Longitudinal phase space tomography at the PITZ facility

1. Overview of the PITZ facility and motivation
2. High momentum measurements at PITZ
3. Idea of the tomography
4. Algebraic reconstruction technique (ART)
5. Simulation of measurements in ASTRA
6. Experimental data, reconstruction results
7. Conclusion

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Humboldt-Universität zu Berlin
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# The Photo Injector Test facility, Zeuthen site (PITZ)

![PITZ Diagram](image-url)

## PITZ main parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bunch charge</td>
<td>1 pC … 4 nC</td>
</tr>
<tr>
<td>Repetition rate</td>
<td>10 Hz</td>
</tr>
<tr>
<td>Beam energy after gun</td>
<td>1 … 7 MeV</td>
</tr>
<tr>
<td>Beam energy after booster</td>
<td>1 … 27 MeV</td>
</tr>
<tr>
<td>Number of bunches</td>
<td>1 … 800</td>
</tr>
<tr>
<td>Bunch spacing</td>
<td>1 us</td>
</tr>
<tr>
<td>Laser pulse temporal shape</td>
<td>2 ps Gauss … 22 ps flat-top</td>
</tr>
</tbody>
</table>

Laser pulse train structure

- Up to 800 pulses
- 1 MHz pulse rep. rate
- 100 ms duration
## Electron bunch characterization

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Originate from</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bunch charge</td>
<td>Electron source</td>
</tr>
<tr>
<td>Bunch energy</td>
<td>Acceleration</td>
</tr>
<tr>
<td>Bunch transverse size</td>
<td>Emittance, transverse phase space</td>
</tr>
<tr>
<td>Bunch length</td>
<td>Energy spread, longitudinal phase space</td>
</tr>
</tbody>
</table>

- **ΔE/E**
- **Tail**
- **Head**

### Trajectories:
- **Lower energy trajectory**
- **Center energy trajectory**
- **Higher energy trajectory**
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HEDDA1 momentum measurements

\[ D_y = \rho (1 - \cos(\theta)) + L_d \sin(\theta) = 2\rho \]

\[ D_y = 0.6 \, m \]

\[ \sigma_y = D_y \frac{\Delta p}{p} \]
HEDA1 momentum resolution, standard measurements

High1.Scr5

Yrms = 0.72 mm

\[ \sigma_\delta = \frac{\sigma_y}{D_y} \]

\[ D_y = \rho (1 - \cos(\theta)) + L \sin(\theta) = 2\rho \]

\[ \rho = 0.3 \text{ m} \rightarrow D_y = 0.6 \text{ m} \]

\[ \sigma_\delta = \frac{0.72 \cdot 10^{-3}}{0.6} = 1.2 \cdot 10^{-3} \]

For 25 MeV/c beam \( \rightarrow \) 30 keV/c
HEDDA2 momentum measurements

\[ D_x = \rho(1 - \cos(\theta)) + L_d \sin(\theta), \quad \theta = 60^\circ \]

\[ \sigma_x = D_x \frac{\Delta p}{p} \]

\[ D_x = 0.9 \, m \]
$\langle p \rangle_{\text{max}} \approx (6.7151 \pm 0.0009) \text{ MeV/c at } -7.0^\circ$

$p_{\text{min}}^{\text{RMS}} \approx (11.65 \pm 0.47) \text{ keV/c at } -5.0^\circ$

$p = 6.7\text{MeV/c} + 18\text{MeV/c} \cdot \cos(\varphi)$

Electron bunch mean momentum after the booster
1. Overview of the PITZ facility
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Tomography

We have unknown object $f(x, y)$. We can measure projection of this object $p_\theta(r)$ at different angle $\theta$. This is called tomography transformation.

Procedure to restore unknown object from set of projections is called inverse tomography transformation.

