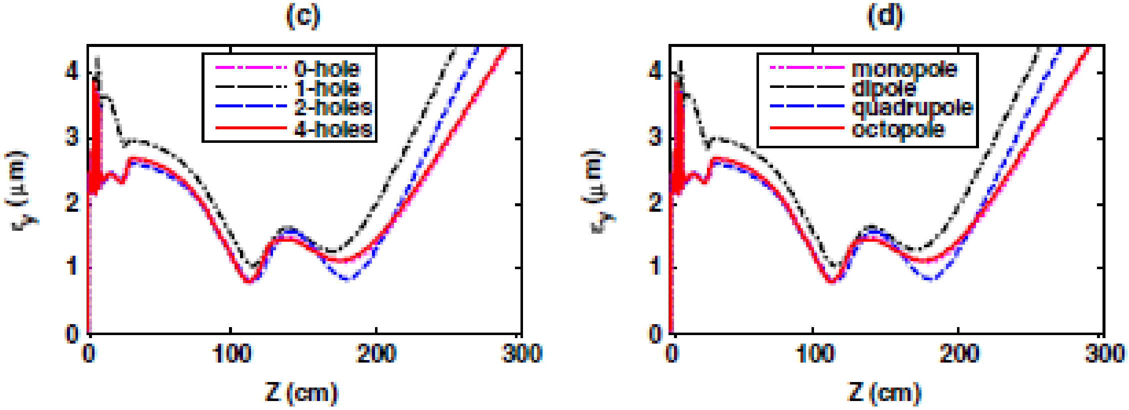
(a) Emittance evolutions in the horizontal (x) direction for each model.

(b) Emittanceevolutions from the dominant multipole field from each model in the horizontal (x) direction.

Simulation results: 1 nC charge with solenoid field

> Emittance evolution in the y direction

- There overall behavior is similar to that in the x direction
- The emittance at the first minimum position in both directions is roughly 0.9μm except the results of one-hole model in the x direction



(c) Emittance evolutions in the vertical (y) direction for each model.

(d) Emittance evolutions from the dominant multipole field from each model in the vertical (y) direction.

> Emittances in the x direction is much higher than the emittance in y direction for the dipole mode with a beam charge 0.1 nC and 1 nC, while the emittances of dipole mode with 0 nC show high values in y direction.(solenoid field)

Conclusion

- The effect of multipole mode induced by asymmetric geometry of rf gun on emittance was studied
  - The emittance in vertical direction was increased in the one-hole model
  - This dipole mode can be reduced by adding one hole in the opposite position(quadrupole brought)
  - Quadrupole mode can be reduced by adding two more holes at 90^o to the previous two holes
- The effects of emittance growth due to field asymmetry almost disappeared when four-hole mode was applied

Thank you for your attention!
Merry Christmas!