This procedures can be applied to the longitudinal phase space image.
Longitudinal phase space, 1 nC simulation

After gun (phase 0)

After booster (phase 0)
Simulated longitudinal phase spaces, 1 nC charge

Simulated electron bunch longitudinal phase spaces for different booster RF phases.

\[ p = 6.7 \text{Mev/c} + 18 \text{Mev/c} \cdot \cos(\varphi) \]

\[ \Delta p_z \approx +147 \frac{\text{keV/c}}{\text{ps}} \cdot \sin(\phi_0) \cdot \frac{\Delta z}{c} \]

Electron bunch mean momentum after the booster

Particle momentum gain by the booster relative to the mean momentum. Particle has position \( \Delta z \) within the bunch.
Longitudinal resolution estimation

\[ \delta z = \frac{\delta p}{k} \]

\[ k \approx +147 \frac{keV/c}{ps} \cdot \sin(\varphi_0) \cdot \frac{1}{c} \]
Estimation of longitudinal resolution

\[ \frac{dp}{dt} = +18 \cdot 2\pi f \cdot \sin(\varphi) = +147 \frac{keV/c}{ps} \cdot \sin(\varphi) \]

for 20° phase offset \[ \frac{dp}{dt} = 50 \frac{keV/c}{ps} \] maximal momentum chirp

3 keV/c momentum resolution + 2 keV/c slice momentum spread \( \Rightarrow 0.1 \text{ ps resolution} \)
How treat the data ????
Pros and cons

Pros:

- Simple measurements via momentum phase scan
- Quite high temporal resolution*

* 0.1 ps resolution ???

Cons:

- Sophisticated data treatment
- Not include 90° rotation
1. Overview of the PITZ facility  
2. High momentum measurements at PITZ  
3. Idea of the tomography  
4. **Algebraic reconstruction technique (ART)**  
5. Simulation of measurements in ASTRA  
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Reconstruction algorithm (ART)

1. Represent 2D image as 1D array – $g_l$

2. Represent phase space rotation via $a_{ijl}$ matrix

$$p_z(z) = p_z(z) + k(\varphi) \cdot z$$

$$p_{ij} = a_{ijl} \cdot g_l$$

- $i$ – phase (rotation)
- $j$ – momentum bin
- $l$ – image pixel index
Reconstruction algorithm, filling “$a_{ijl}$” array

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<tbody>
<tr>
<td>$p_{14}$</td>
<td>$g_{19}$</td>
<td></td>
<td>$g_{24}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_{13}$</td>
<td></td>
<td>$g_{13}$</td>
<td></td>
<td>$g_{18}$</td>
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<tr>
<td>$p_{12}$</td>
<td></td>
<td>$g_{7}$</td>
<td></td>
<td>$g_{12}$</td>
<td></td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>$g_{1}$</td>
<td>$g_{2}$</td>
<td>$g_{3}$</td>
<td>$g_{4}$</td>
<td>$g_{5}$</td>
</tr>
</tbody>
</table>

$p_{ij} = a_{ijl} \cdot g_{l}$

$p_{z}(z) = p_{z}(z) + k(\varphi) \cdot z$

If $\varphi_1$ mean no rotation applied then $a_{1,1,4} = 1$, $a_{1,2,4} = 0$

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<tr>
<td>$p_{24}$</td>
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<tr>
<td>$p_{23}$</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$p_{22}$</td>
<td></td>
<td></td>
<td></td>
<td>$g_{4}$</td>
<td></td>
</tr>
<tr>
<td>$p_{21}$</td>
<td></td>
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</tbody>
</table>

If $\varphi_2$ mean rotation applied then $a_{2,1,4} = 0.3$, $a_{2,2,4} = 0.7$
Reconstruction algorithm, filling “$a_{ijl}$” array

\[ p_{ij} = a_{ijl} \cdot g_l \]

\[ p_z(z) = p_z(z) + k(\varphi) \cdot z \]

more precise representation:

\[ a_{2,1,4} = ???, \ a_{2,2,4} = ???, \ a_{2,3,4} = ??? \]
Reconstruction algorithm, iterations

\[ g_q^{(k+1)} = g_q^{(k)} + \sum_{ij} a_{ijq} \left[ \frac{p_{ij} - \sum_l a_{ijl} \cdot g_l^{(k)}}{\sum_{nm} a^2_{inm}} \right] \]

- \( i \) – phase (Nphase)
- \( j \) – momentum (Npz)
- \( l \) – z coordinate (Nz)
- \( q, l \) – image index (NI = Npz*Nz)
- \( k \) – iteration number

\[ Npz*Nz*Nphase*Npz*(Npz*Nz + Npz*Npz*Nz) = \]

Total calculation time is \( \sim \)

\[ Npz^3Nz^2Nphase(1 + Npz) = \]

\[ Npz^4Nz^2Nphase \]
Convergence criteria

\[ C(k) = \sqrt{\sum_q \left( \frac{g_q^{(k)} - g_q^{(k-1)}}{q_{\text{max}}} \right)^2 / \max(g^{(k)})}, \]

where \( q_{\text{max}} \) and \( \max(g^{(k)}) \) are number of elements and maximal element in the reconstructed image \( g^{(k)} \) respectively.

When \( C(k) \) becomes less than \( 10^{-3} \) we can stop iterations.
Simple reconstruction example, initial image
Example, 10 iteration
Convergence

\[ C(k) = \sqrt{\frac{\sum_q \left( g_q^{(k)} - g_q^{(k-1)} \right)^2}{q_{max}} \max(g^{(k)})} \]
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ASTRA initial parameters

Charge: 1 nC
Laser: 17.5 ps
BSA: 0.4 mm
Main: 377 A
Gun: 6.68 MeV/c
Booster: 22.4 MeV/c
Beam transport and phase spaces along beamline

- **Charge**: 1 nC
- **Laser**: 17.5 ps
- **BSA**: 0.4 mm
- **Main**: 377 A
- **Gun**: 6.68 MeV/c
- **Booster**: 22.4 MeV/c
Momentum phase scan, gun

Beam mean momentum, $max = 6.680 \text{ keV/c at phase} = -1.5$

- Momentum: [6.672, 6.682] MeV/c
- Booster phase: [-5, 1]

Beam RMS momentum spread, $min = 20.9 \text{ keV/c at phase} = -5$

- RMS momentum spread: [20, 50] keV/c
- Booster phase: [-5, 1]

Parameters:
- Charge: 1 nC
- Laser: 17.5 ps
- BSA: 0.4 mm
- Main: 377 A
- Gun: 6.68 MeV/c
- Booster: 22.4 MeV/c
Momentum phase scan, booster

Beam mean momentum, max = 22.381 keV/c at phase = -2

\[ p = 6.68 + (22.38 - 6.68) \cdot \cos (\varphi + 2) \]

The green line is not fit! It is model.

Beam RMS momentum spread, min = 38.4 keV/c at phase = -5

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</tr>
<tr>
<td>Laser</td>
<td>17.5 ps</td>
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<td>BSA</td>
<td>0.4 mm</td>
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</tr>
<tr>
<td>Booster</td>
<td>22.4 MeV/c</td>
</tr>
</tbody>
</table>
Initial data for reconstruction

- Charge: 1 nC
- Laser: 17.5 ps
- BSA: 0.4 mm
- Main: 377 A
- Gun: 6.68 MeV/c
- Booster: 22.4 MeV/c
ART reconstruction (ASTRA data)

Reconstructed longitudinal phase space
100 iterations

Charge: 1 nC
Laser: 17.5 ps
BSA: 0.4 mm
Main: 377 A
Gun: 6.68 MeV/c
Booster: 22.4 MeV/c

ASTRA longitudinal phase space

8.9 m
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Experimental data, machine parameters, setup I

**Laser temporal profile**

FT: FWHM=17.39ps; r1=1.93ps; r2=2.12ps; mod1=2.6%; modP2P=8.2%

\[
\sigma = 0.57 \cdot 10^{-3}
\]

For 22.2 MeV/c beam \(\rightarrow\) 21 keV/c

**Charge**

<table>
<thead>
<tr>
<th>Charge</th>
<th>1 nC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser</td>
<td>17.4 ps</td>
</tr>
<tr>
<td>BSA</td>
<td>0.4 mm</td>
</tr>
<tr>
<td>Main</td>
<td>377 A</td>
</tr>
<tr>
<td>Gun</td>
<td>6.68 MeV/c</td>
</tr>
<tr>
<td>Booster</td>
<td>22.4 MeV/c</td>
</tr>
</tbody>
</table>

**HEDA1 reference screen**

Momentum resolution:

\[
\sigma_\delta = \frac{0.57 \cdot 10^{-3}}{0.6} = 0.95 \cdot 10^{-3}
\]
Data from 14.02.2013 19:27:18, HEDA1 momentum scan

Beam mean momentum, max = 22.242 MeV/c at phase = 35

\[ p = 6.68 + (22.24 - 6.68) \cdot \cos(\varphi - 35) \]

Beam RMS momentum spread, min = 49.4 keV/c at phase = 37
Initial data for reconstruction, HEDA1

Experimental data

ASTRA data

$P_z$, [MeV/c]

Booster phase

$P_z$, [MeV/c]
ART reconstruction, HEDA1

Reconstructed longitudinal phase space

ASTRA longitudinal phase space

8.9 m
HEDA1, 1 nC bunch charge, 80% of charge

Longitudinal phase space, 80% of total charge

Momentum distribution, keV/c

Current distribution

Slice momentum spread, 20 slices
Data from 14.02.2013 20:50:46, HEDA2 momentum scan

Beam mean momentum, max = 22.407 keV/c at phase = 32

\[ p = 6.69 + (22.41 - 6.69) \cdot \cos (\varphi - 33) \]

Beam RMS momentum spread, min = 24.0 keV/c at phase = 38
Initial data for reconstruction, HEDDA2

Experimental data

ASTRA data

\[ p_z, \text{[MeV/c]} \]
ART reconstruction, HEDA2

Reconstructed longitudinal phase space
100 iterations

ASTRA longitudinal phase space

18.6 m
HEDA2, 1 nC bunch charge, 80% of charge

Longitudinal phase space, 80% of total charge

Momentum distribution, keV/c

Current distribution

Slice momentum spread, 20 slices
Experimental data, machine parameters, setup II and III

100 pC

HEDA1 reference screen

Momentum resolution:

\[ \sigma_\delta = \frac{0.078 \cdot 10^{-3}}{0.6} = 0.13 \cdot 10^{-3} \]

For 22.2 MeV/c beam
\[ \rightarrow 3 \text{ keV/c} \]

20 pC

HEDA1 reference screen

Momentum resolution:

\[ \sigma_\delta = \frac{0.136 \cdot 10^{-3}}{0.6} = 0.23 \cdot 10^{-3} \]

For 22.2 MeV/c beam
\[ \rightarrow 5 \text{ keV/c} \]
Reconstructed phase spaces 100 and 20 pC

HEDA1 100 pC

HEDA1 20 pC

HEDA2 100 pC

HEDA2 20 pC
HEDDA1 20 pC bunch charge

Longitudinal phase space, 80% of total charge

Momentum distribution, keV/c

Current distribution

Slice momentum spread, 20 slices
HEDA2 20 pC bunch charge

Longitudinal phase space, 80% of total charge

Momentum distribution, keV/c

Current distribution

Slice momentum spread, 20 slices
Longitudinal profiles at 8.92 for 20 pC bunch charge

- **Magenta line** – laser profile (17.5 ps FWHM)
- **Red line** – simulated bunch profile for 20 pC
- **Blue line** – measured bunch profile for 20 pC
Longitudinal profiles at 8.92 for 1 nC bunch charge

- **Magenta line**: laser profile (17.5 ps FWHM)
- **Red line**: simulated bunch profile for 1 nC
- **Blue line**: measured bunch profile for 1 nC
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Conclusion

- Simulation of the measurements in ASTRA gives results very close to the expected ones. This prove the idea of the longitudinal phase space measurements with the described tomography technique.

- The measured electron bunch longitudinal profiles show the similar shapes to the cathode laser temporal profiles. This demonstrates that the photo cathode has the short response time, less than ps.

- The measured electron bunch length for the 20 pC charge is shorter than the cathode laser pulse, what also can be seen in the ASTRA simulation.

- The measured electron bunch length for the 1 nC charge as well as simulated one is longer than the cathode laser pulse.